The Project Physics Course

Text and Handbook **2**

Motion in the Heavens

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Text and Handbook

UNIT 2 Motion in the Heavens

A Component of the
Project Physics Course

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This Text and Handbook, Unit 2 is one of the many instructional materials developed for the Project Physics Course. These materials include Texts, Handbooks, Teacher Resource Books, Readers, Programmed Instruction booklets, Film Loops, Transparencies, 16mm films, and laboratory equipment.

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(continued on p. 124)

Science is an adventure of the whole human race to learn to live in and perhaps to love the universe in which they are. To be a part of it is to understand, to understand oneself, to begin to feel that there is a capacity within man far beyond what he felt he had, of an infinite extension of human possibilities . . . .

I propose that science be taught at whatever level, from the lowest to the highest, in the humanistic way. It should be taught with a certain historical understanding, with a certain philosophical understanding, with a social understanding and a human understanding in the sense of the biography, the nature of the people who made this construction, the triumphs, the trials, the tribulations.

I. I. RABI
Nobel Laureate in Physics

Preface

Background The Project Physics Course is based on the ideas and research of a national curriculum development project that worked in three phases. First, the authors—a high school physics teacher, a university physicist, and a professor of science education—collaborated to lay out the main goals and topics of a new introductory physics course. They worked together from 1962 to 1964 with financial support from the Carnegie Corporation of New York, and the first version of the text was tried out in two schools with encouraging results.

These preliminary results led to the second phase of the Project when a series of major grants were obtained from the U.S. Office of Education and the National Science Foundation, starting in 1964. Invaluable additional financial support was also provided by the Ford Foundation, the Alfred P. Sloan Foundation, the Carnegie Corporation, and Harvard University. A large number of collaborators were brought together from all parts of the nation, and the group worked together for over four years under the title *Harvard Project Physics.* At the Project's center, located at Harvard University, Cambridge, Massachusetts, the staff and consultants included college and high school physics teachers, astronomers, chemists, historians and philosophers of science, science educators, psychologists, evaluation specialists, engineers, film makers, artists and graphic designers. The teachers serving as field consultants and the students in the trial classes were also of vital importance to the success of Harvard Project Physics. As each successive experimental version of the course was developed, it was tried out in schools throughout the United States and Canada. The teachers and students in those schools reported their criticisms and suggestions to the staff in Cambridge, and these reports became the basis for the subsequent revisions of the course materials. In the Preface to Unit 1 *Text* you will find a list of the major aims of the course.

We wish it were possible to list in detail the contributions of each person who participated in some part of Harvard Project Physics. Unhappily it is not feasible, since most staff members worked on a variety of materials and had multiple responsibilities. Furthermore, every text chapter, experiment, piece of apparatus, film or other item in the experimental program benefitted from the contributions of a great many people. On the preceding pages is a partial list of contributors to Harvard Project Physics. There were, in fact, many other contributors too numerous to mention. These include school administrators in participating schools, directors and staff members of training institutes for teachers, teachers who tried the course after the evaluation year, and most of all the thousands of students who not only agreed to take the experimental version of the course, but who were also willing to appraise it critically and contribute their opinions and suggestions.

The Project Physics Course Today. Using the last of the experimental versions of the course developed by Harvard Project Physics in 1964–68 as a starting point, and taking into account the evaluation results from the tryouts, the three original collaborators set out to develop the version suitable for large-scale publication. We take particular pleasure in acknowledging the assistance of Dr. Andrew Ahlgren of Harvard University. Dr. Ahlgren was invaluable because of his skill as a physics teacher, his editorial talent, his versatility and energy, and above all, his commitment to the goals of Harvard Project Physics.

We would also especially like to thank Miss Joan Laws whose administrative skills, dependability, and thoughtfulness contributed so much to our work. The publisher, Holt, Rinehart and Winston, Inc. of New York, provided the coordination, editorial support, and general backing necessary to the large undertaking of preparing the final version of all components of the Project Physics Course, including texts, laboratory apparatus, films, etc. Damon, a company located in Needham, Massachusetts, worked closely with us to improve the engineering design of the laboratory apparatus and to see that it was properly integrated into the program.

In the years ahead, the learning materials of the Project Physics Course will be revised as often as is necessary to remove remaining ambiguities, clarify instructions, and to continue to make the materials more interesting and relevant to students. We therefore urge all students and teachers who use this course to send to us (in care of Holt, Rinehart and Winston, Inc., 383 Madison Avenue, New York, New York 10017) any criticism or suggestions they may have.

F. James Rutherford
Gerald Holton
Fletcher G. Watson

Contents TEXT SECTION

The Aztec calendar, carved over 100 years before our calendar was adopted, divides the year into eighteen months of twenty days each.

UNIT **2**

Motion in the Heavens

CHAPTER

PROLOGUE Astronomy, the oldest science, deals with objects now known to be the most distant from us. Yet, to early observers, the sun, moon, planets, and stars did not seem to be so far away. Nor were they considered of little importance. On the contrary, even as today, the majestic display of celestial events powerfully stimulated the imagination of curious men. The great variety of objects visible in the sky, the regularity of their motions, the strangely slow changes in their position and brightness—this whole mysterious pattern of motions required some reason, some cause, some explanation.

The discovery of the causes and the meanings is the subject of this unit. It starts with prehistoric attempts to deal with the observations by incorporating them, in disguised form, into myths and tales, some of the best in world literature. It ends with the Scientific Revolution in the seventeenth century, which gave us the explanations that hold to this day. These explanations also provided a whole new set of methods for solving problems in a scientific manner.

Even in modern times outdoorsmen use the sun by day and the stars by night as a clock. Directions are indicated by the sun at rising and setting time, and true south can be determined from the position of the sun at noon. The Pole Star gives a bearing on true north after dark. The sun's position can also be used as a crude calendar. Its noontime altitude varies with the seasons.

Astronomical events affected not only the imagination of the ancients; they had a practical effect on their everyday life. The working day began when the sun rose and ended when the sun set. Before electric lighting, human activity was dominated by the presence or absence of daylight and by the sun's warmth, which changed season by season.

Of all the time units used in common practice, "one day" is probably the most basic and surely the most ancient. For counting longer intervals, a "moon" or month was an obvious unit. Over the centuries, clocks have been devised to subdivide the days into smaller units, and calendars have been devised to record the passage of days into years.

When the early nomadic tribes settled down to live in villages some 10,000 years ago, and became dependent upon agriculture for their food, they needed a calendar for planning their plowing and sowing.

Throughout recorded history, most of the world's population has been involved in agriculture and so has depended on a calendar. If seeds were planted too early, they might rot in the ground, or the young shoots might be killed by a frost. If they were planted too late, the crops would not ripen before winter came. Therefore, a knowledge of the times for planting and harvesting was important for survival. Because religious festivals were often related to the seasons, the making and improving of the calendar by observation of the sun, planets, and stars was often the task of priests. The first astronomers were, therefore, usually priests.

Practical needs and imagination acted together to give astronomy an early importance. Many of the great buildings of ancient times were constructed with careful astronomical orientation. The great pyramids of Egypt, tombs of the Pharaohs, have sides that run due north-south and east-west. The impressive, almost awesome circles of giant stones at Stonehenge in England appear to have been arranged about 2000 B.C. to permit accurate astronomical observations of the positions of the sun and moon. The Mayans and the Incas in America, as well as the Chinese, put enormous effort into buildings from which they could measure the changes in the position of the sun, moon, and planets. At least as early as 1000 B.C. the Babylonians and Egyptians had developed considerable ability in timekeeping. Their recorded observations are still being unearthed.

Thus, for thousands of years, the motions of the heavenly bodies

Stonehenge, England, apparently a prehistoric observatory.

were carefully observed and recorded. In all sciences, no other field has had such a long accumulation of data as astronomy has had.

But our debt is greatest to the Greeks, who began trying to deal in a new way with what they saw. The Greeks recognized the contrast between the apparently haphazard and short-lived motions of objects on the earth and the unending cycles of motions of the objects in the heavens. About 600 B.C. they began to ask a new question: How can we explain these cyclic events in the sky in a simple way? What order and sense can we make of the heavenly happenings? The Greeks' answers, which are discussed in Chapter 5, had an important effect on science. For example, as we shall see, the writings of Aristotle (about 330 B.C.) became widely studied and accepted in western Europe after 1200 A.D. and were important factors in the scientific revolution that followed.

The positions of Jupiter from 132 B.C. to 60 B.C. are recorded on this section of Babylonian clay tablet, now in the British Museum.

After the conquests of Alexander the Great, the center of Greek thought and science shifted to Egypt at the new city of Alexandria, founded in 332 B.C. There a great museum, similar to a modern research institute, was created and flourished for many centuries. But as the Greek civilization gradually declined, the practical-minded Romans captured Egypt, and interest in science died out. In 640 A.D. Alexandria was captured by the Muslims as they swept along the southern shore of the Mediterranean Sea and moved northward through Spain to the Pyrenees. Along the way they seized and preserved many libraries of Greek documents, some of which were later translated into Arabic and carefully studied. During the following centuries the Muslim scientists made new and better observations of the heavens, although they did not make major changes in the explanations or theories of the Greeks.

In western Europe during this period the works of the Greeks were largely forgotten. Eventually they were rediscovered by Europeans through Arabic translations found in Spain after the Muslims were forced out. By 1130 A.D. complete manuscripts of at least one of Aristotle's books were known in Italy and France. After the founding of the University of Paris around 1200, many other writings of Aristotle were acquired and studied both there and at the new English universities, Oxford and Cambridge.

During the next century, the Dominican monk, Thomas Aquinas, blended major elements of Greek thought and Christian theology into a single philosophy. His work was widely studied and accepted in western Europe for several centuries. In achieving this commanding and largely successful synthesis, Aquinas accepted the physics and astronomy of Aristotle. Because the science was blended with theology, any questioning of the science seemed also to be a questioning of the theology. Thus for a time there was little effective criticism of the Aristotelian science.

In the twelfth century, the Muslim scholar Ibn Rashd had attempted a similar union of Aristotelianism and Islam.

The Renaissance movement, which spread out across Europe from Italy, brought new art and music. It also brought new ideas about the universe and man's place in it. Curiosity and a questioning attitude became acceptable, even prized. Men acquired a new confidence in their ability to learn about the world. Among those whose work introduced the new age were Columbus and Vasco da Gama, Gutenberg and da Vinci, Michelangelo and Raphael, Erasmus and Vesalius, Luther, Calvin, and Henry VIII. (The chart in Chapter 6 shows their life spans.) Within this emerging Renaissance culture lived Niklas Koppernigk, later called Copernicus, whose reexamination of astronomical theories is discussed in Chapter 6.

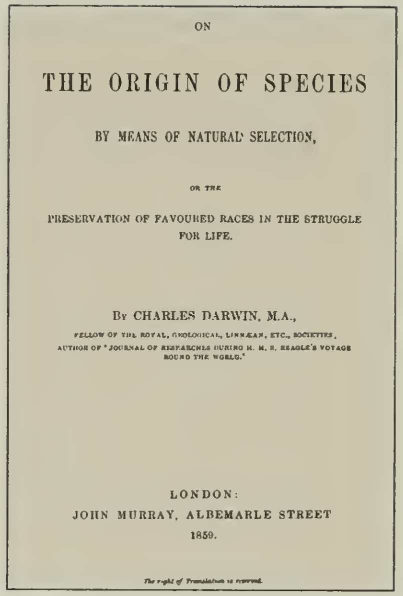
Further improvements in astronomical theory were made in the seventeenth century by Kepler, mainly through mathematical reasoning, and by Galileo through his observations and writings; these are discussed in Chapter 7. In Chapter 8 you shall see that Newton's work, in the second half of the seventeenth century, extended the ideas about the motions of objects on earth to explain the motions observed in the heavens—a magnificent synthesis of terrestrial and celestial dynamics. The results obtained by these men, and by others like them in other sciences such as anatomy and physiology, and the ways in which they went about their work, were so far-reaching that the resulting changes are generally referred to as the *Scientific Revolution.*

Great scientific advances can, and often do, affect ideas outside science. For example, Newton's impressive work helped to bring a new feeling of self-confidence. Man seemed capable of understanding all things in the heavens and on the earth. This great change in attitude

Louis XIV visiting the French Academy of Sciences, which he founded in the middle of the seventeenth century. Seen through the right-hand window is the Paris Observatory, then under construction.

was a major characteristic of the eighteenth century, which has been called the Age of Reason. To a degree, what we think today and how we run our affairs are still based on the effect of scientific discoveries made centuries ago.

Decisive changes in thought developed at the start of the Renaissance and grew during a period of about a century, from the work of Copernicus to that of Newton. In a sense, this era of invention can be compared to the sweeping changes which occurred during the past hundred years. This recent period might extend from the publication in 1859 of Darwin's *Origin of Species* to the first large-scale release of atomic energy in 1945. Within this interval lived such scientists as Mendel and Pasteur, Planck and Einstein, Rutherford and Fermi. The ideas they and others introduced into science during the last century have become increasingly important. These scientific ideas are just as much a part of our time as the ideas and works of people such as Roosevelt, Ghandi, and Pope John XXIII, Marx and Lenin, Freud and Dewey, Picasso and Stravinsky, Shaw and Joyce. If we understand the way in which science influenced the men of past centuries, we shall be better prepared to understand how science influences our thought and lives today. This is clearly one of the essential aims of this course.


ON

THE ORIGIN OF SPECIES

BY MEANS OF NATURAL SELECTION,

OR THE

PRESERVATION OF FAVOURED RACES IN THE STRUGGLE FOR LIFE.

BY CHARLES DARWIN, M.A.,

FELLOW OF THE ROYAL, GEOLOGICAL, LINNÆAN, ETC., SOCIETIES; AUTHOR OF 'JOURNAL OF RESEARCHES DURING H. M. S. BEAGLE'S VOYAGE ROUND THE WORLD.'

LONDON:
JOHN MURRAY, ALBEMARLE STREET
1859.

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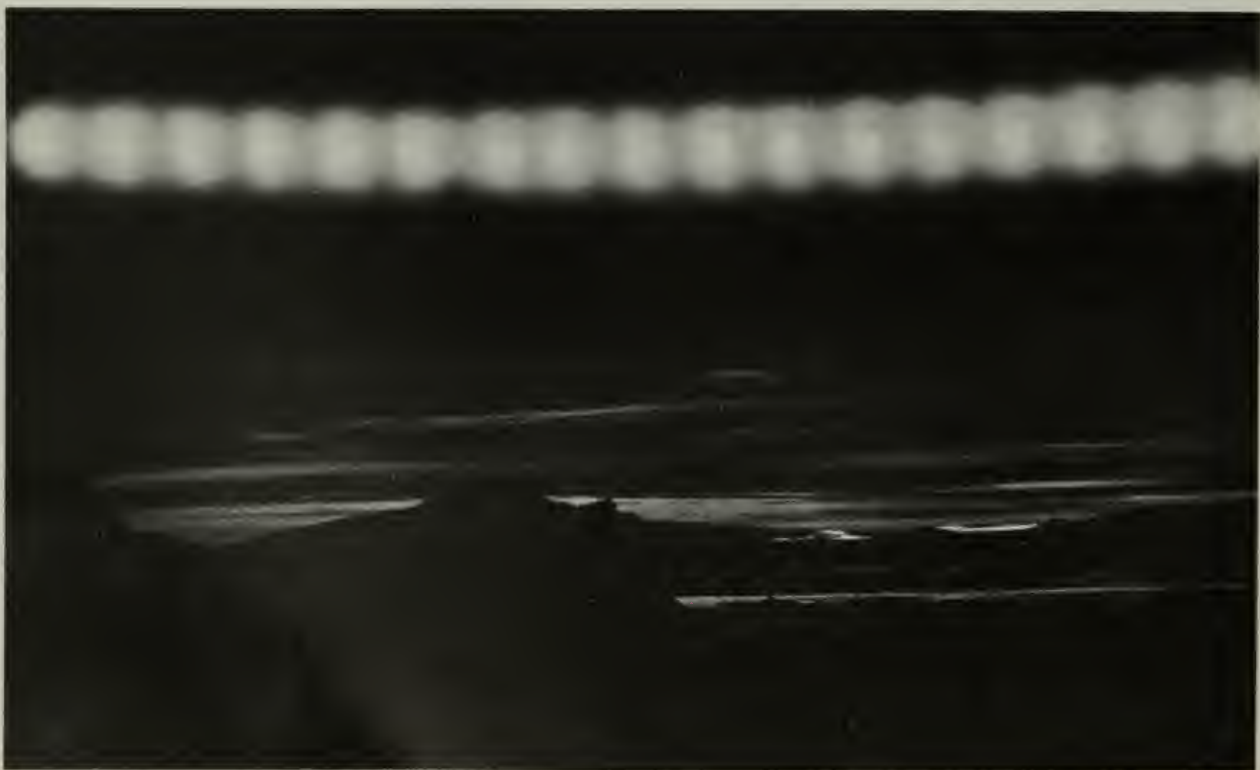
In sum, the material treated in this unit, although historical as well as scientific, is still of the first importance today for anyone interested in an understanding of science. The reasons for presenting the science in its historical context include the following:

> The results that were finally obtained are still valid and rank among the oldest ideas used every day in scientific work. The characteristics of all scientific work are clearly visible: the role of assumptions, of experiment and observations, of mathematical theory; the social mechanisms for collaborating, teaching, and disputing; and the possibility of having one's scientific findings become part of the established lore of the time.
>
> There is an interesting conflict between the rival theories used to explain the same set of astronomical observations. It illustrates what all such disputes have in common down to our day, and helps us to see clearly what standards may be used to judge one theory against another.
>
> This subject matter includes the main reasons for the rise of science as we understand it now. The story of the revolution in science in the seventeenth century and its many effects outside science itself is as crucial to the understanding of this current age of science as is the story of the American Revolution to an understanding of America today.

Midnight sun photographed at five minute intervals over the Ross Sea in Antarctica.



CHAPTER FIVE

Where is the Earth? – The Greeks' Answers

5.1 Motions of the sun and stars

The facts of everyday astronomy, the heavenly happenings themselves, are the same now as in the times of the Greeks. You can observe with your unaided eyes most of what these early scientists saw and recorded. You can discover some of the long-known cycles and rhythms, such as the seasonal changes of the sun's height at noon, the monthly phases of the moon, and the glorious spectacle of the slowly revolving night sky. If our purpose were only to make accurate forecasts of eclipses, planetary positions, and the seasons, we could, like the Babylonians and Egyptians, focus our attention on recording the details of the cycles and rhythms. If, however, like the Greeks, we wish to *explain* these cycles, we must also use these data to construct some sort of simple model or theory with which we can predict the observed variations. But before we explore the several theories proposed in the past, let us review the *major observations* which the theories had to explain: the motions of the sun, moon, planets, and stars.

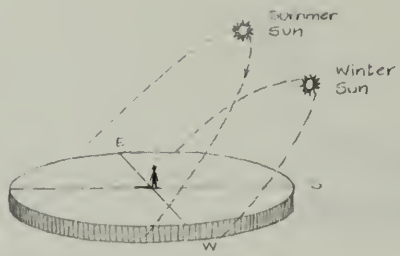
SG 5.1

The motions of these bodies, essentially the same as they were thousands of years ago, are not difficult to observe – you should make a point of doing so. *Handbook 1* has many suggestions for observing the sky, both with the naked eye and with a small telescope.

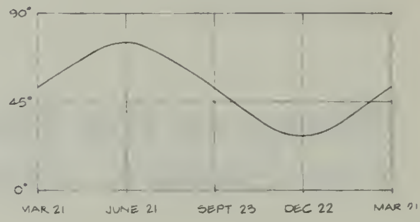
The most basic celestial cycle as seen from our earth is, of course, that of day and night. Each day the sun rises above the local horizon on the eastern side of the sky and sets on the western side. The sun follows an arc across the sky, as is sketched in diagram (a) on the top of the next page. At noon, halfway between sunrise and sunset, the sun is highest above our horizon. Every day, a similar motion can be seen from sunrise to sunset. Indeed all the objects in the sky show this pattern of daily motion. They all rise in the east, reach a highest point, and drop lower in the west (although some stars never actually sink below the horizon).

This description is for observers in the Northern Hemisphere. For observers south of the equator, exchange "north" and "south."

As the seasons change, so do the details of the sun's path across the sky. In our Northern Hemisphere during winter, the sun rises and sets more to the south, its altitude at noon is lower, and hence, its run across the sky lasts for a shorter period of time. In summer the sun rises and sets more toward the north, its height at noon is greater, and its track across the sky lasts a longer time. The whole cycle takes a little less than $365\frac{1}{4}$ days.



(a) Path of the sun through the sky for one day of summer and one day of winter.



(b) Noon altitude of the sun as seen from St. Louis, Missouri, throughout the year.

This year-long cycle north and south is the basis for the seasonal or "solar" year. Apparently the ancient Egyptians thought that the year had 360 days, but they later added five feast days to have a year of 365 days that fitted better with their observations of the seasons. Now we know that the solar year is 365.24220 days long. The decimal fraction of the day, 0.24220, raises a problem for the calendar maker, who works with whole days. If you used a calendar of just 365 days, after four years New Year's Day would come early by one day. In a century you would be in error by almost a month. In a few centuries the date called January 1 would come in the summertime! In ancient times extra days or even whole months were inserted from time to time to keep a calendar of 365 days and the seasons in fair agreement.

SG 5.2

SG 5.3

Such a makeshift calendar is, however, hardly satisfactory. In 45 B.C. Julius Caesar decreed a new 365-day calendar (the Julian calendar) with one extra whole day (a "leap day") being inserted each fourth year. Over many years, the average would therefore be $365\frac{1}{4}$ days per year. This calendar was used for centuries, during which the small difference between $\frac{1}{4}$ and 0.24220 accumulated to a number of days. Finally, in 1582 A.D., under Pope Gregory, a new calendar (the Gregorian calendar) was announced. This had only 97 leap days in 400 years, and the new approximation was close enough that it has lasted satisfactorily to this day without revision.

To reduce the number of leap days from 100 to 97 in 400 years, century years not divisible by 400 were omitted as leap years. Thus the year 1900 was not a leap year, but the year 2000 will be a leap year.

You have noticed that a few stars are bright and many are faint. The brighter stars may seem to be larger, but if you look at them through binoculars, they still appear as points of light. Some bright stars show colors, but most appear whitish. People have grouped many of the brighter stars into patterns, called constellations. Examples include the familiar Big Dipper and Orion.

SG 5.4

You may have noticed a particular pattern of stars overhead, and several hours later, noticed it low in the west. What was happening? More detailed observation, say by taking a time-exposure photograph, would show that the entire bowl of stars had moved from east to west—new stars rising in the east and others setting in the west. During the night, as seen from a point on the Northern

A combination trail and star photograph of the constellation Orion. The camera shutter was opened for several hours while the stars moved across the sky (leaving trails on the photographic plate). Then the camera was closed for a few minutes and reopened while the camera was driven to follow the stars.

Time exposure showing star trails around the north celestial pole. The diagonal line was caused by the rapid passage of an artificial earth satelite.
You can use a protractor to determine the duration of the exposure; the stars move about 15° per hour.

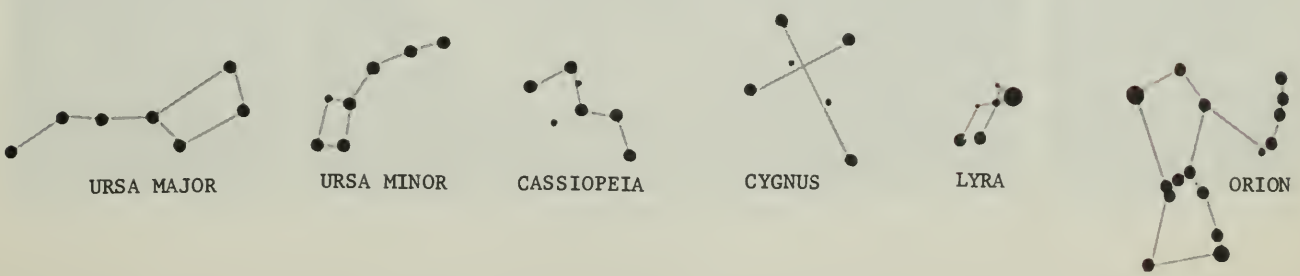
Hemisphere of our earth, the stars appear to move counter-clockwise around a point in the sky called the north celestial pole. This stationary point is near the fairly bright star Polaris, as can be seen in the photograph at the top right of the page.

Some of the star patterns, such as Orion (the Hunter) and Cygnus (the Swan – also called the Northern Cross), were described and named thousands of years ago. Since the star patterns described by the ancients still fit, we can conclude that star positions change very little, if at all, over the centuries. This constancy of relative positions has led to the term "fixed stars."

SG 5.5

See "The Garden of Epicurus" in *Reader 2*.

Thus, we observe in the heavens both stability over the centuries and smooth, orderly, motions. But, although the daily rising and setting cycles of the sun and stars are similar, they are



not identical. Unlike the sun's path, the paths of the stars do not vary in altitude from season to season. Also, stars do not have quite the same rhythm of rising and setting as the sun, but go a little faster. Some constellations seen high in the sky soon after sunset will, at the same time several weeks later, appear to be catching up with the sun. As measured by sun-time, the stars set about four minutes earlier each day.

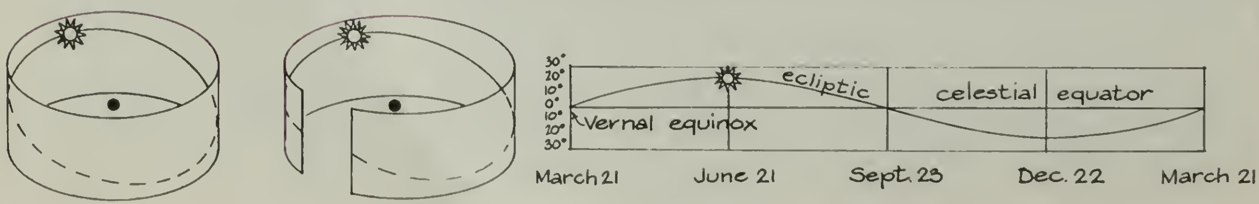
A very easy but precise way to time the motions of the stars is explained in *Handbook 2.*

Thus far, we have described the positions and motions of the sun and stars in relation to the observer's horizon. But, because different observers have different horizons, the horizon is not an unchanging frame of reference from which all observers will see the same positions and motions in the sky. However, a frame of reference which *is* the same for all observers is provided by the fixed stars. The relative positions of these stars do not change as the observer moves over the earth. Also, their daily motions are simple circles with virtually no changes during a year or through the years. For this reason, positions in the heavens are usually described in terms of a frame of reference defined by the stars.

The differences between the two frames of reference—the horizon and the fixed stars—are the basis for establishing a position on the earth, as in navigation.

A description of the sun's motion, using the fixed stars as a reference, must include the daily crossing of the sky, the daily difference in rising and setting times, and the seasonal change in noon altitude. We have already seen that, as measured by sun-time, the stars set about four minutes earlier each day. We can just as well say that, measured by star-time, the sun sets about four minutes *later* each day. That is, the sun appears to be gradually *slipping behind* the daily east-to-west motion of the stars.

The difference in noon altitude of the sun during the year corresponds to a drift of the sun's path north and south on the background of stars. In the first diagram below, the appearance of the middle portion of the sky is represented by a band around the earth. The sun's yearly path against this background of stars is represented by the dark line. If we cut and flatten out this band, as shown in the second and third diagrams, we get a chart of the sun's path during the year. (The 0° line is the *celestial equator*, the imaginary line in the sky that is directly above the earth's equator.) The sun's path against the background of the stars is called the *ecliptic*—its drift north and south of the celestial equator is about $23\frac{1}{2}°$. We also need to define one point on the ecliptic so we can locate the sun, or other objects along it. For centuries this point has been the place where the sun moving eastward on the ecliptic, crosses the equator from south to north—about March 21. This point is called the "vernal (spring) equinox." It is the zero point from which positions among the stars are usually measured.



Thus, there are three apparent motions of the sun: (1) its daily westward motion across the sky, (2) its yearly drift eastward among the stars, (3) its yearly cycle of north-south drift in noon altitude.

These phenomena are all the more intriguing because they repeat unfailingly and precisely. We must try to explain these phenomena by devising a simple model to represent them.

Q1 If you told time by the stars, would the sun set earlier or later each day?

Q2 For what practical purposes were calendars needed?

Q3 What are the observed motions of the sun during one year?

These end-of-section questions are intended to help you check your understanding before going on to the next section.

5.2 Motions of the moon

The moon shares the general east-to-west daily motion of the sun and stars. But the moon slips eastward against the background of the stars faster than the sun does. Each night the moon rises nearly an hour later. When the moon rises in the east at sunset (opposite the sun in the sky) it is bright and shows a full disk (full moon). Each day thereafter, it rises later and appears less round, waning finally to a thin crescent low in the dawn sky. After about fourteen days, when the moon is passing near the sun in the sky and rises with it, we cannot see the moon at all (new moon). After the new moon, we first see the moon as a thin crescent low in the western sky at sunset. As the moon rapidly moves further eastward from the sun, the moon's crescent fattens to a half disk and then within another week goes on to full moon again. After each full moon the cycle repeats.

A "half moon" occurs one-quarter of the way through the monthly cycle, and is therefore called "first quarter" by astronomers. The full moon occurs half way through the cycle, and another "half moon" occurs at "third quarter."

26 days after new moon.

17 days after new moon.

3 days after new moon.

As early as 380 B.C., the Greek philosopher, Plato recognized that the phases of the moon could be explained by thinking of the moon as a globe reflecting sunlight and moving around the earth in about 29 days. Because the moon appears so big and moves so rapidly compared to the stars, people in early times assumed the moon to be quite close to the earth.

The moon's path around the sky is close to the yearly path of the sun; that is, the moon is always near the ecliptic. But the moon's path is tipped a bit with respect to the sun's path; if it were not, the moon would come exactly in front of the sun at every new moon (causing an eclipse of the sun) and be exactly opposite the sun at every full moon, and move into the earth's shadow (causing an eclipse of the moon).

The motions of the moon have been studied with great care for
SG 5.6 centuries, partly because of interest in predicting eclipses, and have been found to be very complicated. The precise prediction of the moon's position is an exacting test for any theory of motion in the heavens.

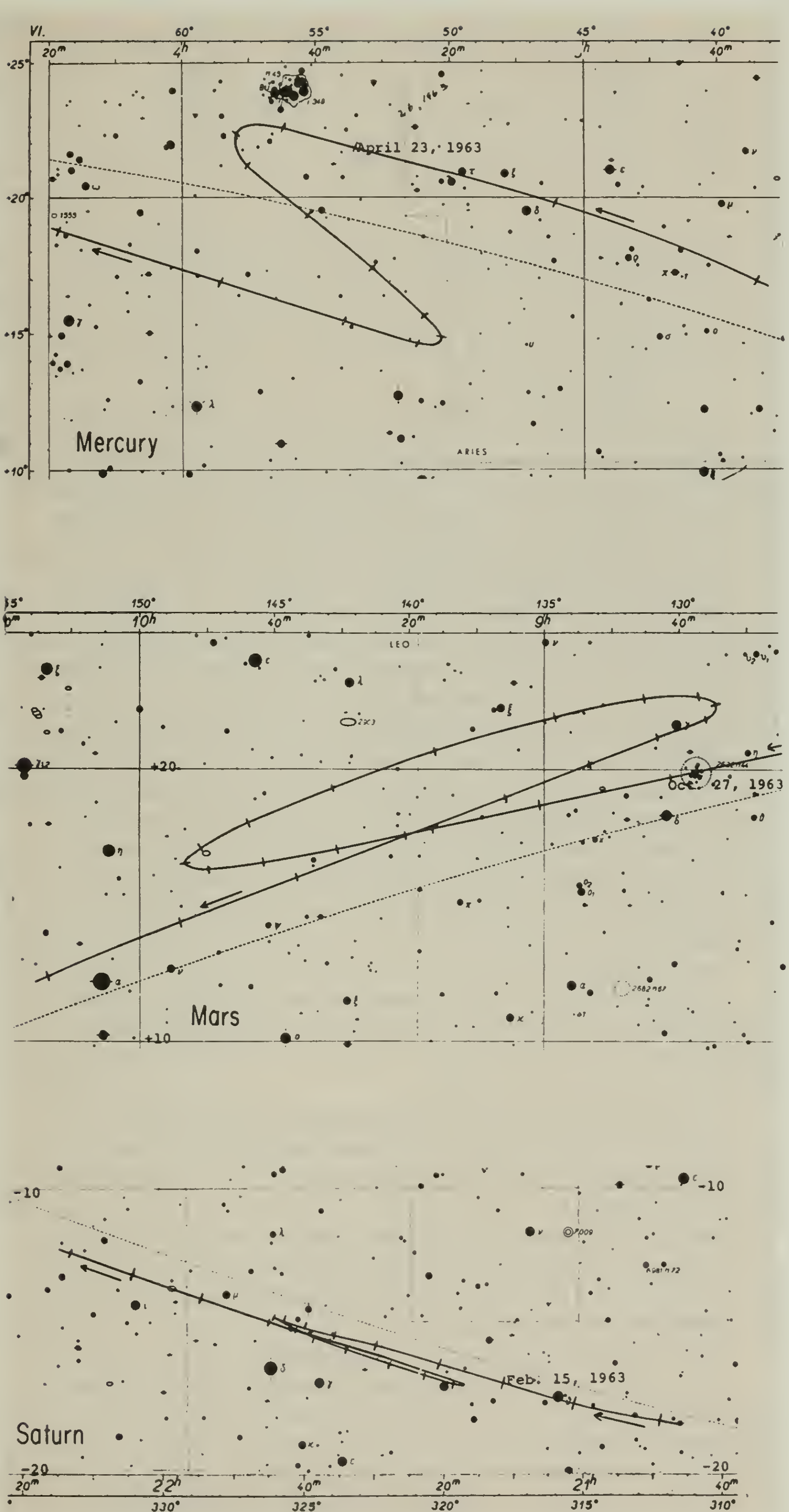
Q4 Draw a rough diagram to show the relative positions of the sun, earth, and moon during each of the moon's four phases.

Q5 Why don't eclipses occur each month?

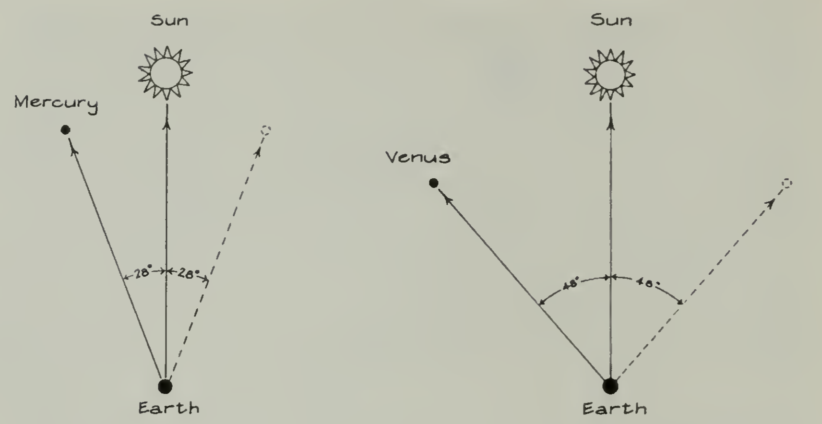
5.3 The "wandering stars"

Without a telescope we can see, in addition to the sun and moon, five rather bright objects which move among the stars. These are the "wanderers," or planets: Mercury, Venus, Mars, Jupiter, and Saturn. With the aid of telescopes, three more planets have been discovered: Uranus, Neptune, and Pluto; but none of these were known for nearly a century after the time of Isaac Newton. Like the sun and moon, all the planets rise daily in the east and set in the west. Also like the sun and moon, the planets generally move slowly eastward among the stars. But they have another remarkable and puzzling motion of their own: at certain times each planet stops moving eastward among the stars and for some months loops back westward. This westward or "wrong-way" motion is called *retrograde motion*. The retrograde loops made by Mercury, Mars, and Saturn during 1963 are plotted on the next page. Saturn, Jupiter, and Mars can at one time or another be anywhere in the sky, although always very near the ecliptic. The retrograde motion of one of these planets occurs when it is nearly opposite to the sun (that is, halfway across the sky at midnight). Mercury and Venus, however, have limits to how far away from the sun they can be;

When a planet is observed directly opposite from the sun, the planet is said to be *in opposition*. Retrograde motions of Mars, Jupiter, and Saturn are observed about the time they are in opposition.



The retrograde motions of Mercury (marked at 5-day intervals), Mars (at 10-day intervals), and Saturn (at 20-day intervals) in 1963, plotted on a star chart. The dotted line is the annual path of the sun, called the ecliptic.



The maximum angles from the sun at which we observe Mercury and Venus. Both planets can, at times, be observed at sunset or at sunrise. Mercury is never observed to be more than 28° from the sun, and Venus is never more than 48° from the sun.

as the above figures show, the greatest angular distance in either direction from the sun is 28° for Mercury and 48° for Venus. The retrograde motion of Venus or Mercury begins after the planet is furthest east of the sun and visible in the evening sky.

SG 5.7

The planets change considerably in brightness. When Venus is first seen in the evening sky as the "evening star," the planet is only fairly bright. But during the following four to five months, as it moves farther eastward from the sun, Venus gradually becomes so bright that it often can be seen in the daytime if the air is clear. A few weeks later, when Venus scoots westward toward the sun, it fades rapidly, passes the sun, and soon reappears in the morning sky before sunrise as the "morning star." Then it goes through the same pattern of brightness changes, but in the opposite order: bright, then gradually fading. The variations of Mercury follow much the same pattern. But because Mercury is always seen near the sun (that is, only during twilight), Mercury's changes are difficult to observe.

Typical Retrograde Motions of the Planets

PLANET	DAYS	WESTWARD DISPLACEMENT
Mercury	34	15
Venus	43	19
Mars	83	22
Jupiter	118	10
Saturn	139	7
Uranus	152	4
Neptune	160	3
Pluto	156	2

Mars, Jupiter, and Saturn are brightest about the time that they are in retrograde motion and opposite to the sun. Yet over many years their maximum brightness differs. The change is most noticeable for Mars; the planet is brightest when it is opposite the sun during August or September.

Not only do the sun, moon, and planets generally move eastward among the stars, but the moon and planets (except Pluto) are always found within a band only 8° wide on either side of the sun's path.

These, then, are some of the main observations of celestial phenomena. All of them were known to the ancients, and in their day as in ours, the puzzling regularities and variations seemed to cry out for some explanation.

Q6 In what part of the sky must you look to see the planets Mercury and Venus?

Q7 In what part of the sky would you look to see the planet which is in opposition?

Q8 When do Mercury and Venus show retrograde motion?

Q9 When do Mars, Jupiter, and Saturn show retrograde motion?

Q10 Can Mars, Jupiter, and Saturn appear any place in the sky?

5.4 Plato's problem

In the fourth century B.C., Greek philosophers asked a new question: How can we explain the cyclical changes observed in the sky? That is, what model can consistently and accurately account for all celestial motions? Plato sought a theory to account for what was seen, or, as he phrased it, "to save the appearances." The Greeks were among the first people to desire explanations for natural phenomena that did not require the intervention of gods and other supernatural beings. Their attitude was an important step toward science as we know it today.

How did the Greeks begin their explanation of the motions observed in the heavens? What were their assumptions?

Any answers to these questions must be tentative. Although many scholars over the centuries have devoted themselves to the study of Greek thought, the documents, on which our knowledge of the Greeks is based, are mostly copies of copies and translations of translations, in which errors and omissions occur. In some cases all we have are reports from later writers on what certain philosophers did or said, and these accounts may be distorted or incomplete. The historians' task is difficult. Most of the original Greek writings were on papyrus or cloth scrolls which have decayed through the ages. Many wars and much plundering and burning have also destroyed many important documents. Especially tragic was the burning of the famous library of Alexandria in Egypt, which contained several hundred thousand documents. (It was burned three times: in part by Caesar's troops in 47 B.C.; then in the fourth century A.D. by Christians; and the third time about 640 A.D. by early Muslims when they overran the country.) Thus, while the general picture of Greek culture seems to be rather well established, many interesting details are not known.

Several centuries later, a more mature Islamic culture led to extensive study and scholarly commentary on the remains of Greek thought. Several centuries later still, a more mature Christian culture used the ideas preserved by the Muslims to evolve early parts of modern science.

The approach taken by the Greeks and their intellectual followers for many centuries was already implied in a statement by Plato in the fourth century B.C. He defined the problem to his students in this way: the stars—representing eternal, divine, unchanging beings—move at a uniform speed around the earth, as we observe, in that most regular and perfect of all paths, the endless circle. But a few celestial objects, namely the sun, moon, and planets, wander across the sky and trace out complex paths, including even retrograde motions. Yet, being heavenly bodies, surely they too must really move in a way that suits their exalted status. Their motions, if not in a perfect circle, must therefore be in some combination of perfect circles. What combinations of circular motions at uniform speed can account for the peculiar variations in the overall regular motions in the sky?

Notice that the problem is concerned only with the changing *apparent* positions of the sun, moon, and planets. The planets appear to be only points of light moving against the background of stars. From two observations at different times we obtain a rate of motion: a value of so many degrees per day. The problem then is to find a "mechanism," some combination of motions, that will reproduce the observed angular motions and lead to predictions which agree with observations. The ancient astronomers had no observational evidence about the distances of the planets from the earth; all they had were directions, dates, and rates of angular motion. Although changes in brightness of the planets were known to be related to their positions relative to the sun, these changes in brightness were not included in Plato's problem.

Plato and many other Greek philosophers assumed that there were a few basic "elements" that mixed together to cause the apparent variety of materials observed in the world. Although not everyone agreed as to what these elements were, gradually four were accepted as the explanation of phenomena taking place on earth. These elements were Fire, Air, Water, and Earth. Because substances found on earth were supposed to contain various mixtures of these elements, these compound substances would have a wide range of properties. (See Unit 1, Chapter 2.) Only perfection could exist in the heavens, which were separate from the earth and were the abode of the gods. Just as motions in the heavens must be eternal and perfect, so also the unchanging heavenly objects could not be composed of elements normally found on or near the earth. Hence, they were supposed to be composed of a changeless fifth element of their own—the ether.

In Latin the ether became *quinta essentia* (fifth element), whence our "quintessence."

Plato's problem in explaining the motion of planets remained the most significant problem for theoretical astronomers for nearly two thousand years. To appreciate the later efforts and consequences of the different interpretations developed by Kepler, Galileo, and Newton, we will first examine the solutions to Plato's problems as they were developed by the Greeks. Let us confess right away that for their time these solutions were useful, ingenious, and indeed beautiful.

Q11 What was Plato's problem of planetary motion?

Q12 Why is our knowledge of Greek science incomplete?

Q13 Why did the Greeks feel that they should use only uniform circular motion to explain celestial phenomena?

5.5 The Greek idea of "explanation"

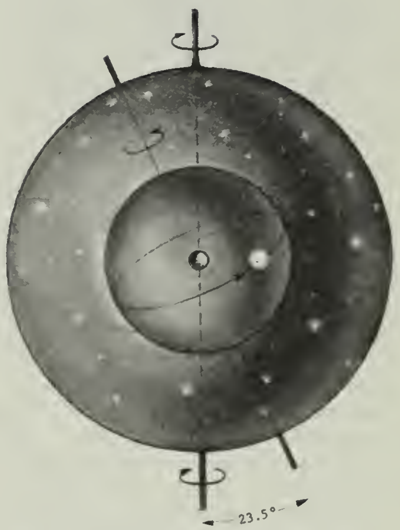
Plato's statement of this historic problem of planetary motion illustrates three contributions of Greek philosophers which, with modifications, are still basic to our understanding of the nature of physical theories:

1. A theory should be based on simple ideas. Plato regarded it

not merely as simple, but also as self-evident, that heavenly bodies must move uniformly along circular paths. Only in recent centuries have we come to understand that such common-sense beliefs may be misleading. More than that—we have learned that unproved assumptions are often necessary, but must be critically examined and should at first be accepted only tentatively. As we shall often see in this course, the identification of hidden assumptions in science has been extremely difficult. Yet in many cases, when the assumptions have been identified and questioned, entirely new theories have followed.

2. Physical theory must incorporate the measured results of observation of *phenomena*, such as the motions of the planets. Furthermore, our purpose in making a theory is to discover the uniformity of behavior, the hidden simplicities underlying apparent irregularities. For organizing our observations, the language of number and geometry is useful. This use of mathematics, widely accepted today, was derived in part from the Pythagoreans, a group of mathematicians who lived in southern Italy about 500 B.C. and believed that "all things are numbers." Actually the use of mathematics and measurement became important only in the later development of science. Plato stressed the fundamental role of numerical data only in his astronomy, while Aristotle largely avoided detailed measurements. This was unfortunate because, as the Prologue reported, the arguments of Aristotle, which did not include the idea of measurement of change as a tool of knowledge, were adopted by such influential philosophers as Thomas Aquinas.

3. To "explain" complex phenomena means to develop or invent a scheme (a physical model, or a geometrical or other mathematical construction) which shows the same features as the phenomena to be explained. Thus, for example, if one actually constructs a model of interlocking spheres, as has been often done, a point on one of the spheres has the same motions as the planet which the point represents.



The annual north-south (seasonal) motion of the sun was explained by having the sun on a sphere whose axis was tilted $23\frac{1}{2}°$ from the axis of the eternal sphere of the stars.

5.6 The first earth-centered solution

The Greeks observed that the earth was large, solid, and permanent, while the heavens seemed to be populated by small, remote, ethereal objects that were continually in motion. What was more natural than to conclude that our big, heavy earth was the steady, unmoving center of the universe? Such an earth-centered viewpoint is called *geocentric*. From this viewpoint the daily motion of the stars could easily be explained: they were attached to, or were holes in, a large, dark, spherical shell surrounding the earth and were all at the same distance from us. Daily, this celestial sphere turned once around on an axis through the earth. As a result, all the stars fixed on it would move in circular paths around the pole of rotation. Thus, a simple model of a rotating celestial sphere and a stationary earth could account for the daily motions of the stars.

The three observed motions of the sun require a somewhat more complex model. To explain the sun's motion with respect to the stars, a separate invisible shell was imagined that carried the sun around the earth. This shell was fixed to the celestial sphere and shared its daily motion but had also a slow, contrary motion of its own, namely one cycle of 360° per year.

The yearly north-south motion of the sun was accounted for by tipping the axis of this sphere, for the sun was tipped from the axis of the dome of the stars.

The motions of the visible planets—Mercury, Venus, Mars, Jupiter, and Saturn—were more difficult to explain. They share generally the daily motion of the stars, but they also have peculiar motions of their own. Because Saturn moves most slowly among the stars (it revolves once in 30 years), its sphere was assumed to be largest and closest to the stars. Inside the sphere for Saturn would be spheres that carried the faster-moving Jupiter (12 years) and Mars (687 days). Since they all require more than a year for a complete trip among the stars, these three planets were believed to be beyond the sphere of the sun. Venus, Mercury, and the moon were placed between the sun and the earth. The fast-moving moon was assumed to reflect sunlight and to be closest to the earth.


DE LA COSMOGRAPH.
La Figure & nombre
des Spheres.

Des Cercles de la Sphere. Chap. III.

Quelle chose est la Sphere.

LA Sphere est vn corps cõtenu d'une superfice ronde, au millieu du quel est vn poinct: toutes les lignes qui en sont protractes iusques à la circunference sont egaulx.

Quelle chose est l'exieu de la Sphere.

L'Exieu de la Sphere (cõme dict Diodochus) est le diametre qui passe au trauers la Sphere, sur lequel elle se tourne.

B

A geocentric cosmological scheme. The earth is fixed at the center of concentric rotating spheres. The sphere of the moon *(lune)* separates the terrestrial region (composed of concentric shells of the four elements Earth, Water, Air, and Fire) from the celestial region. In the latter are the concentric spheres carrying Mercury, Venus, Sun, Mars, Jupiter, Saturn, and the stars. To simplify the diagram, only one sphere is shown for each planet. (From the DeGolyer copy of Petrus Apianus' *Cosmographia*, 1551.)

Such an imaginary system of transparent shells or spheres can provide a rough "machine" to account for the general motions of heavenly objects. By choosing the sizes of the spheres and their rates and directions of motions, a rough match could be made between the model and the observations. If additional observations revealed other cyclic variations, more spheres could be added to make the necessary adjustment in the model.

Plato's friend Eudoxus concluded that 26 spheres would account for the general pattern of motions. Later Aristotle added 29 more. (An interesting description of this general system or cosmological scheme is given by the poet Dante in the *Divine Comedy*, written about 1300 A.D., shortly after Aristotle's writings became known in Europe.) Yet even Aristotle knew that this system did not get the heavenly bodies to their observed positions at quite the right time. Moreover, it did not account at all for the observed variations in brightness of the planets.

You may feel that Greek science was bad science because it was different from our own or less accurate. But you should understand from your study of this chapter that such a conclusion is not justified. The Greeks were just beginning the development of scientific theories and inevitably made assumptions that we now consider invalid. Their science was not "bad science," but in many ways it was a different kind of science from ours. And ours is not the last word, either. We must realize that to scientists 2000 years from now our efforts may seem strange and inept.

Even today's scientific theory does not and cannot account for every detail of each specific situation. Scientific concepts are idealizations which treat only selected aspects of observations rather than the totality of the raw data. Also, each period in history has its own limits on the range of human imagination. As you already have seen in Unit 1, important general concepts, such as force or acceleration, are specifically invented to help organize observations. They are not given to us in final form by some supernatural genius.

As you might expect, the history of science contains many examples in which certain factors overlooked by one researcher turn out later to be very important. But how would better systems for making predictions be developed without first trials? Theories are improved through tests and revisions, and sometimes are completely replaced by better ones.

Q14 What is a geocentric system? How does it account for the motions of the sun?

Q15 Describe the first solution to Plato's problem.

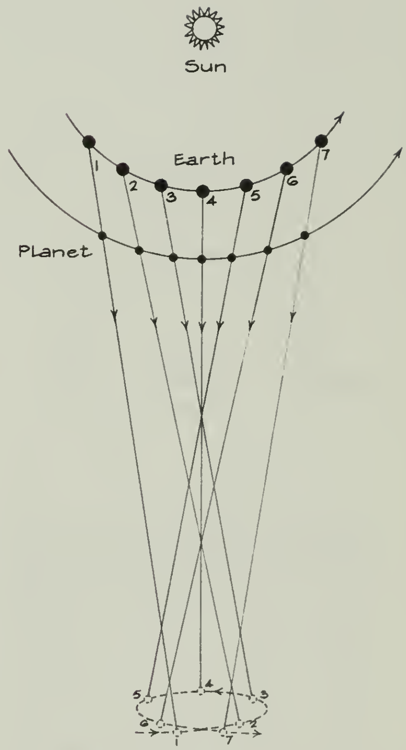
5.7 A sun-centered solution

For nearly two thousand years after Plato and Aristotle, the geocentric model was generally accepted. However, a radically different model, based on different assumptions, had been proposed in the third century B.C. The astronomer Aristarchus, (perhaps influenced by the writings of Heracleides, who lived a century earlier) suggested that a simpler explanation of heavenly motion would result if the *sun* were considered to be at the center, with the earth, planets, and stars all revolving around it. A sun-centered system is called *heliocentric*.

Because the major writings of Aristarchus have been lost, our knowledge of his work is based mainly on comments made by other writers. According to Archimedes, Aristarchus taught that the sun must be at least eighteen times farther away than the moon, and that the larger body, which was also the source of light, should be at the center of the universe.

Aristarchus proposed that all the daily motions observed in the sky could be explained by assuming that the celestial sphere is motionless and that the earth rotates once daily on an axis of its own. The apparent tilt of the paths of the sun, moon, and all the planets is accounted for simply by the tilt of the earth's own axis. Furthermore, the yearly changes in the sky, including the retrograde motions of the planets, could be explained by assuming that the earth and the five visible planets revolve around the sun. In this model, the motion previously assigned to the sun around the earth was assigned to the earth moving around the sun. Also, the earth became just one among several planets.

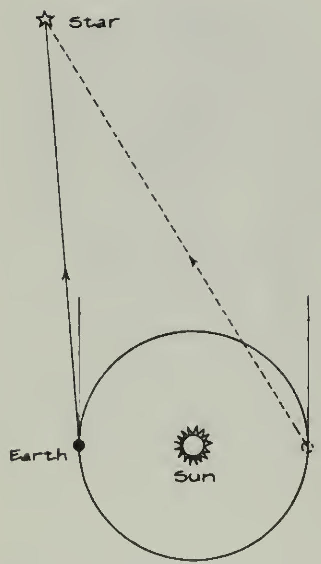
How such a system can account for the rectrograde motions of Mars, Jupiter, and Saturn can be seen from the diagram in the margin in which an outer planet and the earth are assumed to be moving around the sun in circular orbits. The outer planet moves



As the earth passes a planet in its orbit around the sun, the planet appears to move backwards in the sky. The arrows show the sight lines toward the planet for the different numbered positions of the earth. The lower numbered circles indicate the resulting apparent positions of the planet against the background of distant stars.

more slowly than the earth. As a result, when the earth is directly between the sun and the planet, the earth moves rapidly past the planet. To us the planet appears for a time to be moving backward or in retrograde motion across the sky.

Gone are all the interlocking concentric spheres. The heliocentric (sun-centered) hypothesis, which also uses only uniform circular motions, has one further advantage. It explains the bothersome observation that the planets are brighter during their retrograde motion, since at that time the planets are nearer to the earth. Even so, the proposal by Aristarchus was essentially neglected in antiquity. It was severely criticized for three basic reasons. One reason was that the idea of making a moving earth was unacceptable. It contradicted the philosophical doctrines that the earth is different from the celestial bodies and that the natural place of the earth, both physically and theologically, is the center of the universe. In fact, his contemporaries considered Aristarchus impious for even suggesting that the earth moved. Also, this new picture of the solar system contradicted common sense and everyday observations: the earth certainly seemed to be at rest rather than rushing through space.



If the earth goes around the sun, then the direction in which we have to look for a star should change during the year. A shift in the relative observed positions of objects that is caused by a displacement of the observer is called a *parallax*. The greatest observed parallax of a star caused by the earth's annual motion around the sun is about 1/2400°. This is explained by the fact that the distance to this nearest star is not just hundreds of millions of miles but 25 *million* million miles.

Another criticism was that certain observational evidence seemed to refute Aristarchus. If the earth were moving in an orbit around the sun, it would also be moving back and forth under the fixed stars. As shown in the sketch in the margin, the angle from the vertical at which we have to look for any star would be different when seen from the various points in the earth's annual path. This annual shift of the fixed stars should occur if the earth moves around the sun. But it was not observed by the Greek astronomers. This awkward fact could be explained in two ways either (1) the earth does *not* go around the sun and so there is no shift, or (2) the earth does go around the sun but the stars are so far away that the shift is *too small to observe*. But as the Greeks realized, for the shift to be undetectably small, the stars would have to be enormously far away—perhaps hundreds of millions of miles.

Today we can observe the annual shift of the stars with telescopes, so we know that Aristarchus' model is in fact useful. The shift is too small to be seen with the naked eye using the best sighting instruments—and indeed so small that even with telescopes it was not measured until 1838. The largest annual shift is an angle of only about 1/2400 of a degree of arc. The smallest angle observable by the human eye under ideal conditions is about 1/25°, so the actual angular shift is about 100 times smaller than could possibly have been observed. The shift exists, but we can sympathize with the Greeks, who rejected the heliocentric theory partly because the shift required by the theory could not be observed. Only Aristarchus imagined that the stars might be as immensely distant as we now know them to be.

Finally, Aristarchus was criticized because he did not develop his system in detail or use it for making predictions of planetary positions. His work seems to have been purely qualitative, a general scheme of how things might be.

The geocentric and heliocentric systems were two different ways to account for the same observations. But the heliocentric proposal required such a drastic change in man's image of the universe that Aristarchus' heliocentric hypothesis had little influence on Greek thought. Fortunately his arguments were recorded and reported and eighteen centuries later stimulated the thoughts of Copernicus. Ideas are not bound by space or time.

Q16 What two radically new assumptions were made by Aristarchus? What simplification resulted?

Q17 How can the heliocentric model proposed by Aristarchus explain retrograde motion?

Q18 What change predicted by Aristarchus' theory was not observed by the Greeks?

Q19 Why was Aristarchus considered impious? Why was his system neglected?

5.8 The geocentric system of Ptolemy

Disregarding the heliocentric model suggested by Aristarchus, the Greeks continued to develop their planetary theory as a geocentric system. As we noted, the first solution in terms of concentric spheres lacked accuracy. During the 500 years after Plato and Aristotle, astronomers began to sense the need for a more accurate theory for the heavenly timetables. To fit the observations, a complex mathematical theory was required for each planet.

Several Greek astronomers made important contributions which culminated about 150 A.D. in the geocentric theory of Claudius Ptolemy of Alexandria. Ptolemy's book on the motions of the heavenly objects is a masterpiece of analysis.

Ptolemy wanted a system that would predict accurately the positions of each planet. The type of system and the motions he accepted were based on the assumptions of Aristotle. In the preface to his *Almagest*, Ptolemy defines the problem and states his assumptions:

The Arabic title given to Ptolemy's book, the *Almagest*, means "the greatest."

> . . . we wish to find the evident and certain appearances from the observations of the ancients and our own, and applying the consequences of these conceptions by means of geometrical demonstrations.
>
> And so, in general, we have to state, that the heavens are spherical and move spherically; that the earth, in figure, is sensibly spherical . . .; in position, lies right in the middle of the heavens, like a geometrical center; in magnitude and distance, [the earth] has the ratio of a point with respect to the sphere of the fixed stars, having itself no local motion at all.

Ptolemy then argues that each of these assumptions is necessary and fits with all our observations. The strength of his belief is illustrated by his statement ". . . it is once for all clear from the very appearances that the earth is in the middle of the world and all weights move towards it." Notice that he has supported his interpretation of the astronomical observations with the physics of falling bodies. This mixture of astronomy and physics, when applied to the earth itself and its place in the scheme, became highly important when he referred to the proposal of Aristarchus that the earth might rotate and revolve:

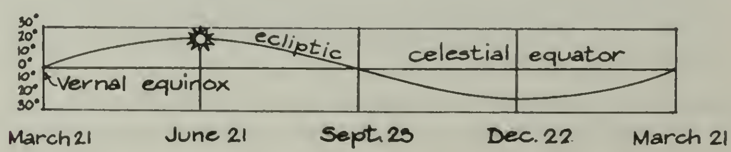
> Now some people, although they have nothing to oppose to these arguments, agree on something, as they think, more plausible. And it seems to them there is nothing against their supposing, for instance, the heavens immobile and the earth as turning on the same axis [as the stars] from west to east very nearly one revolution a day. . . .
>
> But it has escaped their notice that, indeed, as far as the appearances of the stars are concerned, nothing would perhaps keep things from being in accordance with this simpler conjecture, but that in the light of what happens around us in the air such a notion would seem altogether absurd.

Ptolemy believed that if the earth rotated it would not take its blanket of air around with it, with the result that all clouds would fly past toward the west, and all birds or other things in the air would be carried away to the west. If, however, the air was carried with the earth, the objects in the air would still be left behind by the earth and air together.

The paragraphs quoted above contain a main theme in Unit 2. Ptolemy recognized that the two systems were equally successful in *describing* motion—in the kinematics; but the one was to be preferred over the others because it better fit the *causes* of motion—the dynamics—as understood at the time. Much later, when Newton had developed a completely different dynamics, the choice would fall the other way.

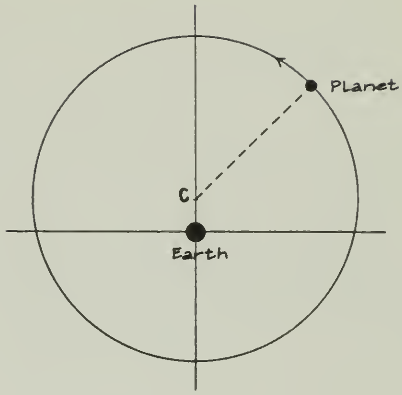
Ptolemy developed very clever and rather accurate procedures by which the positions of each planet could be derived on a geocentric model. In the solutions he went far beyond the scheme of concentric spheres of the earlier Greeks, constructing a model
SG 5.8 out of circles and three other geometrical devices. Each device provided for variations in the rate of angular motion as seen from the earth. To appreciate Ptolemy's ingenious solution, let us examine one of the many small variations he was attempting to explain.

We can divide the sun's yearly 360° path across the background of stars into four 90° parts. If the sun is at the 0 point on March 21, it will be 90° farther east on June 21, 90° further still on September



The annual path of the sun against the celestial sphere.

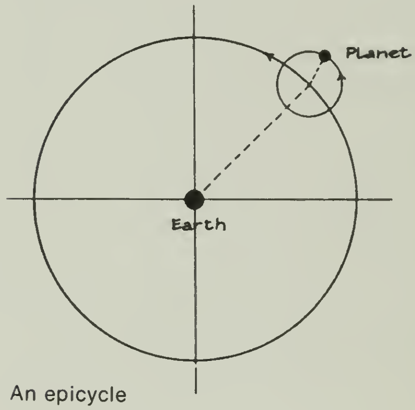
23, 90° farther on December 22, and back at the starting point on March 21, a year later. If the sun moves uniformly on a circle around the earth, the times between these dates ought to be equal. But, as you will find by consulting a calendar, they aren't equal. The sun takes longer to move 90° in spring and summer than it does in fall and winter. So any simple circular system based on motion with constant speed will not work for the sun.



An eccentric

The three devices that Ptolemy used in proposing an improved geocentric theory were the *eccentric*, the *epicycle* and *equant*.

In agreement with Plato, astronomers had held previously that the motion of a celestial object must be at a uniform angular rate and at a constant distance from the center of the earth. Although Ptolemy believed that the earth was at the center of the universe, he did not insist that it be at the geometrical centers of all the perfect circles. He proposed that the center C of the circle could be off-center from the earth, in an *eccentric* position. Thus, motion that was really uniform around the center C would not appear to be uniform motion when observed from the earth. An eccentric orbit of the sun will therefore account for the type of seasonal irregularity observed in the sun's rate of motion.



An epicycle

While the eccentric can also account for small variations in the rate of motion of planets, it cannot describe any such radical change as retrograde motion of the planets. To account for retrograde motion, Ptolemy used another device, the *epicycle* (see the figure at the right). The planet is considered to be moving at a uniform rate on the circumference of a small circle, called the epicycle. The center of the epicycle moves at a uniform rate on the large circle, called the *deferent,* around the earth.

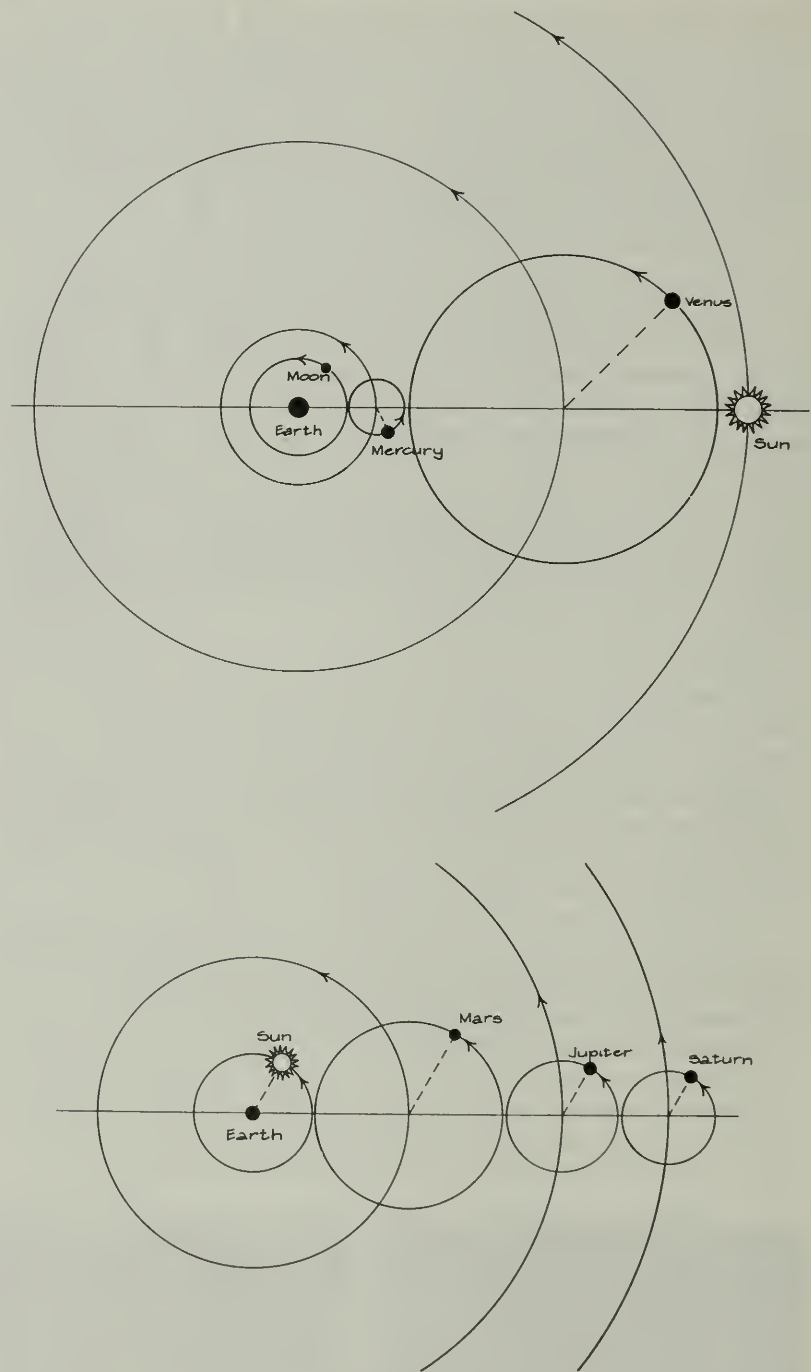
If a planet's speed on the epicycle is greater than its speed on the large circle, the planet as seen from *above* the planetary system would appear to move through loops. When observed from a location near the center, these loops would look like the retrograde motions actually observed for planets. The photographs below show two views of the motions produced by a simple mechanical model, an "epicycle machine" with a small lamp in place of the planet. The photo on the left was taken from "above," like the diagram in the margin; the photograph on the right was taken "on edge,"

SG 5.9


a

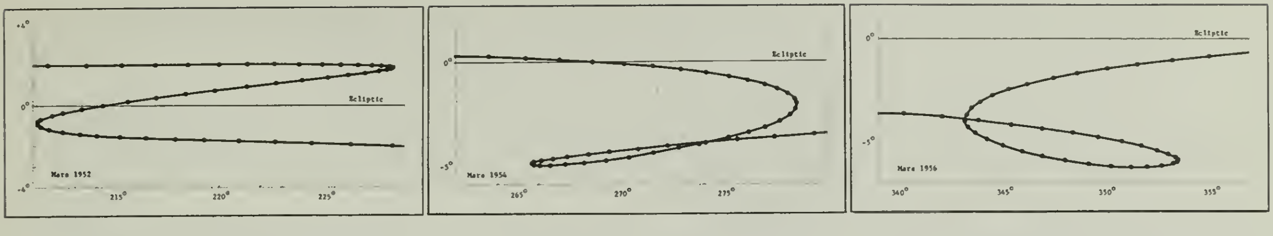

b

Retrograde motion created by a simple epicycle machine.
(a) Stroboscopic photograph of epicyclic motion. The flashes where made at equal time intervals. Note that the motion is slowest in the loop.
(b) Loop seen from near its plane.



Ptolemy did not picture the planetary motions as those in an interlocking machine where each planet determined the motion of the next. Because there was no information about the distances of the planets, Ptolemy adopted the old order of distances from the earth: stars being the most remote, then Saturn, Jupiter, Mars, the sun, Venus, Mercury, and the moon. The orbits were usually shown nested inside one another so that their epicycles did not overlap.

Simplified representation of the Ptolemaic system. The scale of the upper drawing, which shows the planets between the earth and the sun, is eight times that of the lower drawing, which shows the planets that are farther than the sun. The planets' epicycles are shown along one straight line to emphasize the relative sizes of the epicycles.



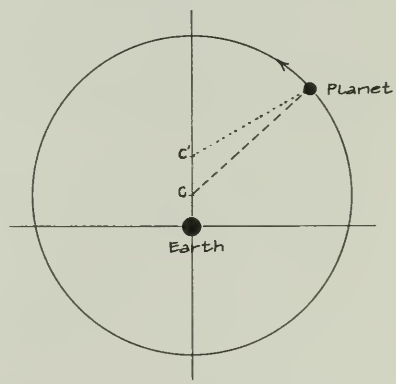
Mars plotted at four-day intervals on three consecutive oppositions. Note the different sizes and shapes of the retrograde curves.

almost in the plane of the motion; thus, the appearance of the loop is very much like it would be if viewed from near the center.

Epicycles can be used to describe many kinds of motion. So it was not too difficult to produce a system that had all the main features of observed planetary motion. One particularly interesting feature of Ptolemy's system was the epicycles for the outer planets all had the same period: exactly one year! Moreover, as the sketches on the opposite page indicate, the positions of the outer planets on their epicycles always matched the position of the sun relative to the earth. This correspondence of epicycles to the relative motion of sun and earth was, fourteen centuries later, to be a key point of concern to Copernicus.

So far, the system of epicycles and deferents "works" well enough. It explains not only retrograde motion, but also the greater brightness of the planets when they are in retrograde motion. Since the planet is on the inside of its epicycle during retrograde motion, it is closest to the earth, and so appears brightest. This is an unexpected bonus, since the model was not designed to explain this feature.

But even with combinations of eccentrics and epicycles, Ptolemy was not able to fit the motions of the five planets precisely. For example, as we see in the three figures above, the retrograde motion of Mars is not always of the same angular size or duration. To allow for these variations, Ptolemy used a third geometrical device, called the *equant*, which is a modification of an eccentric. As shown in the margin, the earth is again off-center from the geometric center C of the circle, but the motion along the circle is not uniform around C; instead, it is uniform as seen from another point C′, which is as far off-center as the earth is, but on the other side of the center.



An equant. C is the center of the circle. The planet P moves at a uniform rate around the off-center point C′.

5.9 Successes and limitations of the Ptolemaic model

Ptolemy's model always used a uniform rate of angular motion around some center, and to that extent stayed close to the assumptions of Plato. But Ptolemy was willing to displace the centers of motion from the center of the earth, as much as was necessary to fit the observations. By a combination of eccentrics, epicycles, and equants he described the positions of each planet separately. For each planet, Ptolemy had found a combination of motions that predicted its observed positions over long periods of time to within about two degrees (roughly four diameters of the moon)—a considerable improvement over earlier systems.

SG 5.10

Astronomical observations were all observations of *angles*—a small loop in the sky could be a small loop fairly near, or a larger loop much farther away.

The success of Ptolemy's model, especially the unexpected explanation of variation in brightness, might be taken as proof that objects in the sky actually moved on epicycles and deferents around off-center points. It seems, however, that Ptolemy himself did not believe he was providing an actual physical model of the universe. He was content to give a mathematical model for the computation of positions.

Of course, some difficulties remained. For example, to explain the motions of the moon, Ptolemy had to use such large epicycles that during a month the moon would appear to grow and shrink in size appearing to have at some times twice the diameter than at other times! Ptolemy surely knew that this was predicted by his model—and that it does not happen in actual observation. But, his model was not intended to be "real," it was only a basis for computing positions.

The Ptolemaic description was a series of mathematical devices to match and predict the motion of each planet separately. His geometrical analyses were equivalent to finding a complicated equation of motion for each individual planet. Nevertheless, in the following centuries most scholars, including the poet Dante,
SG 5.11 believed that the planets really moved on some sort of crystalline spheres as Eudoxus had suggested earlier.

Although now discarded, the Ptolemaic form of the geometric model of the planetary system, proposed in 150 A.D., was used for about 1500 years. There were good reasons for this long acceptance.

It predicted fairly accurately the positions of the sun, moon, and planets.

It explained why the fixed stars do not show an annual shift when observed with the naked eye.

It agreed in most details with the philosophical doctrines developed by the earlier Greeks, including the idea of "natural motion" and "natural place."

It had common-sense appeal to all who saw the sun, moon, planets, and stars moving around them.

It agreed with the comforting assumption that we live on an immovable earth at the center of the universe.

Also, later, it fitted into Thomas Aquinas' widely accepted synthesis of Christian belief and Aristotelian physics.

Yet, Ptolemy's system was eventually displaced by a heliocentric one. Why did this occur? What advantages did the new theory have
SG 5.12 over the old? From this epic argument about competing theories what can we learn about the relative value of rival theories in
SG 5.13 science today? These are some of the questions to consider in the next chapter.

5.1 The Project Physics learning materials particularly appropriate for Chapter 5 include the following:

Experiments
- Naked-Eye Astronomy (cont.)
- Size of the Earth
- Height of Piton, A Mountain on the Moon

Activities
- Making Angular Measurements
- Celestial Sphere Model
- How Long is a Sidereal Day?
- Scale Model of the Solar System
- Build a Sundial
- Plot an Analemma
- Stonehenge
- Moon Crater Names
- Literature
- Size of the Earth—Simplified Version

Reader Articles
- The Boy Who Redeemed His Father's Name
- Four Poetic Fragments About Astronomy

Film Strip
- Retrograde Motion of Mars

Film Loops
- Retrograde Motion of Mars and Mercury
- Retrograde Motion—Geocentric Model

Transparencies
- Stellar Motion
- Celestial Sphere
- Retrograde Motion
- Eccentrics and Equants

In addition, the following *Reader* articles are of general interest for Unit 2:
- The Black Cloud
- Roll Call
- A Night at the Observatory
- The Garden of Epicurus
- The Stars Within 22 Light-years That Could Have Habitable Planets
- Scientific Study of UFO's

5.2 How could you use the shadow cast by a vertical stick on horizontal ground to find
(a) the local noon?
(b) which day was June 21st?
(c) the length of a solar year?

5.3 What is the difference between 365.24220 days and $365\frac{1}{4}$ days (a) in seconds (b) in percent?

5.4 (a) List the observations of the motions of heavenly bodies that you might make which would also have been possible in ancient Greek times.
(b) For each observation, list some reasons why the Greeks thought these motions were important.

5.5 Which of the apparent motions of the stars could be explained by a flat earth and stars fixed to a bowl that rotated around it?

5.6 Describe the motion of the moon during one month. (Use your own observations if possible.)

5.7 Mercury and Venus show retrograde motion after they have been farthest east of the sun and visible in the evening sky. Then they quickly move ahead westward toward the sun, pass it, and reappear in the morning sky. During this motion they are moving westward relative to the stars, as is shown by the plot of Mercury on page 13. Describe the rest of the cyclic motion of Mercury and Venus.

5.8 Center a protractor on point C in the top diagram on page 23 and measure the number of degrees in the four quadrants. Consider each 1° around C as one day. Make a table of the days needed for the planet to move through the four arcs as seen from the earth.

5.9 (a) How many degrees of longitude does the sun move each hour?
(b) Tell how you could roughly obtain the diameter of the earth from the following information:
i. Washington, D.C. and San Francisco have about the same latitude. How can one easily test this?
ii. A non-stop jet plane, going upwind at a ground speed of 500 mph from Washington, D.C., to San Francisco, takes 5 hours to get there.
iii. When it is just sunset in Washington, D.C., a man there turns on his TV set to watch a baseball game that is just beginning in San Francisco. The game goes into extra innings. After three hours the announcer notes that the last out occurred just as the sun set.

5.10 In Ptolemy's theory of the planetary motions there were, as in all theories, a number of assumptions. Which of the following did Ptolemy assume?
(a) the vault of stars is spherical in form
(b) the earth has no motions
(c) the earth is spherical
(d) the earth is at the center of the sphere of stars
(e) the size of the earth is extremely small compared to the distance to the stars
(f) uniform angular motion along circles (even if measured from an off-center point) is the only proper behavior for celestial objects

5.11 As far as the Greeks were concerned, and indeed as far as we are concerned, a reasonable argument can be made for either the geocentric or the heliocentric theory of the universe.
(a) In what ways were both ideas successful?
(b) In terms of Greek science, what are some advantages and disadvantages of each system?
(c) What were the major contributions of Ptolemy?

5.12 Why was astronomy the first successful science, rather than, for example, meteorology or zoology?

Copernicus' diagram of his heliocentric system (from his manuscript, of *De Revolutionibus*, 1543). This simplified representation omits the many small epicycles actually used in the system.

CHAPTER SIX

Does the Earth Move? – The Work of Copernicus and Tycho

6.1 The Copernican system

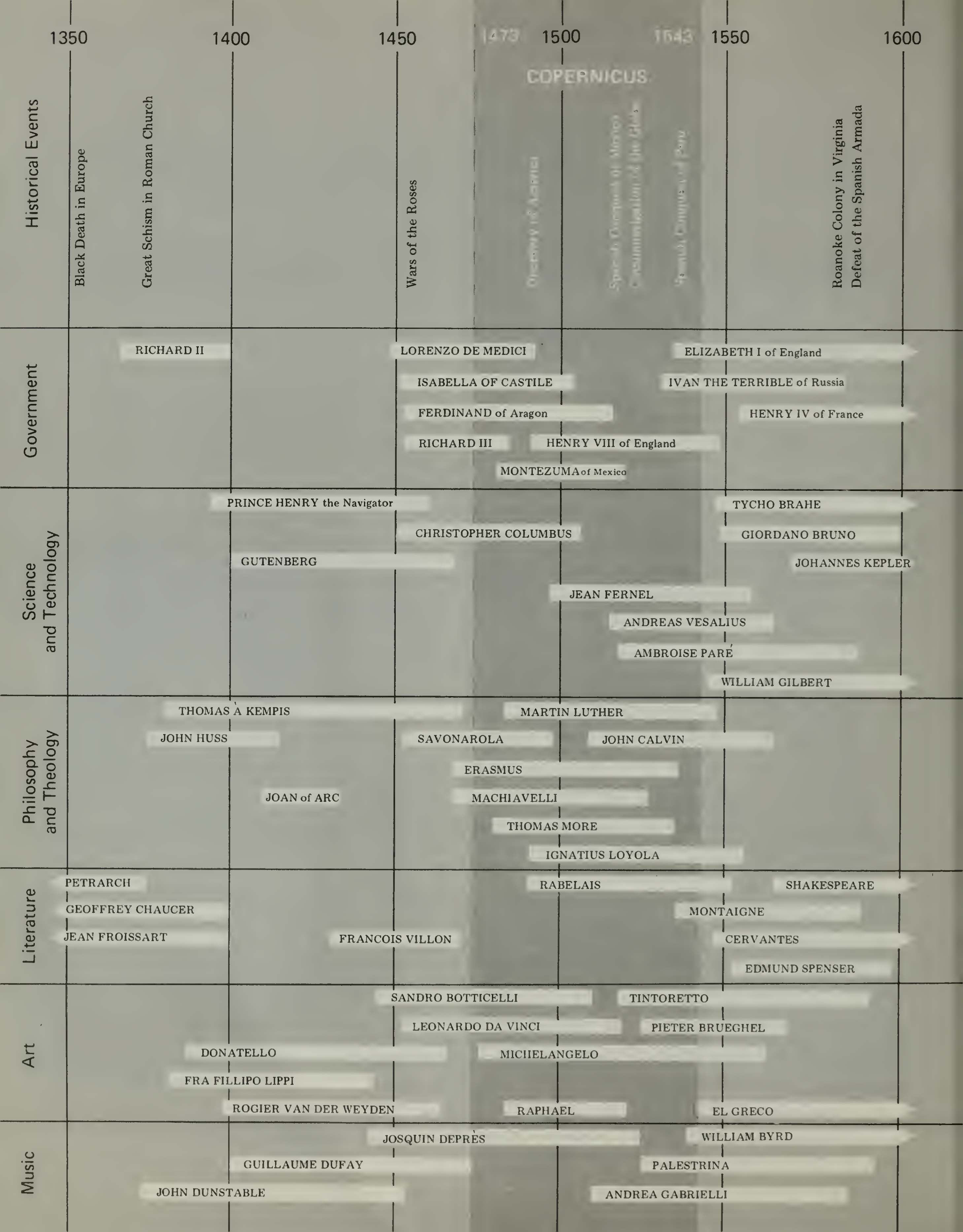
Nicolaus Copernicus (1473-1543) was a young student in Poland when America was discovered. An outstanding astronomer and mathematician, Copernicus was also a talented and respected churchman, jurist, administrator, diplomat, physician, and economist. During his studies in Italy he read the writings of Greek and other earlier philosophers and astronomers. As Canon of the Cathedral of Frauenberg he was busy with civic and church affairs and also worked on calendar reform. It is said that on the day of his death in 1543, he saw the first copy of his great book, on which he had worked most of his life and which opened a whole new vision of the universe.

SG 6.1

Nicolas Copernicus (1473-1543). (In Polish his name was Koppernigk, but, in keeping with the scholarly tradition of the age, he gave it the Latin form Copernicus.)

Copernicus titled his book *De Revolutionibus Orbium Coelestium*, or *On the Revolutions of the Heavenly Spheres*, which suggests the early Greek notion of concentric spheres. Copernicus was indeed concerned with the old problem of Plato: the construction of a planetary system by combinations of the fewest possible uniform circular motions. He began his study to rid the Ptolemaic system of the equants, which were contrary to Plato's assumptions. In his words, taken from a short summary written about 1530,

> . . . the planetary theories of Ptolemy and most other astronomers, although consistent with the numerical data, seemed likewise to present no small difficulty. For these theories were not adequate unless certain equants were also conceived; it then appeared that a planet moved with uniform velocity neither on its deferent nor about the center of its epicycle. Hence a system of this sort seemed neither sufficiently absolute nor sufficiently pleasing to the mind.
>
> Having become aware of these defects, I often considered whether there could perhaps be found a more reasonable arrangement of circles, from which every apparent inequality would be derived and in which everything would move uniformly about its proper center.


1350
1400
1450
1473
1500
1543
1550
1600
COPERNICUS
Historical Events
Black Death in Europe
Great Schism in Roman Church
Wars of the Roses
Roanoke Colony in Virginia
Defeat of the Spanish Armada
Government
RICHARD II
LORENZO DE MEDICI
ELIZABETH I of England
ISABELLA OF CASTILE
IVAN THE TERRIBLE of Russia
FERDINAND of Aragon
HENRY IV of France
RICHARD III
HENRY VIII of England
MONTEZUMA of Mexico
Science and Technology
PRINCE HENRY the Navigator
TYCHO BRAHE
CHRISTOPHER COLUMBUS
GIORDANO BRUNO
GUTENBERG
JOHANNES KEPLER
JEAN FERNEL
ANDREAS VESALIUS
AMBROISE PARÉ
WILLIAM GILBERT
Philosophy and Theology
THOMAS A KEMPIS
MARTIN LUTHER
JOHN HUSS
SAVONAROLA
JOHN CALVIN
ERASMUS
JOAN of ARC
MACHIAVELLI
THOMAS MORE
IGNATIUS LOYOLA
Literature
PETRARCH
RABELAIS
SHAKESPEARE
GEOFFREY CHAUCER
MONTAIGNE
JEAN FROISSART
FRANCOIS VILLON
CERVANTES
EDMUND SPENSER
Art
SANDRO BOTTICELLI
TINTORETTO
LEONARDO DA VINCI
PIETER BRUEGHEL
DONATELLO
MICHELANGELO
FRA FILLIPO LIPPI
ROGIER VAN DER WEYDEN
RAPHAEL
EL GRECO
Music
JOSQUIN DEPRÈS
WILLIAM BYRD
GUILLAUME DUFAY
PALESTRINA
JOHN DUNSTABLE
ANDREA GABRIELLI

In *De Revolutionibus* he wrote:

> We must however confess that these movements [of the sun, moon, and planets] are circular or are composed of many circular movements, in that they maintain these irregularities in accordance with a constant law and with fixed periodic returns, and that could not take place, if they were not circular. For it is only the circle which can bring back what is past and over with. . . .
>
> I found first in Cicero that Nicetas thought that the Earth moved. And afterwards I found in Plutarch that there were some others of the same opinion. . . . Therefore I also . . . began to meditate upon the mobility of the Earth. And although the opinion seemed absurd, nevertheless, because I knew that others before me had been granted the liberty of constructing whatever circles they pleased in order to demonstrate astral phenomena, I thought that I too would be readily permitted to test whether or not, by the laying down that the Earth had some movements, demonstrations less shaky than those of my predecessors could be found for the revolutions of the celestial spheres. . . . I finally discovered by the help of long and numerous observations that if the movements of the other wandering stars are correlated with the circular movement of the Earth, and if the movements are computed in accordance with the revolution of each planet, not only do all their phenomena follow from that but also this correlation binds together so closely the order and magnitudes of all the planets and of their spheres or orbital circles and the heavens themselves that nothing can be shifted around in any part of them without disrupting the remaining parts and the universe as a whole.

After nearly forty years of study, Copernicus had proposed a system of more than thirty eccentrics and epicycles which would "suffice to explain the entire structure of the universe and the entire ballet of the planets." Like Ptolemy's *Almagest, De Revolutionibus* uses long geometrical analyses and is difficult to read. Comparison of the two books strongly suggests that Copernicus thought he was producing an improved version of the *Almagest*. He used many of Ptolemy's observations plus some more recent ones. Yet his system differed from that of Ptolemy in several fundamental ways. Above all, he adopted a sun-centered system which in general outline was the same as that of Aristarchus (who was still discredited, and whom Copernicus did not think it wise to mention in print).

See the preface to Copernicus' *De Revolutionibus* in *Reader 2*.

Like all scientists, Copernicus made a number of assumptions in his system. In his own words (rendered in modern equivalent in several places), his assumptions were:

> 1. There is no one precise, geometrical center of all the celestial circles or spheres.
> 2. The center of the earth is not the center of the

universe, but only of gravitation and of the lunar sphere.

3. All the spheres revolve about the sun . . . and therefore the sun has a central location in the universe.

4. The distance from the earth to the sun is inperceptible in comparison with the distance to the stars.

5. Whatever motion appears in the sky arises not from any motion of the sky, but from the earth's motion. The earth together with its water and air performs a complete rotation on its fixed poles in a daily motion, while the sky remains unchanged.

6. What appear to us as motions of the sun arise not from its motion but from the motion of the earth and . . . we revolve about the sun like any other planet. The earth has, then, more than one motion.

7. The apparent retrograde motion of the planets arises not from their motion but from the earth's. The motions of the earth alone, therefore, are sufficient to explain so many apparent motions in the sky.

SG 6.2

Comparison of this list with the assumptions of Ptolemy, given in Chapter 5, will show some close similarities and important differences.

Notice that Copernicus proposed that the earth rotates daily. As Aristarchus and others had realized, this rotation would account for all the daily risings and settings observed in the sky. Copernicus also proposed, as Aristarchus had done, that the sun was stationary and occupied the central position of the universe. The earth and each of the other planets moved about different central points near the sun.

The figure at the left shows the main concentric spheres carrying the planets around the sun (*sol*). His text explains the basic features of his system:

The ideas here stated are difficult, even almost impossible, to accept; they are quite contrary to popular notions. Yet with the help of God, we will make everything as clear as day in what follows, at least for those who are not ignorant of mathematics. . . .

The first and highest of all the spheres is the sphere of the fixed stars. It encloses all the other spheres and is itself self-contained; it is immobile; it is certainly the portion of the universe with reference to which the movement and positions of all the other heavenly bodies must be considered. If some people are yet of the opinion that this sphere moves, we are of contrary mind; and after deducing the motion of the earth, we shall show why we so conclude. Saturn, first of the planets, which accomplishes its revolution in thirty years, is nearest to the first sphere. Jupiter, making its revolution in twelve years, is next. Then comes Mars, revolving once in two years. The fourth place in the series is occupied by the sphere which contains the earth and the sphere of the moon, and which performs an

> annual revolution. The fifth place is that of Venus, revolving in nine months. Finally, the sixth place is occupied by Mercury, revolving in eighty days. . . . In the midst of all, the sun reposes, unmoving.

Already we see an advantage in Copernicus' system that makes it "pleasing to the mind." The rates of rotation for the heavenly spheres increase in order from the motionless sphere of stars to speedy Mercury. SG 6.3

Q1 What reasons did Copernicus give for rejecting the use of equants?

Q2 In the following list of propositions, mark with a *P* those made by Ptolemy and with a *C* those made by Copernicus.

(a) The earth is spherical.
(b) The earth is only a point compared to the distance to the stars.
(c) The heavens rotate daily around the earth.
(d) The earth has one or more motions.
(e) Heavenly motions are circular.
(f) The observed retrograde motion of the planets results from the earth's motion around the sun.

6.2 New conclusions

A new way of looking at old observations—a new theory—can suggest quite new kinds of observations to make, or new uses of old data. Copernicus used his moving-earth model to obtain two important results which were not possible with the Ptolemaic theory. He was able to calculate (a) the period of motion of each planet around the sun, and (b) the sizes of each planet's orbit compared to the size of the earth's orbit. This, for the first time, gave a scale for the dimensions of the universe.

To calculate the periods of the planets around the sun, Copernicus used observations that had been recorded over many centuries. The method of calculation is similar to the "chase problem" of how often the hands on a clock pass one another. The details of the calculation are shown on page 34. In Table 6.1 below, Copernicus' results are compared with the currently accepted values. SG 6.4, 6.5

Table 6.1

PLANET	COPERNICUS' VALUE	MODERN VALUE
Mercury	0.241 y (88 d)	87.97 d
Venus	0.614 y (224 d)	224.70 d
Mars	1.88 y (687 d)	686.98 d
Jupiter	11.8 y	11.86 y
Saturn	29.5 y	29.46 y

Copernicus was also able, for the first time in history, to derive relative distances between the planets and the sun. Remember that the Ptolemaic system had no distance scale; it provided only a way

The Periods of Revolution of the Planets

The problem is to find the rate at which a planet moves around the sun by using observations made from the earth—which is itself moving around the sun. Say, for example, that a planet closer to the sun than the earth is, goes around the sun at the frequency (rate) of $1\frac{1}{4}$ cycles per year. The earth moves around the sun also, in the same direction, at the rate of 1 cycle per year. Because the earth follows along behind the planet, the planet's motion around the sun would, as seen from the earth, appear to be at a rate less than $1\frac{1}{4}$ cycles per year. In fact, as the diagrams below suggest, the planet's apparent rate of motion around the sun would be the difference between the planet's rate and the earth's rate around the sun: $1\frac{1}{4}$ cycle per year − 1 cycle per year = $\frac{1}{4}$ cycle per year. In general, if an inner planet moves around the sun at frequency f_p and the earth moves around the sun with frequency f_e, then the planet's apparent rate of motion, f_{pe}, as seen from the earth, will be $f_{pe} = f_p - f_e$.

A similar argument holds for planets farther from the sun than the earth is. (See diagram B.) Since these outer planets revolve about the sun more slowly than the earth, the earth repeatedly leaves the planets behind. Consequently, for the outer planets the sign in the equation for f_{pe} is reversed: $f_{pe} = f_p - f_e$.

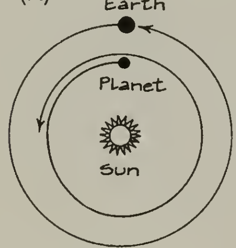
The apparent frequency f_{pe} is what is actually observed and f_e is by definition 1 cycle per year, so either equation is easily solved for the unknown actual rate f_p of the planet around the sun:

For inner planets: $f_p = 1$ cycle/yr $+ f_{pe}$

For outer planets: $f_p = 1$ cycle/yr $- f_{pe}$

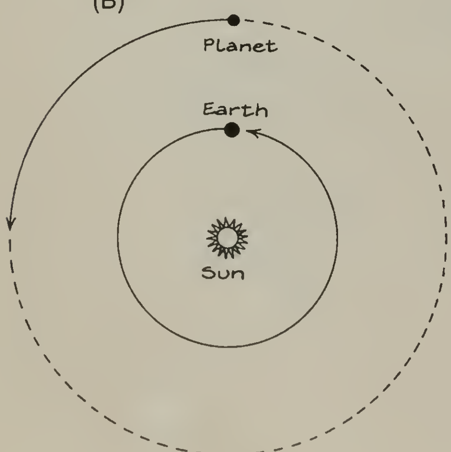
Copernicus used some observations by Ptolemy and some of his own. A typical data statement in *De Revolutionibus* is "Jupiter is outrun by the earth 65 times in 71 solar years minus 5 days 45 minutes 27 seconds . . . ." In the table below, Copernicus' data have been rounded off to the nearest year (but they were very near to whole years to begin with). The cycle used for the inner planets is from one position of greatest eastern displacement from the sun to the next. The cycle used for the outer planets is from one opposition to the next.

(A)



A planet that is inside the Earth's orbit and moves $1\frac{1}{4}$ revolutions around the sun in a year would, as seen from the earth, appear to have made only $\frac{1}{4}$ cycle.

(B)



A planet that is outside the Earth's orbit and moves only $\frac{1}{4}$ revolution around the sun in a year would, as seen from the earth, appear to make about $1\frac{3}{4}$ revolution.

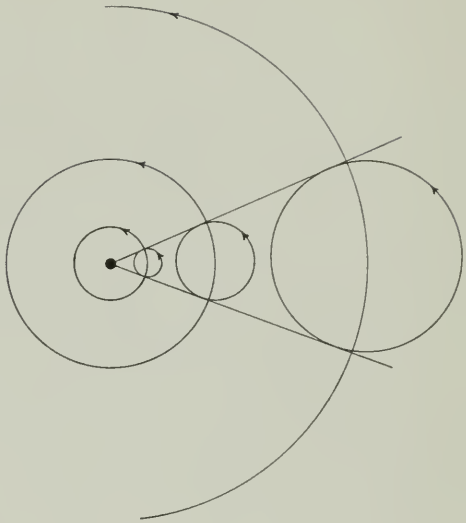
TABLE 6.2

	NUMBER OF YEARS OF OBSERVATION (t)	APPARENT NUMBER OF CYCLES WITH RESPECT TO SUN (n)	APPARENT FREQUENCY f_{pe} IN CYCLES PER YEAR (n/t)	FREQUENCY f_p AROUND SUN IN CYCLES PER YEAR	PERIOD AROUND SUN ($1/f_p$) IN YEARS
Mercury	46	145	3.15	4.15	.241
Venus	8	5	.625	1.625	.614
Mars	79	37	.468	.532	1.88
Jupiter	71	65	.915	.085	11.8
Saturn	59	57	.966	.034	29.4

of deriving the directions to the planets or the angle through which they move.

So in Ptolemy's system, only the *relative* sizes of epicycle and deferent circle were known, separately for each planet. In Copernicus' system, the motions of the sun and five planets that had previously been attributed to one-year epicycles or deferent circles were all replaced by the *single* motion of the earth's yearly revolution *around the sun*. The details of how this can be done are given on pages 36 and 37. Thus, it became possible to compare the radii of the planet's orbit with that of the earth. Because the distances were all compared to the distances between the sun and the earth, the sun-earth distance is conveniently called 1 *astronomical unit*, abbreviated 1 AU.

Table 6.3 below compares Copernicus' values of the orbit radii with the currently accepted values.



In the Ptolemaic system, only the relative size of epicycle and deferent was specified. Then size could be changed at will, so long as they kept the same proportions.

TABLE 6.3

PLANET	RADII OF PLANETARY ORBITS COPERNICUS' VALUES	MODERN VALUE
Mercury	0.38	0.39 AU
Venus	0.72	0.72
Earth	1.00	1.00
Mars	1.52	1.52
Jupiter	5.2	5.20
Saturn	9.2	9.54

Notice that Copernicus now had one system in which the size of each planet's orbit was related to the sizes of all the other planets' orbits. Contrast this to Ptolemy's solutions which were completely independent for each planet. No wonder that Copernicus said, in the quote on p. 31 that: "nothing can be shifted around in any part of them without disrupting the remaining parts and the universe as a whole."

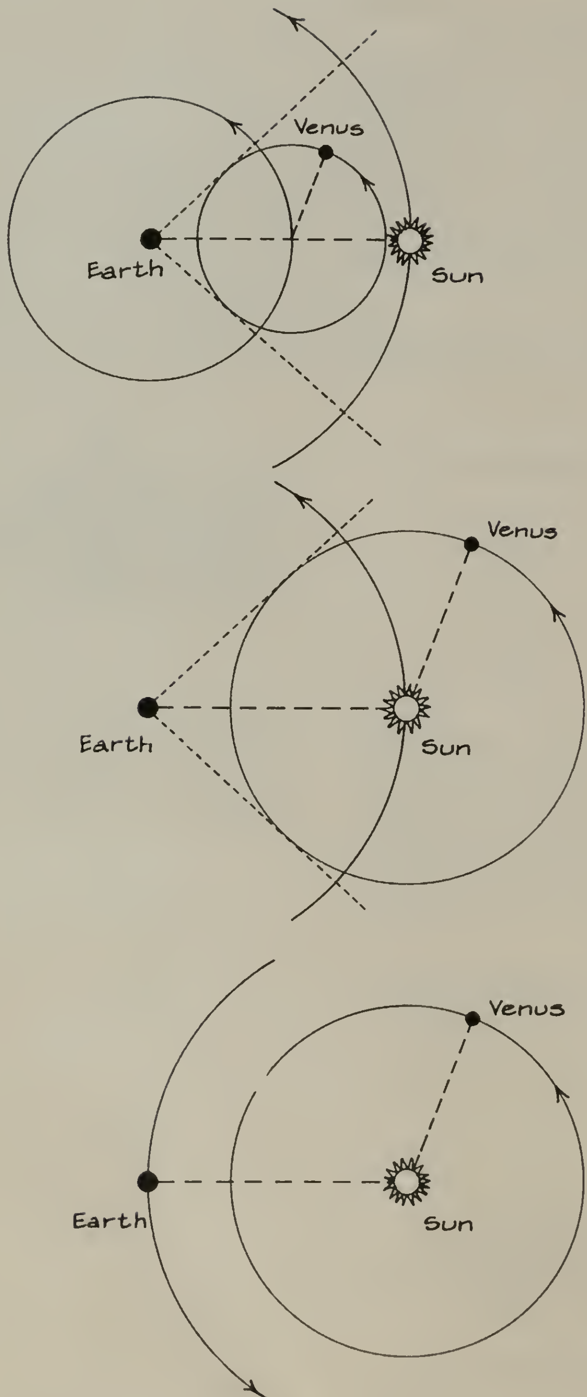
SG 6.6

Q3 What new kinds of results did Copernicus obtain with a moving-earth model which were not possible with a geocentric model for the planetary system?

6.3 Arguments for the Copernican system

Since Copernicus knew that to many his work would seem absurd, "nay, almost contrary to ordinary human understanding," he tried in several ways to meet the old arguments against a moving earth.

1. Copernicus argued that his assumptions agreed with theological dogma at least as well as Ptolemy's. Copernicus' book has many sections on the limitations of the Ptolemaic system (most of which had been known for centuries). Other sections pointed out the harmony of his own system and how well it reflects the thought



Changing Frame of Reference from the Earth to the Sun

The change of viewpoint from Ptolemy's system to Copernicus' involved what today would be called a shift in frame of reference. The apparent motion previously attributed to the deferent circles and epicycles was attributed by Copernicus to the earth's orbit and the planet's orbits around the sun.

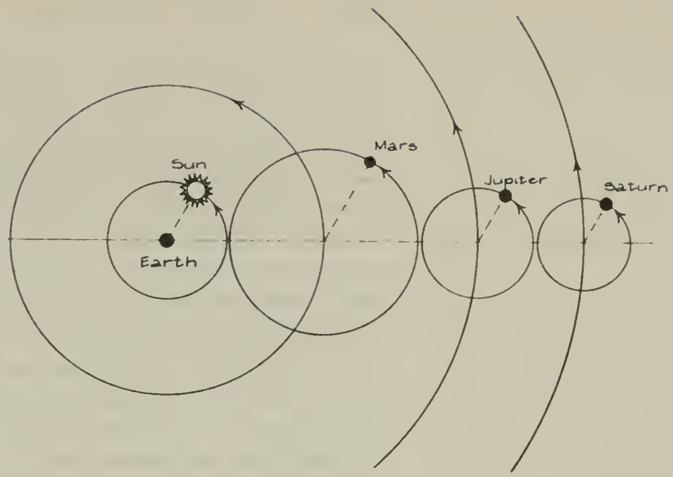
For example, consider the motion of Venus. In Ptolemy's earth-centered system the center of Venus' epicycle was locked to the motion of the sun, as shown in the top diagram at the left. The size of Venus' deferent circle was thought to be smaller than the sun's, and the epicycle was thought to be entirely between the earth and sun. However, the observed motions to be explained by the system *required* only a certain *relative* size of epicycle and deferent. The deferent could be changed to any size, as long as the epicycle was changed proportionally.

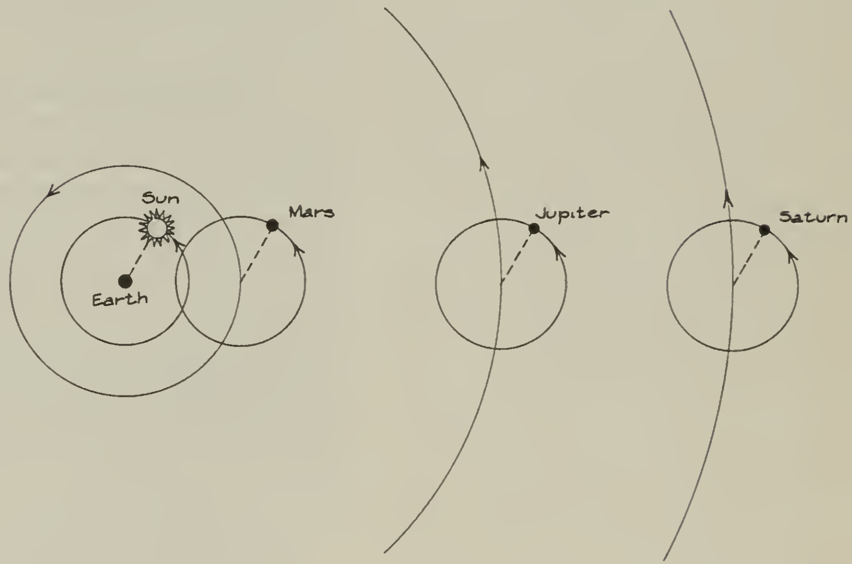
The first step toward a sun-centered system is taken by moving the center of Venus' 1-year deferent out to the sun and enlarging Venus' epicycle proportionally, as shown in the middle diagram at the left. Now the planet moves about the sun, while the sun moves about the earth. Tycho actually proposed such a system with all the observed planets moving about the moving sun.

Copernicus went further and accounted for the relative motion of the earth and sun by considering the earth to be moving around the sun, instead of the sun moving about the earth. In the Copernican system, Venus' epicycle becomes its orbit around the sun and Venus' deferent is replaced by the earth's orbit around the sun, as shown in the bottom diagram at the left. All three systems, Ptolemy's, Tycho's and Copernicus', explain the same observations. For the outer planets the argument is similar, but the roles of epicycle and deferent circle are reversed.

For the outer planets, it was the epicycles instead of the deferent circles which had a 1-year period and which were synchronized with the sun's orbit. The sizes of the deferents were chosen so that the epicycle of each planet would just miss the epicycles of the planets next nearest and next furthest from the sun. (This was a beautiful example of a simplifying assumption—it filled the space with no overlap and no gaps.) This system is represented in the top diagram at the right (in which the planets are shown in the unlikely condition of having their epicycle centers along a single line.)

The first step in shifting to an earth-centered view was to adjust the sizes of the deferent circles, keeping the epicycles in proportion, until the 1-year epicycles were the same size as the sun's 1-year orbit. This adjustment is shown in the middle diagram at the right. Next, the sun's apparent yearly motion around the earth is accounted for just as well by having the earth revolve around the sun. Also, the same earth orbit would account for the retrograde loops associated with all the outer planets' matched 1-year epicycles. So all the synchronized epicycles of the outer planets and the sun's orbit are replaced by the *single* device of the earth's orbit around the sun. This shift is shown in the bottom diagram at the right. The deferent circles of the outer planets became their orbits around the sun.





of the Divine Architect. To Copernicus, as to many scientists, the complex events he saw were merely symbols of God's thinking. To find order and symmetry in the observed changes was to Copernicus an act of piety. To him the symmetry and order were another proof of the existence of the Deity. As an important church dignitary, he would have been appalled if he had been able to foresee that his theory would contribute to the conflict, in Galileo's time, between religious dogma and the interpretations that scientists gave to their experiments.

2. Copernicus' analysis was as thorough as that of Ptolemy. The relative radii and speeds of the circular motions in his system were calculated so that tables of planetary motion could be made. Actually the theories of Ptolemy and Copernicus were about equally accurate in predicting planetary positions, which for both theories often differed from the observed positions by as much as 2° (about four diameters of the moon).

3. Copernicus tried to answer several other objections that were certain to be raised, as they had been against Aristarchus' heliocentric system nearly nineteen centuries earlier. In reply to the argument that the rapidly rotating earth would surely fly apart, he asked, "Why does the defender of the geocentric theory not fear the same fate for his rotating celestial sphere—so much faster because so much larger?" It was argued that birds and clouds in the sky would be left behind by the earth's rotation and revolution. He answered this objection by indicating that the atmosphere is dragged along with the earth. To the lack of observable annual shift for the fixed stars, he could only give the same answer that Aristarchus had proposed, namely:

> . . . though the distance from the sun to the earth appears very large as compared with the size of the spheres of some planets, yet compared with the dimensions of the sphere of the fixed stars, it is as nothing.

4. Copernicus claimed that the greatest advantage of his scheme was its simple description of the general motions of the planets. There certainly is a basic overall simplicity to his system, as is shown in his own diagram on p. 28. (Yet for precise computations, because Copernicus would not use equants, he needed *more* small motions than did Ptolemy to account for the observations. A diagram from Copernicus' manuscript that shows more detail is reproduced on page 42.)

5. Last of all, Copernicus pointed out that the simplicity of his system was not merely convenient, but also beautiful and "pleasing to the mind." It is not often stressed in textbooks that this sort of esthetic pleasure which a scientist finds in his models' simplicity is one of the most powerful experiences in the actual practice of science. Far from being a "cold," merely logical exercise, scientific work is full of such recognitions of harmony and therefore of beauty. One beauty that Copernicus saw in his system was the central place given to the sun, the biggest, brightest object—the giver of light and warmth and life. As Copernicus himself put it:

> In the midst of all, the sun reposes, unmoving. Who, indeed, in this most beautiful temple would place the light-giver in any other part than whence it can illumine all other parts? So we find underlying this ordination an admirable symmetry in the Universe and a clear bond of the harmony in the motion and magnitude of the spheres, such as can be discovered in no other wise.

Look again at SG 6.2

Q4 Which of these arguments did Copernicus use in favor of his system?

(a) it was obvious to ordinary common sense
(b) it was consistent with Christian theology
(c) it was much more accurate in predicting planet positions
(d) its simplicity made it beautiful
(e) the stars showed an annual shift in position due to the earth's motion around the sun

Q5 What were the largest differences between observed planetary positions and those predicted by Ptolemy? by Copernicus?

Q6 Did the Copernican system allow simple calculations of where the planets should be seen?

6.4 Arguments against the Copernican system

Copernicus' hopes for acceptance of his theory were not quickly fulfilled. More than a hundred years passed before the heliocentric system was generally accepted even by astronomers—and then, as we shall see, the acceptance came on the basis of arguments quite different from those of Copernicus. In the meantime the theory and its few champions met powerful opposition. Most of the arguments were the same as those used by Ptolemy against the heliocentric system of Aristarchus.

1. Apart from its apparent simplicity, the Copernican system had no clear *scientific* advantages over the geocentric theory. There was no known observation that was explained by one system and not by the other. Copernicus had a different viewpoint but no new types of observations, no experimental data that could not be explained by the old theory. Furthermore, the accuracy of his predictions of planetary positions was little better than that by Ptolemy. As Francis Bacon wrote in the early seventeenth century: "Now it is easy to see that both they who think the earth revolves and they who hold the old construction are about equally and indifferently supported by the phenomena."

Basically, the rival systems differed in their choice of reference systems used to describe the observed motions. Copernicus himself stated the problem clearly:

> Although there are so many authorities for saying that the Earth rests in the centre of the world that people think the contrary supposition . . . ridiculous; . . . if, however, we consider the thing attentively, we will see

> that the question has not yet been decided and accordingly is by no means to be scorned. For every apparent change in place occurs on account of the movement either of the thing seen or of the spectator, or on account of the necessarily unequal movement of both. For no movement is perceptible relatively to things moved equally in the same directions—I mean relatively to the thing seen and the spectator. Now it is from the Earth that the celestial circuit is beheld and presented to our sight. Therefore, if some movement should belong to the Earth . . . it will appear, in the parts of the universe which are outside, as the same movement but in the opposite direction, as though the things outside were passing over. And the daily revolution . . . is such a movement.

Ptolemy too had recognized the possibility of alternative frames of reference. (Reread the quotation on p. 22 in Ch. 5.) Most of Ptolemy's followers did not share this insight

In this statement Copernicus invites the reader to shift the frame of reference from that of an observer on the earth to one at a remote position looking upon the whole system with the sun at the center. As you may know from personal experience, such a shift is not easy, and we can sympthize with those who preferred to hold to an earth-centered system for describing what they saw.

Physicists now generally agree that *all* systems of reference are in principle equally valid for describing phenomena, although some will be easier and others more complex to use or think about. Copernicus and those who followed him felt that the heliocentric system was right in some absolute sense—that the sun was really fixed in space, and the same claim was made for the earth by his opponents. But the modern attitude is that the choice of a frame of reference depends mainly on which frame will allow the simplest discussion of the problem being studied. We should not speak of reference systems as being right or wrong, but rather as being convenient or inconvenient. (To this day, navigators use a geocentric model for their calculations; see the page of a navigation book shown on p. 41.) Yet even if it is recognized that different frames of reference are possible mathematically, a reference system that is acceptable to one person may involve philosophical assumptions that are unacceptable to another.

2. The lack of an observable annual shift for the fixed stars spoke against Copernicus' model. His only possible reply was unacceptable because it meant that the stars were at an enormous distance away from the earth. The naked-eye instruments allowed positions in the sky to be measured to a precision of about 1/10°; for an annual shift to be less than 1/10°, the stars would have to be more than 1000 times further from the sun than the earth is! To us this is no shock, because we have been raised in a society that accepts the idea of enormous extensions in space and in time. Even so, such distances do strain our imagination. To the opponents of Copernicus such distances were absurd. Indeed we may well speculate that even if an annual shift in star position had then been observable, it would not have been taken as unmistakable evidence against one and for the other theory. One can usually modify a

SG 6.8

SG 6.7

theory—more or less pleasingly—to accomodate a bothersome finding.

The Copernican system led to other conclusions that were also puzzling and threatening. Copernicus determined the actual distances between the sun and the planetary orbits. Perhaps, then, the Copernican system was not just a mathematical procedure for predicting the observable positions of the planets! Perhaps Copernicus was describing a real system of planetary orbits in space (as he thought he was). But this would be difficult to accept, for the orbits were far apart. Even the small epicycles which Copernicus still needed to account for the variations in the motions did not fill up the spaces between the planets. Then what did fill up these spaces? Because Aristotle had stated that "nature abhors a vacuum," it was agreed there had to be something in all that space. As you might expect, even many of those who believed Copernicus' system felt that space should be full of something and invented various sorts of invisible fluids and ethers to fill up the emptiness. More recently, similar fluids were used in theories of chemistry and of heat, light, and electricity, as you will see in later units.

3. Since no definite decision between the Ptolemaic and the Copernican theories could be made on the astronomical evidence, attention focused on the argument concerning the central, immovable position of the earth. Despite his efforts, Copernicus was unable to persuade most of his readers that his heliocentric system reflected the mind of God at least as closely as did the geocentric system. All religious faiths in Europe, including the new Protestants, found enough biblical quotations (for example, *Joshua* 10:12-13) to assert that the Divine Architect must have worked from a Ptolemaic blueprint. Indeed, Martin Luther called Copernicus "the fool who would overturn the whole science of astronomy."

Eventually, in 1616, when storm clouds were raised by the case of Galileo, the Papacy put *De Revolutionibus* on the *Index* of forbidden books as "false and altogether opposed to Holy Scriptures." Some Jewish communities also prohibited the teaching of his theory. It seems that man, believing himself central to God's plan, had to insist that his earth be the center of the physical universe.

The assumption that the earth was not the center of the universe was offensive enough. But worse, the Copernican system suggested that the other planets were similar to the earth. Thus, the concept of the distinctly different heavenly matter was threatened; who knew but what some rash person might next suggest that the sun and possibly even the stars were made of earthly materials? If the other celestial bodies, either in our solar system or beyond, were similar to the earth, they might even be inhabited. And the inhabitants might be heathens, or beings as well-beloved by God as man is, or possibly even more beloved! Thus, the whole Copernican scheme led to profound philosophical questions which the Ptolemaic scheme avoided.

4. The Copernican theory conflicted with the basic propositions

CELESTIAL OBSERVATIONS

By PHILIP KISSAM, C. E.
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I. The Principles upon which Celestial Observations are Based.

A. CONCEPTS.

1. The Celestial Sphere. To simplify the computations necessary for the determinations of the direction of the meridian, of latitude, and of longitude or time, certain concepts of the heavens have been generally adopted. They are the following:

a. The earth is stationary.

b. The heavenly bodies have been projected outward, along lines which extend from the center of the earth, to a sphere of infinite radius called the ***celestial sphere***.

The celestial sphere has the following characteristics:

a. Its center is at the center of the earth.

b. Its equator is on the projection of the earth's equator.

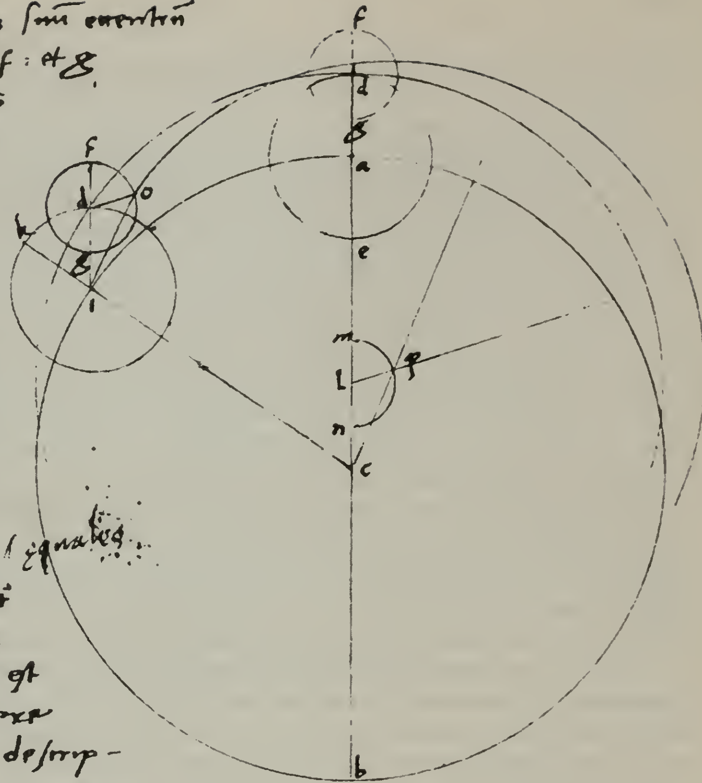
c. With respect to the earth, the celestial sphere rotates from east to west about a line which coincides with the earth's axis. Accordingly, the poles of the celestial sphere are at the prolongations of the earth's poles.

d. The speed of rotation of the celestial sphere is 360° 59.15′ per 24 hours.

e. With the important exception of bodies in the solar system, which change position slowly, all heavenly bodies remain practically fixed in their positions on the celestial sphere, never changing more than negligible amounts in 24 hours, and accordingly are often called *fixed stars*.

Celestial navigation involves comparing the apparent position of the sun (or star) with the "actual" position as given in a table called an "ephemeris." Above is an excerpt from the introduction to the tables in the *Solar Ephemeris* for 1950. (Keuffel and Esser Co.)

que ipsi d i fuerit aequalis. f quoque sive excentrum
secundum distantiam c l m aequalem ipsi i d f : et g
similiter secundum i g et c n distantias
aequales. Interea si centrum terrae
iam emensum fuerit utrumque
f o circumferentia scilicet ac sui
epicycli, iam ipsum o non describit
excentrum q cuius centrum in a c
linea contingat: sed in ea que
ipsi d o parallelus fuerit qualis
est l p. Quod si etiam coniungantur
o i et c p erunt et ipse aequales
minores ante ipsis i f et c m et
angulus d i o angulo l c p per 4   aequales.
primi Euclid.: et pro tanto videbitur
Solis apogeum in c p linea procedere
ipsum a. Hinc etiam manifestum est
per excentrepicyclum idem contingere
f Solo   Quoniam in persistente excentro quem descrip-
serit d epicyclum circa l centrum, centrum terrae
volvatur in f o circumferentia predictis revolutionibus: hoc est
plus modico quam fuerit annua revolutio. Superinducet enim ~~et~~ quae annua
alterum excentrum priori circa p centrum: accidentque prorsus
eadem. Cumque tot modi ad eundem numerum sese conferrant
quis locum habeat haut facile dixerim: nisi quod illa nu-
merorum ac apparentium perpetua consonantia credere cogit
eorum esse aliquem



Quanta sit secunda Solaris inaequalitatis differentia Cap. xx

Cum igitur iam visum fuerit: quod ista secunda inaequalitas
prima ac simplicem illam anomaliam obliquitatis signiferi vel
eius similitudinem sequeretur: certas habebimus eius diffe-
rentias, si non obstiterit error aliquis observatorum preteritorum
Habemus enim ipsam simplicem anomaliam anno Christi MD~~xv~~
secundum numerationem grad. clxv ~~~~ fere : et eius   xxxix
principium facta retrorsum supputatione ~~sexaginta~~ lxiiii fere
annis ante Christum natum, ~~et~~ colliguntur anni MDlxxx a quo ~~principio~~ tempore ad nos usque
... aut ~~a quo~~ principio inventa excentrotes maxima ~~partium 4 1 6~~ 417
quarum quae ex centro orbis esset 10000: nunc vero, ut
a nobis

of Aristotelian physics. This conflict is well described by H. Butterfield in *Origins of Modern Science*:

> . . . at least some of the economy of the Copernican system is rather an optical illusion of more recent centuries. We nowadays may say that it requires smaller effort to move the earth round upon its axis than to swing the whole universe in a twenty-four hour revolution about the earth; but in the Aristotelian physics it required something colossal to shift the heavy and sluggish earth, while all the skies were made of a subtle substance that was supposed to have no weight, and they were comparatively easy to turn, since turning was concordant with their nature. Above all, if you grant Copernicus a certain advantage in respect of geometrical simplicity, the sacrifice that had to be made for the sake of this was tremendous. You lost the whole cosmology associated with Aristotelianism—the whole intricately dovetailed system in which the nobility of the various elements and the hierarchical arrangement of these had been so beautifully interlocked. In fact, you had to throw overboard the very framework of existing science, and it was here that Copernicus clearly failed to discover a satisfactory alternative. He provided a neater geometry of the heavens, but it was one which made nonsense of the reasons and explanations that had previously been given to account for the movements in the sky.

In short, although the sun-centered Copernican scheme was scientifically equivalent to the Ptolemaic in explaining the astronomical observations, to abandon the geocentric hypothesis seemed philosophically false and absurd, dangerous, and fantastic. Most learned Europeans at that time recognized the Bible and the writings of Aristotle as their two supreme sources of authority. Both appeared to be challenged by the Copernican system. Although the freedom of thought that marked the Renaissance was beginning, the old image of the universe provided security and stability to many. So, to believe in a sun-centered rather than an earth-centered universe in Copernicus' time required that a partial gain in simplicity be considered more important than common sense and observation, the teaching of philosophy and religion, and physical science. No wonder Copernicus had so few believers!

Similar conflicts between the assumptions underlying accepted beliefs and the philosophical content of new scientific theories have occurred many times, and are bound to occur again. During the last century there were at least two such conflicts. Neither is completely resolved today. In biology, the evolutionary theory based on Darwin's work has caused major philosophical and religious reactions. In physics, as Units 4, 5, and 6 indicate, evolving theories of atoms, relativity, and quantum mechanics have challenged other long-held assumptions about the nature of the world and our knowledge of reality. As the dispute between Copernicans and Ptolemaists illustrates, the assumptions which common sense holds

Opposite: A page from Copernicus' manuscript of *De Revolutionibus*, showing detail of some epicycles in his model.

SG 6.9 so dearly and defends so fiercely are often only the remnants of an earlier, less complete scientific theory.

Q7 Why were many people, such as Francis Bacon, undecided about the correctness of the Ptolemaic and Copernican systems?

Q8 How did the astronomical argument become involved with religious beliefs?

Q9 From a modern viewpoint, was the Ptolemaic or the Copernican system of reference more valid?

6.5 Historical consequences

Eventually the moving-earth model of Copernicus was accepted. The slowness with which that acceptance came is illustrated by a passage in the published diary of John Adams (who later became the second president of the United States). He wrote that he attended a lecture at Harvard College in which the correctness of the Copernican viewpoint was argued—on June 19, 1753.

Soon we shall follow the work which gradually led to the general acceptance of the heliocentric viewpoint—a heliocentric viewpoint without, however, the detailed Copernican system of uniform circular motions with eccentrics and epicycles. We shall see that the real scientific significance of Copernicus' work lies in the fact that a heliocentric view opened a new approach to understanding planetary motion. This new way became dynamic, rather than just kinematic—it involved the laws relating force and motion, developed in the 150 years after Copernicus, and the application of these laws to motions in the heavens.

The Copernican model with moving earth and fixed sun opened a floodgate of new possibilities for analysis and description. According to this model the planets could be considered as real bodies moving along actual orbits. Now Kepler and others could consider these planetary paths in quite new ways. In science, the sweep of possibilities usually cannot be foreseen by those who begin the revolution—or by their critics.

Today the memory of Copernicus is honored not so much because of the details of his theory, but because he challenged the prevailing world-picture, and because his theory became a principal force in the intellectual revolution which shook man out of his self-centered view of the universe. As men gradually accepted the Copernican system, they necessarily found themselves accepting the view that the earth was only one among several planets circling the sun. Thus, it became increasingly difficult to assume that all creation centered on mankind. At the same time, the new system
SG 6.10 stimulated a new self-reliance and curiosity about the world.

Acceptance of a revolutionary idea based on quite new assumptions, such as Copernicus' shift of the frame of reference, is always slow. Sometimes compromise theories are proposed as attempts to unite two conflicting alternatives, to "split the difference." As you will see in later units, such compromises are rarely successful. Often the new ideas do stimulate new observations and concepts that in turn may lead to a very useful development or restatement of the original revolutionary theory.

Such a restatement of the heliocentric theory came during the 150 years after Copernicus. Many men provided observations and ideas, and in Chapters 7 and 8 we will follow the major contributions made by Kepler, Galileo, and Isaac Newton. But first, we will consider here the work of Tycho Brahe, who devoted his life to improvements in the precision with which planetary positions were observed and to the proposal of a compromise theory of planetary motion.

Q10 In terms of our historical perspective, what were the greatest contributions of Copernicus to modern planetary theory?

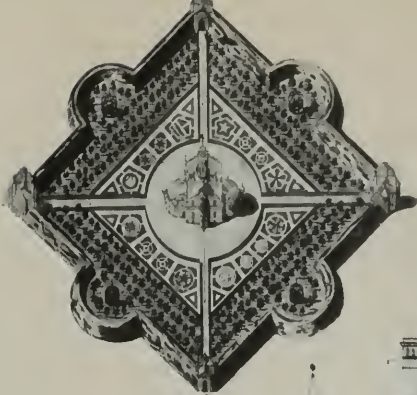
6.6 Tycho Brahe

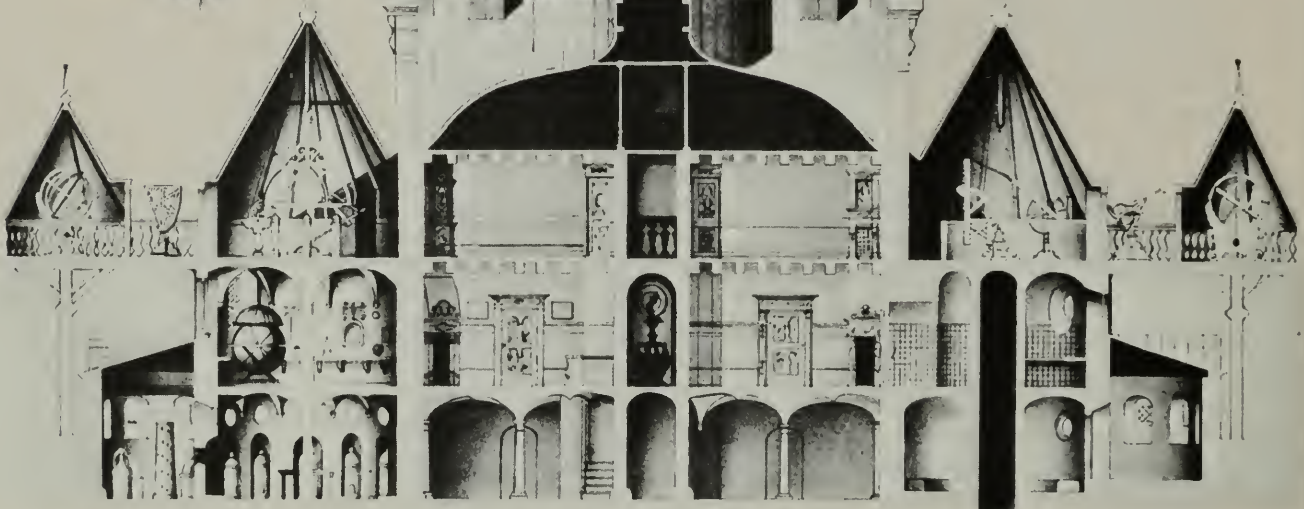
Tycho Brahe was born in 1546 of a noble, but not particularly rich, Danish family. By the time Tycho was thirteen or fourteen, he had become intensely interested in astronomy. Although he was studying law, he secretly spent his allowance money on astronomical tables and books such as the *Almagest* and *De Revolutionibus*. Soon he discovered that both Ptolemy and Copernicus had relied upon tables of planetary positions that were inaccurate. He concluded that before a satisfactory theory of planetary motion could be created, new astronomical observations of the highest possible accuracy, gathered over many years, would be necessary.

Tycho's interest in studying the heavens was increased by an exciting observation in 1572. Although the ancients had taught that the stars were unchanging, Tycho observed a "new star" in the constellation Cassiopeia. It soon became as bright as Venus and could be seen even during the daytime. Then over several years it faded until it was no longer visible. To Tycho these events were astonishing—changes in the starry sky! Evidently at least one assumption of the ancients was wrong. Perhaps other assumptions were wrong, too.

After observing and writing about the new star, Tycho traveled through northern Europe where he met many other astronomers and collected books. Apparently he was considering moving to Germany or Switzerland where he could easily meet other astronomers. To keep the young scientist in Denmark, King Frederick II made Tycho an offer that was too attractive to turn down. Tycho was given an entire small island and also the income derived from various farms to allow him to build an observatory on the island and to staff and maintain it. The offer was accepted, and in a few years Uraniborg ("Castle of the Heavens") was built. It was a large structure, having four large observatories, a library, a laboratory, shops, and living quarters for staff, students, and observers. There was even a complete printing plant. Tycho estimated that the observatory cost Frederick II more than a ton of gold. For that time in history this magnificent laboratory was at

Although there were precision sighting instruments, all observations were with the naked eye—the telescope was not to be invented for another 50 years





At the top left is a plan of the observatory and gardens built for Tycho Brahe at Uraniborg, Denmark.

The cross section of the observatory, above center, shows where most of the important instruments including large models of the celestial spheres were housed.

The picture at the left shows the room containing Tycho's great quadrant. On the walls are pictures of some of his instruments. He is making an observation, aided by assistants.

Above is a portrait of Tycho, painted about 1597.

The bright comet of 1965.

least as significant, complex, and expensive as some of the great research establishments of our own time. Primarily a research center, Uraniborg was a place where scientists, technicians, and students from many lands could gather to study astronomy. Here was a unity of action, a group effort under the leadership of an imaginative scientist to advance the boundaries of knowledge in one science.

In 1577 Tycho observed a bright comet, a fuzzy object whose motion across the sky seemed to be erratic, unlike the orderly motions of the planets. To find the distance to the comet, Tycho compared its position as observed from Denmark with its positions as observed from elsewhere in Europe. At any given time, the comet was found to have the same position with respect to the stars, even though the observing places were many hundreds of miles apart. By contrast, the moon's position in the sky was measurably different when observed from places so far apart. Therefore, Tycho concluded, the comet must be at least several times farther away than the moon.

Two articles on comets appear in *Reader 2:* "The Great Comet of 1965" and "The Boy Who Redeemed His Father's Name."

This was an important conclusion. Up to that time comets had been believed to be some sort of local event, like clouds or lightning, rather than something in the realm of eternal things beyond the moon. Now comets had to be considered distant astronomical objects which seemed to move right through the crystalline spheres that were still generally believed to carry the planets. Tycho's book on this comet was widely read and helped to undermine belief in the old assumptions about the nature of the heavens.

SG 6.11

Q11 What event stimulated Tycho's interest in astronomy?

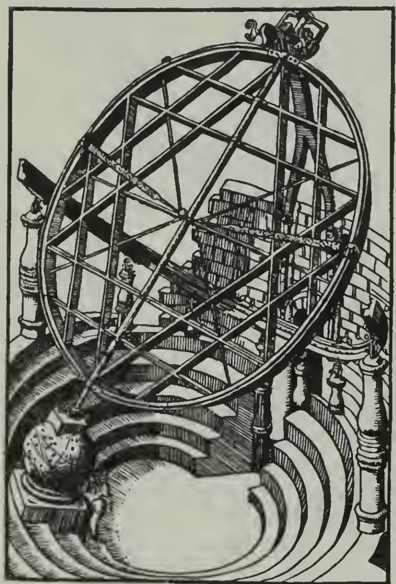
Q12 In what ways was Tycho's observatory like a modern research institute?

Q13 Why were Tycho's conclusions about the comet of 1577 important?

6.7 Tycho's observations

Tycho's fame results from his lifelong devotion to making unusually accurate observations of the positions of the stars, sun, moon, and planets. He did this before the telescope was invented. Over the centuries many talented observers had been recording the positions of the celestial objects, but the accuracy of Tycho's work was much greater than that of the best astronomers before him. How was Tycho Brahe able to do what no others had done before?

For a more modern example of this same problem of instrumentation, you may wish to read about the development and construction of the 200-inch Hale telescope on Mt. Palomar. Also see "A Night at the Observatory" in *Reader 2*.



One of Tycho's sighting devices. Unfortunately Tycho's instruments were destroyed in 1619 during the Thirty Years War.

Singleness of purpose was certainly one of Tycho's assets. He knew that highly precise observations must be made during many years. For this he needed improved instruments that would give consistent readings. Fortunately he possessed both the mechanical ingenuity to devise such instruments and the funds to pay for their construction and use.

Tycho's first improvement on the astronomical instruments of the day was to make them larger. Most of the earlier instruments had been rather small, of a size that could be moved by one person. In comparison, Tycho's instruments were gigantic. For instance, one of his early devices for measuring the angular altitude of planets (shown in the etching in the margin) had a radius of about six feet. This wooden instrument was so large that it took several men to set it into position. Tycho also put his instruments on heavy, firm foundations or else attached them to a wall that ran exactly north-south. By increasing the stability of the instruments, Tycho increased the reliability of the readings over long periods of time. Thoughout his career Tycho also created better sighting devices, more precise scales, and stronger support systems and made dozens of other changes in design which increased the precision of the observations.

Apparent distortion of the setting sun. The light's path through the earth's atmosphere is bent, making the sun appear flattened and rough-edged.

Not only did Tycho devise better instruments for making his observations, but he also determined and specified the actual limits of precision of each instrument. He realized that merely making larger and larger instruments does not always result in greater precision; ultimately, the very size of the instrument introduces errors, since the parts will bend under their own weight. Tycho therefore tried to make his instruments as large and strong as he could without at the same time introducing errors due to bending. Furthermore, in modern style, Tycho calibrated each instrument and determined its range of error. (Nowadays many commercially available scientific instruments designed for precision work are accompanied by a measurement report, usually in the form of a table, of corrections to be applied to the readings.)

Refraction, or bending, of light from a star by the earth's atmosphere. The amount of refraction shown in the figure is greatly exaggerated over what actually occurs.

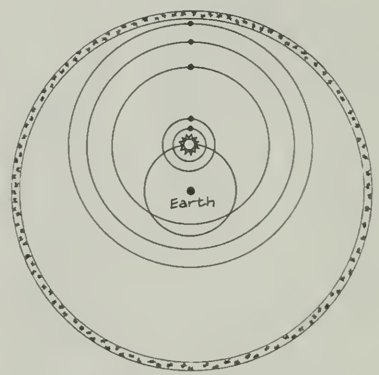
Like Ptolemy and the Muslim astronomical observers, Tycho knew that the light coming from each celestial body was bent downward by the earth's atmosphere and increasingly so as the object neared the horizon. To increase the precision of his observations, Tycho carefully determined the amount of refraction so that each observation could be corrected for refraction effects. Such careful work was essential if improved records were to be made.

Tycho worked at Uraniborg from 1576 to 1597. After the death of King Frederick II, the Danish government became less interested in helping to pay the cost of Tycho's observatory. Yet Tycho was unwilling to consider any reduction in the costs of his activities. Because he was promised support by Emperor Rudolph of Bohemia, Tycho moved his records and several instruments to Prague. There, fortunately, he hired as an assistant an able, imaginative young man named Johannes Kepler. When Tycho died in 1601, Kepler obtained all his records of observations of the motion of Mars. As Chapter 7 reports, Kepler's analysis of Tycho's observations solved many of the ancient problems of planetary motion.

Q14 What improvements did Tycho make in astronomical instruments?

Q15 In what way did Tycho correct his observations to provide records of higher accuracy?

6.8 Tycho's compromise system



Main spheres in Tycho Brahe's system of the universe. The earth was fixed and was at the center of the universe. The planets revolved around the sun, while the sun, in turn, revolved around the fixed earth.

Tycho's observations were intended to provide a basis for a new theory of planetary motion which he had outlined in an early publication. Tycho saw the simplicity of the Copernican system by which the planets moved around the sun, but he could not accept the idea that the earth had any motion. In Tycho's system, all the planets except the earth moved around the sun, but the sun moved around the stationary earth, as shown in the sketch in the margin. Thus he devised a compromise model which, as he said, included the best features of both the Ptolemaic and the Copernican systems.



Buildings and instruments of modern observatories. Top: the 200-inch telescope and its dome on Mt. Palomar. Bottom: the complex of buildings on Mt. Wilson.

However, he did not live to publish quantitative details of his theory.

The compromise Tychonic system was accepted by only a few people. Those who accepted the Ptolemaic model objected to Tycho's proposal because he had the planets move around the sun. Those who accepted the Copernican model objected to having the earth held stationary. So the argument continued between those holding the seemingly self-evident position that the earth was stationary and those who accepted, at least tentatively, the strange, exciting proposals of Copernicus that the earth might rotate and revolve around the sun. The choice for one or the other was based on philosophical or esthetic preferences, for each of the three systems could account about equally well for the observational evidence.

All planetary theories up to that time had been developed only to provide some system by which the positions of the planets could be predicted fairly precisely. In the terms used in Unit 1, these would be called kinematic descriptions. The causes of the motions—what we now call dynamics—had not been considered in any detail. The angular motions of objects in the heavens were, as Aristotle said and everyone (including Ptolemy, Copernicus, and Tycho) agreed, "natural"; the heavens were still considered to be completely different from earthly materials and to behave in quite different ways. That a single theory of dynamics could describe both earthly and heavenly motions was a revolutionary idea yet to be proposed.

As long as there was no explanation of the causes of motion, there remained a question of whether the orbits proposed for the planets in the various systems were actual paths of real objects in space or only convenient imaginary devices for making computations. The status of the problem in the early part of the seventeenth century was later described well by the English poet John Milton in *Paradise Lost*:

> . . . He his fabric of the Heavens
> Hath left to their disputes, perhaps to move
> His laughter at their quaint opinions wide
> Hereafter, when they come to model Heaven
> And calculate the stars, how they will wield
> The mighty frame, how build, unbuild, contrive
> To save appearances, how gird the sphere
> With centric and eccentric scribbled o'er
> Cycle and epicycle, orb in orb.

You will see that the eventual success of Newton's universal dynamics led to the belief, one which was held confidently for about two centuries, that scientists were describing the "real world." Later chapters of this text that deal with recent discoveries and theories will indicate that today scientists and philosophers are much less certain that the common-sense notion of "reality" is so useful in science.

SG 6.12

Q16 In what ways did Tycho's system for planetary motions resemble the Ptolemaic and the Copernican systems?

6.1 The Project Physics learning materials particularly appropriate for Chapter 6 include the following:

Experiments
The Shape of the Earth's Orbit
Using Lenses to Make a Telescope

Activities
Two Activities on Frames of Reference

Reader Articles
The Boy Who Redeemed His Father's Name
The Great Comet of 1965
A Night at the Observatory

Film Loop
Retrograde Motion—Heliocentric Model

6.2 The first diagram on the next page shows numbered positions of the sun and Mars (on its epicycle) at equal time intervals in their motion around the earth, as described in the Ptolemaic system. You can easily redraw the relative positions to change from the earth's frame of reference to the sun's. Mark a sun-sized circle in the middle of a thin piece of paper; this will be a frame of reference centered on the sun. Place the circle over each successive position of the sun, and trace the corresponding numbered position of Mars and the position of the earth. (Be sure to keep the piece of paper straight.) When you have done this for all 15 positions, you will have a diagram of the motions of Mars and the earth as seen in the sun's frame of reference.

6.3 What reasons did Copernicus give for believing that the sun is fixed at or near the center of the planetary system?

6.4 Consider the short and long hands of a clock or watch. If, starting from 12:00 o'clock, you rode on the slow short hand, how many times in 12 hours would the long hand pass you? If you are not certain, slowly turn the hands of a clock or watch, and keep count. From this information, can you derive a relation by which you could conclude that the period of the long hand around the center was one hour?

6.5 The diagram at the upper right section of the next page shows the motions of Mercury and Venus east and west of the sun as seen from the earth during 1966-1967. The time scale is indicated at 10-day intervals along the central line of the sun's position.

(a) Can you explain why Mercury and Venus appear to move from farthest east to farthest west more quickly than from farthest west to farthest east?
(b) From this diagram can you find a period for Mercury's apparent position in the sky relative to the sun?
(c) Can you derive a period for Mercury's actual orbital motion around the sun?
(d) What are the major sources of uncertainty in the results you derived?
(e) Similarly, can you estimate the orbital period of Venus?

6.6 From the sequence of orbital radii from Mercury to Saturn, guess what the orbital radius would be for a new planet if one were discovered. What is the basis for your guess?

6.7 If you have had some trigonometry, try this problem: the largest observed annual shift in star position is about 1/2400 of a degree. What is the distance (in AU's) to this closest star?

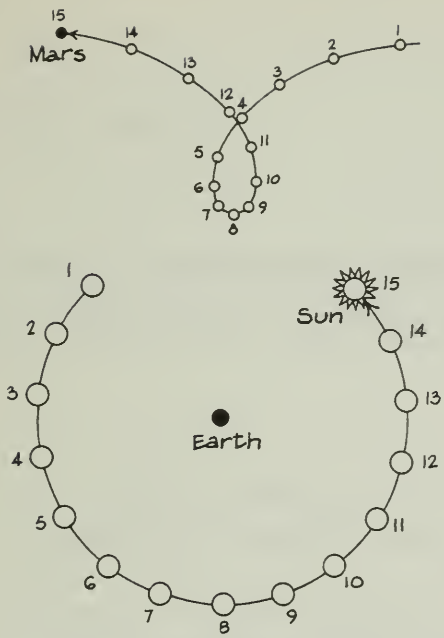
6.8 How might a Ptolemaic astronomer have modified the geocentric system to account for observed stellar parallax?

6.9 Do you know of any conflicts between scientific theories and common sense today?

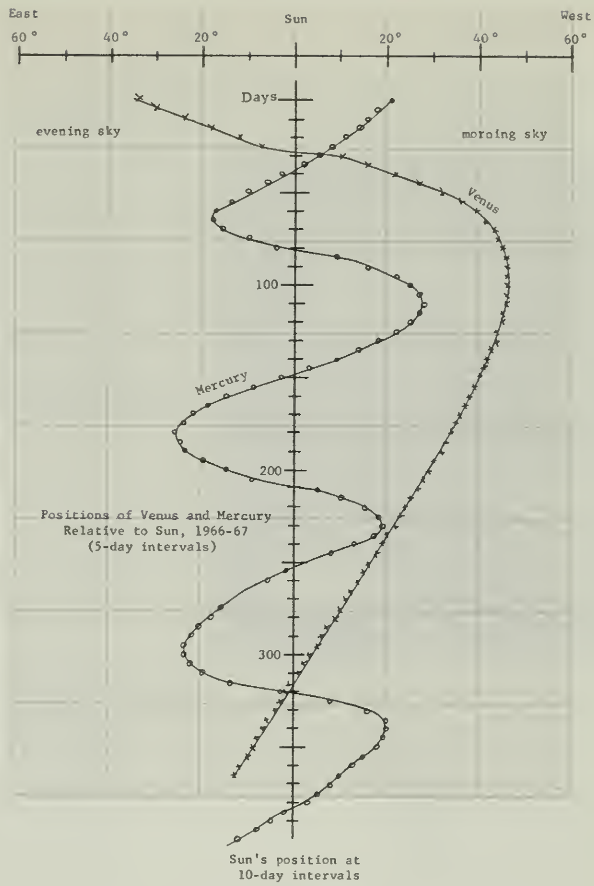
6.10 How did the Copernican system encourage the suspicion that there might be life on objects other than the earth? Is such a possibility seriously considered today? What important kinds of questions would such a possibility raise?

6.11 How can you explain the observed motion of Halley's comet during 1909-1910, as shown on the star chart on the next page?

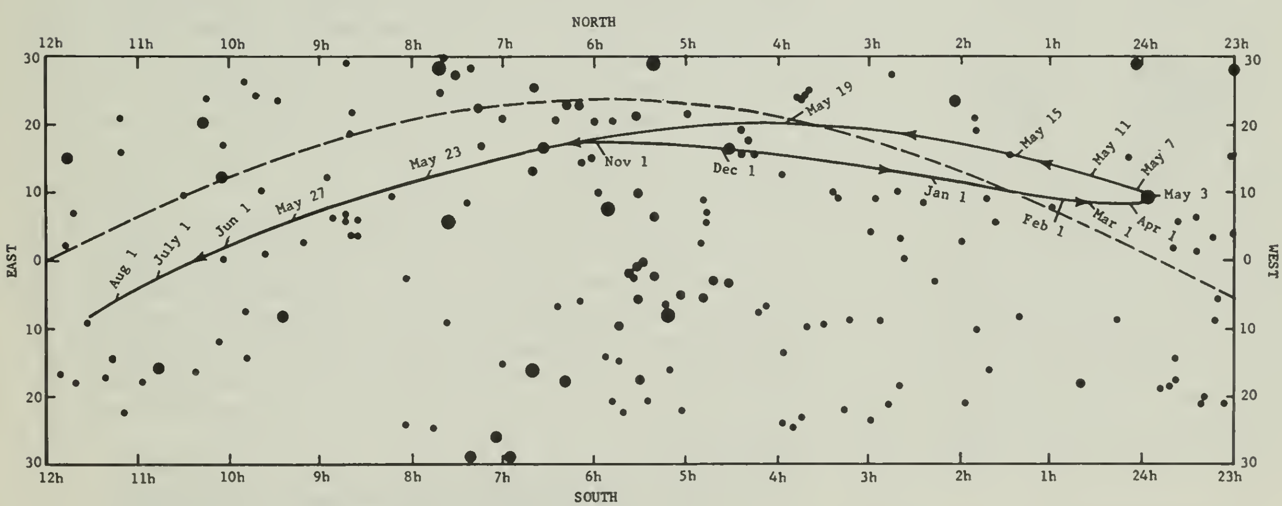
6.12 To what extent do you feel that the Copernican system, with its many motions in eccentrics and epicycles, reveals real paths in space, rather than being only another way of computing planetary positions?



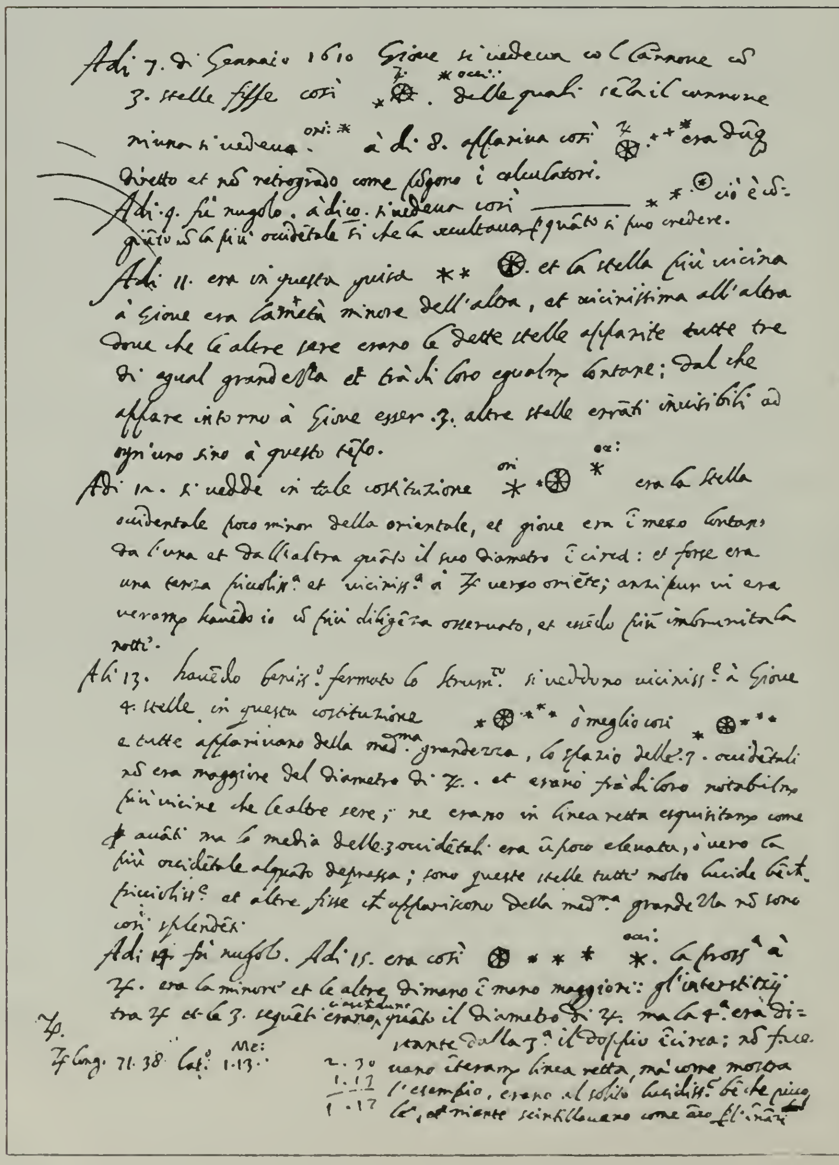
Apparent motion of Mars and Sun around the earth.



Positions of Venus and Mercury relative to Sun.



Observed motion of Halley's comet during 1909-1910.



Galileo's notes on his observations beginning "on day 7 of January 1610."

CHAPTER SEVEN

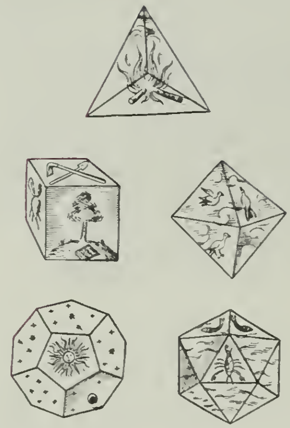
A New Universe Appears – The Work of Kepler and Galileo

7.1 The abandonment of uniform circular motion

Kepler's lifelong desire was to perfect the heliocentric theory. He viewed the harmony and simplicity of that theory with "incredible and ravishing delight." To Kepler, such patterns of geometric order and numerical relation were clues to God's mind. To unfold these patterns further through the heliocentric theory, Kepler attempted in his first major work to explain the spacing of the planetary orbits, as calculated by Copernicus (p. 35 in Chapter 6).

SG 7.1

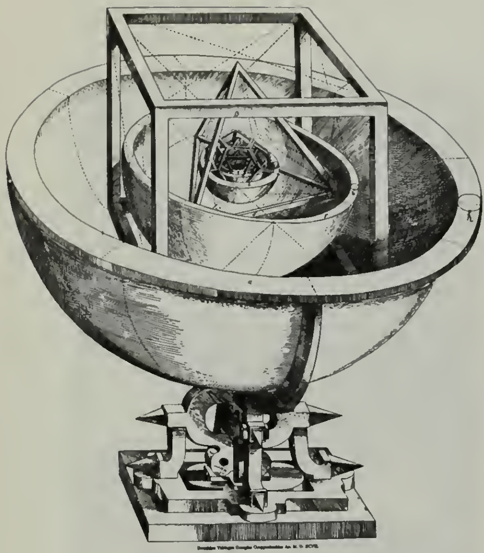
Kepler was searching for the reasons just six planets (including the earth) were visible and were spaced as they are. These are excellent scientific questions, though even today too difficult to answer. Kepler felt the key lay in geometry, and he began to wonder whether there was any relation between the six known planets and the five "regular solids." A regular solid is a polyhedron whose faces all have equal sides and angles. From the time of the Greeks, it was known that there were just five regular geometrical solids. Kepler imagined a model in which these five regular solids could be nested, one inside the other, somewhat like a set of mixing bowls. Between the five solids would be spaces for four planetary spheres. A fifth sphere could be inside the whole nest and a sixth sphere could be around the outside. Kepler then sought some sequence of the five solids; just touching the spheres, that would space the spheres at the same relative distances from the center as were the planetary orbits. Kepler said:



The five "perfect solids" taken from Kepler's *Harmonices Mundi* (Harmony of the World). The cube is a regular solid with six square faces. The dodecahedron has twelve five-sided faces. The other three regular solids have faces which are equilateral triangles: the tetrahedron has four triangular faces, the octahedron has eight triangular faces, and the icosahedron has twenty triangular faces.

> I took the dimensions of the planetary orbits according to the astronomy of Copernicus, who makes the sun immobile in the center, and the earth movable both around the sun and upon its own axis; and I showed that the differences of their orbits corresponded to the five regular Pythagorean figures. . . .

For Kepler, this geometric view was related to ideas of harmony. (See "Kepler's Celestial Music" in *Reader 2*.)



A model of Kepler's explanation of the spacing of the planetary orbits by means of the regular geometrical solids. Notice that the planetary spheres were thick enough to include the small epicycle used by Copernicus.

By trial and error Kepler found a way to arrange the solids so that the spheres fit within about five percent of the actual planetary distances. To Kepler this remarkable (but as we now know, entirely accidental) arrangement explained both the spacings of the planets and the fact that there were just six. Also it indicated the unity he expected between geometry and scientific observations. Kepler's results, published in 1597, demonstrated his imagination and computational ability. Furthermore, it brought him to the attention of major scientists such as Galileo and Tycho. As a result, Kepler was invited to become one of Tycho's assistants at his new observatory in Prague in 1600.

There Kepler was given the task of determining in precise detail the orbit of Mars. This unusually difficult problem had not been solved by Tycho and his other assistants. As it turned out, the investigation of the motion of Mars was the start from which Kepler could redirect the study of celestial motion, just as Galileo used the motion of falling bodies to redirect the study of terrestrial motion.

Kepler began his study of Mars by trying to fit the observations with motions on an eccentric circle and an equant. Like Copernicus, Kepler eliminated the need for the large epicycle by putting the sun motionless at the center and having the earth move around it (see p. 31). But Kepler made an assumption which differed from that of Copernicus. Recall that Copernicus had rejected the equant as an improper type of motion, but he used small epicycles. Kepler used an equant, but refused to use even a single small epicycle. To Kepler the epicycle seemed "unphysical" because the center of the epicycle was empty, and empty space could not exert any force on a planet. Thus, from the start of his study on Mars, Kepler was assuming that the orbits were real and that the motion had some physical causes. Even though Kepler's teacher advised him to make only "astronomical" (observational) and not physical assumptions, Kepler stubbornly stuck to his idea that the motions must be produced and explained by forces. When finally he published his results on Mars in his book *Astronomia Nova*, the *New Astronomy*, it was subtitled *Celestial Physics*.

In keeping with Aristotelian physics Kepler believed that force was necessary to drive the planets along their circles, not to hold them in circles.

For a year and a half Kepler struggled to fit Tycho's observations of Mars by various arrangements of an eccentric and an equant. When after 70 trials success finally seemed near, he made a discouraging discovery. Although he could represent fairly well the motion of Mars in longitude (east and west along the ecliptic), he failed markedly with the latitude (north and south of the ecliptic). However, even in longitude his very best fit still had differences of eight minutes of arc between Tycho's observed positions and the positions predicted by the model.

SG 7.2

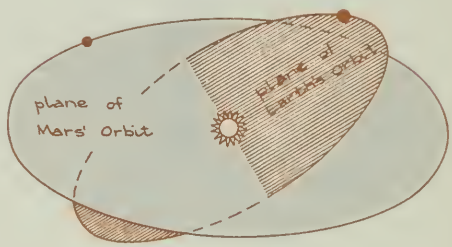
Eight minutes of arc, about a fourth of the moon's diameter, may not seem like much of a difference. Others might have been tempted to explain it away, perhaps charging it to observational error. But Kepler knew from his own studies that Tycho's instruments and observations were rarely in error by even as much as two minutes of arc. Those eight minutes of arc meant to Kepler that his best system, using the old, accepted devices of eccentric and equant, would never be adequate to match the observations.

In his *New Astronomy*, Kepler wrote:

> Since divine kindness granted us Tycho Brahe, the most diligent observer, by whose observations an error of eight minutes in the case of Mars is brought to light in this Ptolemaic calculation, it is fitting that we recognize and honor this favor of God with gratitude of mind. Let us certainly work it out, so that we finally show the true form of the celestial motions (by supporting ourselves with these proofs of the fallacy of the suppositions assumed). I myself shall prepare this way for others in the following chapters according to my small abilities. For if I thought that the eight minutes of longitude were to be ignored, I would already have corrected the hypothesis which he had made earlier in the book and which worked moderately well. But as it is, because they could not be ignored, these eight minutes alone have prepared the way for reshaping the whole of astronomy, and they are the material which is made into a great part of this work.

Fortunately Kepler had made a major discovery earlier which was crucial to his later work. He found that the orbits of the earth and other planets were in planes which passed through the sun. Ptolemy and Copernicus required special explanations for the motion of planets north and south of the ecliptic, but Kepler found that these motions were simply the result of the orbits lying in planes tilted to the plane of the earth's orbit.

Kepler concluded that the orbit was not a circle and there was no point around which the motion was uniform. So Plato's aim of fitting perfect circles to the heavens, which had for twenty centuries engaged the minds of brilliant men, had to be abandoned. Kepler had in his hands the finest observations ever made, but now he had no theory by which they could be explained. He would have to start over to account for two altogether new questions: what *is* the shape of the orbit followed by Mars, and how does the speed of the planet change as it moves along the orbit?



The diagram depicts a nearly edge-on view of orbital planes of earth and another planet, both intersecting at the sun.

Q1 When Kepler joined Tycho Brahe what task was he assigned?

Q2 Why did Kepler conclude that Plato's problem, to describe the motions of the planets by combinations of circular motions, could not be solved?

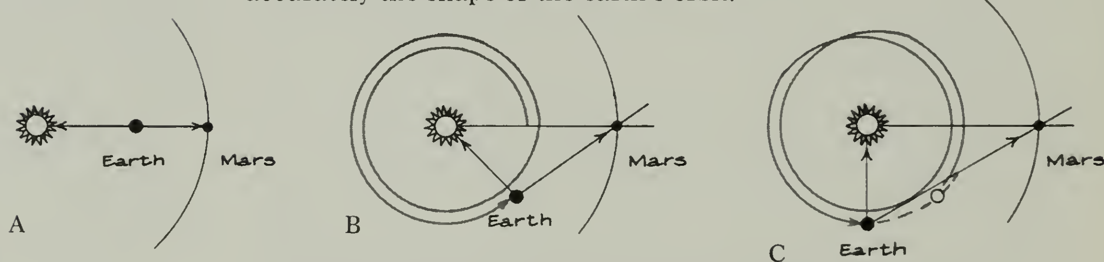
7.2 Kepler's law of areas

Kepler's problem was immense. To solve it would demand the utmost of his imagination and computational skills.

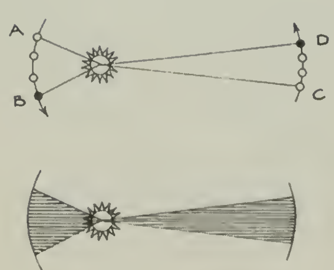
As the basis for his study, Kepler had Tycho's observed directions to Mars and to the sun on certain dates. But these observations were made from a moving earth whose orbit was not well known. Kepler realized that he must first determine more accurately the shape of the earth's orbit so that he would know where it was on the dates that the various observations of Mars had been made. Then he might be able to use the observations to determine the shape and size of the orbit of Mars. Finally, to predict positions for Mars he would need to discover some rule or regularity that described how fast Mars moved along different parts of its orbit.

As we follow his brilliant analysis here, and particularly if we repeat some of this work in the laboratory, we will see the series of problems that he solved.

To derive the earth's orbit he began by considering the moments when the sun, earth, and Mars are essentially in a straight line (Fig. A). After 687 days, as Copernicus had found, Mars would return to the same place in its orbit (Fig. B). Of course, the earth at that time would *not* be at the same place in its own orbit as when the first observation was made. But as Figs. B and C indicate, the directions to the sun and Mars as they might be seen from the earth against the fixed stars would be known. The crossing point of the sight-lines to the sun and to Mars must be a point on the earth's orbit. By working with several groups of observations made 687 days apart (one Mars "year"), Kepler was able to determine fairly accurately the shape of the earth's orbit.



The orbit Kepler found for the earth appeared to be almost a circle, with the sun a bit off center. From his plotted shape and the record of the apparent position of the sun for each date of the year, he could locate the position of the earth on its orbit, and its speed along the orbit. Now he had an orbit and a timetable for the earth's motion. You made a similar plot in the experiment The Shape of the Earth's Orbit.



In Kepler's plot of the earth's motion around the sun, it was evident that the earth moves fastest when nearest the sun. (Kepler wondered why this occurred and speculated that the sun might exert some force that drove the planets along their orbits; his concern with the physical cause of planetary motion marked a change in attitude toward motion in the heavens.) The drawings at the left represent (with great exaggeration) the earth's motion for two parts of its orbit. The different positions on the orbit are separated by equal time intervals. Between points A and B there is a relatively large distance, so the planet is moving rapidly; between points C and D it moves more slowly. Kepler noticed, however, the two areas swept over by a line from the sun to the planet are equal. Kepler, it is believed, had actually calculated such areas only for the nearest and farthest positions of two planets, Earth and Mars, yet the beautiful simplicity of the relation led him to conclude that it was generally true, for all parts of orbits. In its general form the Law of Areas states: *The line from the sun to the moving planet sweeps over areas that are proportional to the time intervals.* Later, when Kepler found the exact shape of orbits, his law of areas became a powerful tool for predicting positions along the orbit. In the next section we shall use both laws and see in detail how they work.

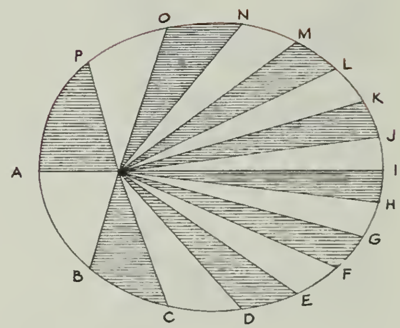
You may be surprised that the first rule we have encountered about the motions of the planets is concerned with the areas swept over by the line from the sun to the planet. After we considered circles, eccentric circles, epicycle, and equants, we come upon a quite unexpected property: the area swept over per unit time is the first property of the orbital motion to remain constant. (As we shall see in Chapter 8, this major law of nature applies to all orbits in the solar system and also to double stars.) Here was something that, besides being new and different, also drew attention to the central role of the sun, and so bolstered Kepler's faith in the still widely neglected Copernican idea of a heliocentric system.

As you will see, Kepler's other labors would have been of little use without this basic discovery, although the rule does not give any hint why this regularity should exist. The law of areas describes the relative rate at which the earth and, Mars (and, Kepler thought, any other planet) move at any point of their orbits. Kepler could not fit the rule to Mars by assuming a circular orbit, and so he set out to find what shape Mars' orbit was.

SG 7.4

Another way to express this relationship for the nearest and farthest positions would be to say the speeds were inversely proportional to the distance; but this rule does *not* generalize to any other points on the orbit. (A modification of the rule that *does* hold is explained on pages 64 and 65.)

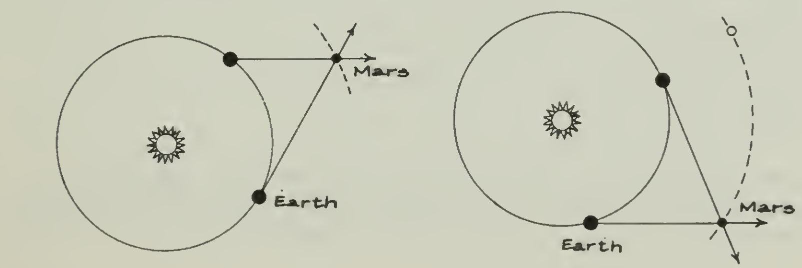
Q3 What observations did Kepler use to plot the earth's orbit?
Q4 State Kepler's law of areas.
Q5 Where in its orbit does a planet move the fastest?



Kepler's Law of Areas. A planet moves along its orbit at a rate such that the line from the sun to the planet sweeps over areas which are proportional to the time intervals. The time taken to cover AB is the same as that for BC, CD, etc.

7.3 Kepler's law of elliptical orbits

With the orbit and timetable of the earth known, Kepler could reverse the analysis and find the shape of Mars' orbit. For this purpose he again used observations separated by one Martian year. Because this interval is somewhat less than two earth years, the earth is at different positions in its orbit at the two times, so the two directions from the earth toward Mars differ. Where they cross is a point on the orbit of Mars. From such pairs of observations Kepler fixed many points on the orbit of Mars. The diagrams below illustrate how two such points might be plotted. From a curve drawn through such points, he obtained fairly accurate values

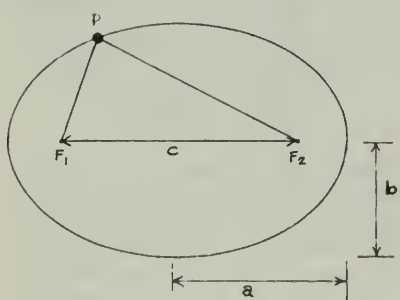


In this experiment the orbit of Mars is plotted from measurements made on pairs of sky photographs taken one Martian year apart.

for the size and shape of Mars' orbit. Kepler saw at once that the orbit of Mars was not a circle around the sun. You will find the same result from the experiment, The Orbit of Mars. But what sort of path was this? How could it be described most simply? As Kepler said, "The conclusion is quite simply that the planet's path is not a circle—it curves inward on both sides and outward again at opposite ends. Such a curve is called an oval." But what kind of oval?

Many different closed curves can be called ovals. Kepler thought for a time that the orbit was egg-shaped. Because such a shape did not agree with Kepler's ideas of physical interaction between the sun and the planet, he rejected that possibility. Kepler concluded that there must be some better way to describe the orbit, and that he could find it. For many months, Kepler struggled with the question. Finally he was able to show that the orbit was a simple curve which had been studied in detail by the Greeks two thousand years before. The curve is called an *ellipse*. It is the shape you see when you view a circle at a slant.

SG 7.5



An ellipse showing the major axis *a* the minor axis *b*, and the two foci F_1 and F_2. The shape of an ellipse is described by its eccentricity e, where $e = c/a$.

Ellipses can differ greatly in shape. They have many interesting properties. For example, you can draw an ellipse by looping a piece of string around two thumb tacks pinned to a drawing board at points F_1 and F_2 as shown at the left. Pull the loop taut with a pencil point (P) and run the pencil once around the loop. You will have drawn an ellipse. (If the two thumb tacks had been together, what curve would you have drawn? What results do you get as you separate the two tacks more?)

Each of the points F_1 and F_2 is called a *focus* of the ellipse. The greater the distance between F_1 and F_2, the flatter, or more "eccentric" the ellipse becomes. As the distance between F_1 and F_2 shrinks to zero, the ellipse becomes more nearly circular. A measure of the eccentricity of the ellipse is the ratio of the distance F_1F_2 to the long axis. If the distance between F_1 and F_2 is *c* and length of the long axis is *a*, then the eccentricity *e* is defined as $e = c/a$.

SG 7.6

SG 7.7

In the Orbit of Mercury Experiment, you can plot the shape of Mercury's very eccentric orbit from observational data. See also SG 7.8.

The eccentricities are given for each of the ellipses shown in the series of photographs in the margin of the next page. You can see that a circle is the special case of an ellipse with $e = 0$, and that the greatest possible eccentricity for an ellipse is $e = 1.0$.

SG 7.9

What Kepler discovered was not merely that the orbit of Mars is an ellipse—a remarkable enough discovery in itself—but also that the sun is at one focus. (The other focus is empty.) Kepler stated these results in his *Law of Elliptical Orbits: The planets move in orbits which are ellipses and have the sun at one focus.*

As Table 7.1 shows, the orbit of Mars has the largest eccentricity of all the orbits that Kepler could have studied, namely those of Venus, Earth, Mars, Jupiter, and Saturn. Had he studied any planet other than Mars, he might never have noticed that the orbit was an ellipse! Even for the orbit of Mars, the difference between the elliptical orbit and an off-center circle is quite small. No wonder Kepler later wrote that "Mars alone enables us to penetrate the secrets of astronomy which otherwise would remain forever hidden from us."

Table 7.1 The Eccentricities of Planetary Orbits

PLANET	ORBITAL ECCENTRICITY	NOTES
Mercury	0.206	Too few observations for Kepler to study
Venus	0.007	Nearly circular orbit
Earth	0.017	Small eccentricity
Mars	0.093	Largest eccentricity among planets Kepler could study
Jupiter	0.048	Slow moving in the sky
Saturn	0.056	Slow moving in the sky
Uranus	0.047	Not discovered until 1781
Neptune	0.009	Not discovered until 1846
Pluto	0.249	Not discovered until 1930

The work of Kepler illustrates the enormous change in outlook in Europe that had begun well over two centuries earlier. Kepler still shared the ancient idea that each planet had a "soul," but he refused to rest his explanation of planetary motion on this idea. Instead, he began to search for physical causes. When Copernicus and Tycho were willing to settle for geometrical models by which planetary positions could be predicted, Kepler was one of the first to seek dynamic *causes* for the motions. This new desire for physical explanations marks the beginning of one of the chief characteristics of modern physical science.

Like Kepler, we believe that our observations represent some aspects of a reality that is more stable than the changing emotions and wishes of human beings. Like Plato and all subsequent scientists, we assume that nature is basically orderly and consistent and, therefore, understandable in a simple way. This faith has led to great theoretical and technical gains. Kepler's work illustrates one of the scientific attitudes—to regard a wide variety of phenomena as better understood when they can be summarized by simple law, preferably one expressed in mathematical form.

After Kepler's initial joy over the discovery of the law of elliptical paths, he may have asked himself the question: why are the planetary orbits elliptical rather than some other geometrical

e = 0.3

e = 0.5

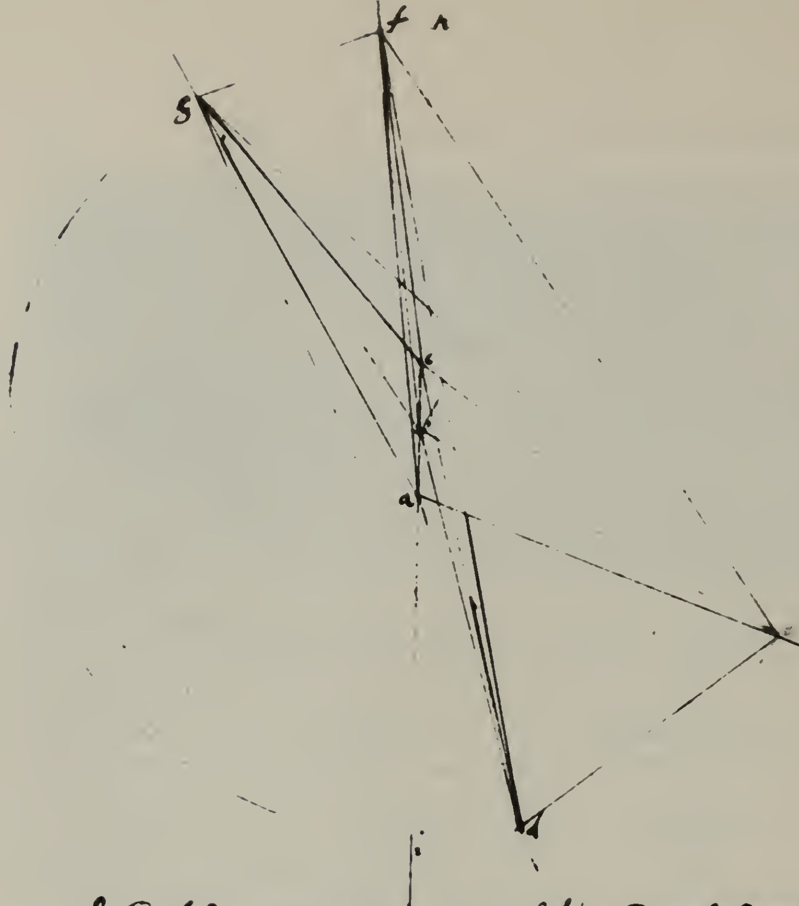
e = 0.7

e = 0.8

e = 0.94

e = 0.98

Ellipses of different eccentricities. (The pictures were made by photographing a saucer at different angles.)



B Centrum orbitae ♂. A corpus solis. C centrum aequalitatis Martis GFHED1 orbita Martis. HAL Linea apogaei.

F Situs Martis an 87. in 26 ♍
G Situs Martis an 91 in 28 ♐
D Situs Martis an 93 in 17 ♒
E Situs Martis an 95 in 18 ♓

F. 147. 26. 0.
G. 238. 36. 24
91 10 24

G 238. 36. 24
D 314. 6. 15
75. 29 51

D 314 6. 15
E. 19. 15. 57.
65 9. 42

E. 19. 15. 57
G. ~~238. 36. 24~~
~~19. 15. 57~~
147. 26. 0
128 10 3

FAG. 91. 10. 24 FCG. 94. 59. 4.
GAD. 75 29. 51. GCD 64. 8. 36.
DAE. 65. 9. 42 DCE. 57. 16. 12.
EAF. 128. 10. 3. ECF 143. 36. 8
360. 0. 0.

Summa CGA, ADC. 11. 21. 15
Summa CFA, AEC. 15. 26. 5.
Summa CGA, AEC. 19. 14. 45.
Summa CFA, ADC 7. 32. 35.

Excessus ~~CGA~~ sup ~~GAD~~ CGA, ADC: — 7. 53. 30.
Excessus CFA, AEC — 3. 48. 40.
AEC CFA CFA

... certa et data sunt. Jam oportet Apogaeum H sic ordinari, ut anguli quatuor linea FB, GB, DB, EB, sint ... ita in ... circulo ... linea ... necessarius ...

GCD 64. 8. 36
DCE 57. 16. 12
GCE 121. 24. 48

GAD. 75. 29. 51
DAE 65. 9. 42
GAE. 140 39 33
19. 14. 45.

FCG 94. 59. 4
GCD 64. 8. 36
FCD 159. 7. 40.

FAG. 91. 10. 24
GAD. 75. 29. 51.
FAD 166 40 15
7. 32. 35.

DAE 65. 9. 42
DCE. 57. 16. 12
7. 53. 30

GAF 91. 10. 24
GCF 94. 59. 4
3. 48. 40

shape? While we might understand Plato's desire for uniform circular motions, nature's insistence on the ellipse is a surprise.

In fact, there was no satisfactory answer to this question until Newton showed, almost eighty years later, that these elliptical orbits were necessary results of a more general law of nature. Let us accept Kepler's laws as rules that contain the observed facts about the motions of the planets. As *empirical laws*, they each summarize the data obtained by observation of the motion of any planet. The law of orbits, which describes the paths of planets as elliptical around the sun, gives us all the possible positions each planet can have if we know the size and eccentricity. That law, however, does not tell us *when* the planet will be at any one particular position on its ellipse or how rapidly it will be moving then. The law of areas does not specify the shape of the orbit, but does describe how the angular speed changes as the distance from the sun changes. Clearly these two laws complement each other. With these two general laws, and given the values for the size and eccentricity of the orbit (and a starting point), we can determine both the position and angular speed of a given planet at any time, past or future. Since we can also find where the earth is at the same instant, we can calculate the position of the planet as it would have been or will be seen from the earth.

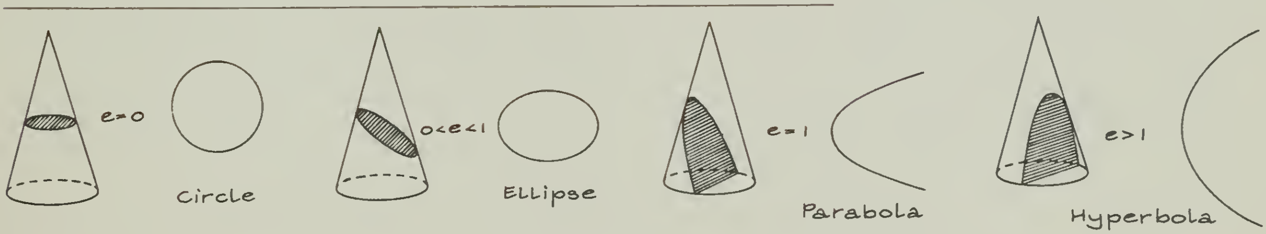
Empirical means based on observation, but not on theory.

The elegance and simplicity of Kepler's two laws are impressive. Surely Ptolemy and Copernicus would have been amazed that the solution to the problem of planetary motions could be given by such short statements. But we must not forget that these laws were distilled from Copernicus' idea of a moving earth, the great labors and expense that went into Tycho's fine observations, and the imagination and devotion, agony and ecstasy of Kepler.

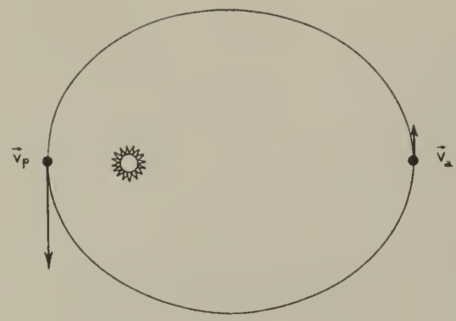
Q6 What was special about Mars' orbit that made Kepler's study of it so fortunate?

Q7 If the average distance and eccentricity of a planet's orbit are known, which of the following can be predicted from the law of areas alone? From the law of Elliptical orbits alone? Which require both? (Mark *A*, E, or A + E).
(a) All possible positions in the orbit,
(b) speed at any point in an orbit,
(c) position at any given time.

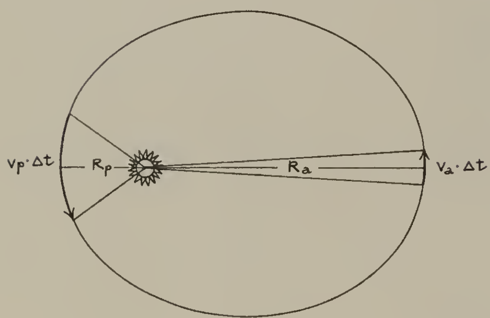
Conic Sections are figures produced by cutting a cone with a plane—the eccentricity of a figure is related to the angle of the cut. In addition to circles and ellipses, parabolas and hyperbolas are conic sections, with eccentricities greater than ellipses. Newton eventually showed that all of these shapes are possible paths for a body moving under the influence of the sun.



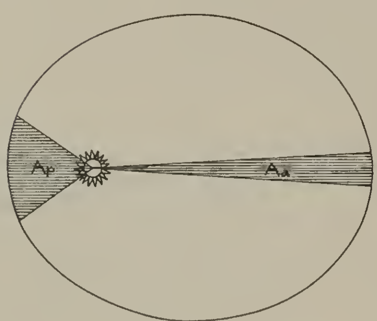
Opposite: A page from Kepler's notebooks.



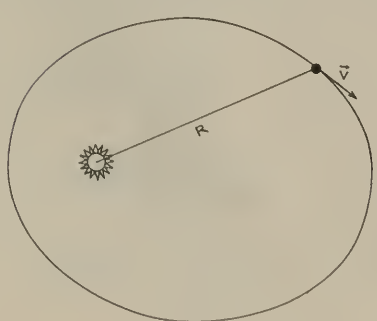
(A)



(B)



(C)



(D)

A General Equation for Orbital Speed

Figure A represents the elliptical orbit of a planet, with the sun at one focus. By a short analysis we can find the ratio of the speeds at the position nearest to the sun (perihelion) and farthest from the sun (aphelion).

Figure B shows a small part of the planet's path around perihelion, during a time interval Δt. If Δt is very short, then the average speed along the path will be virtually equal to the instantaneous speed at perihelion, v_p, and the path length will be $v_p \times \Delta t$.

Also, if Δt is very short, the section of orbit is almost straight, so it can be considered the base of a long, thin triangle of altitude R_p, shaded in Figure C. The area of any triangle is $\frac{1}{2}$ *base* $\times$ *altitude*, so the area A_p of this triangle is $\frac{1}{2}(v_p \times \Delta t)R_p$.

Similarly, the area A_a of a triangle swept out during Δt at aphelion is $\frac{1}{2}(v_a \times \Delta t)R_a$. By Kepler's law of areas, equal areas are swept out in equal times, so $A_a = A_p$. Then

$$\tfrac{1}{2}v_a \times \Delta t \times R_a = \tfrac{1}{2}v_p \times \Delta t \times R_p$$

and, dividing both sides by $\frac{1}{2}\Delta t$,

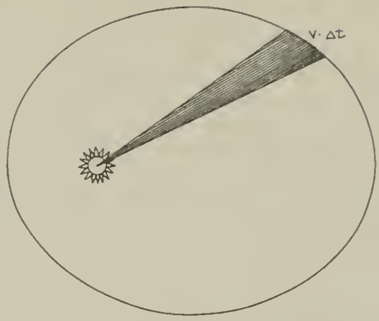
$$v_a R_a = v_p R_p$$

We can rearrange the equation to the form

$$\frac{v_a}{v_p} = \frac{R_p}{R_a}$$

which shows that the speeds at aphelion and perihelion are inversely proportional to the distances from the sun: at a larger distance the speed is smaller.

The derivation for these two points was easy, because at these points the velocity is perpendicular to the line drawn to the sun. When the planet is at some



(E)

position other than perihelion or aphelion, the velocity vector is not perpendicular, as shown in Figure D. However, we can approximate the area swept out, shaded in Figure E, by a triangle of altitude R, as shown in Figure F. Notice that it includes a tiny corner of extra area, but also leaves out a tiny corner. For a very short time interval Δt, the triangle will be very thin and the difference between the two tiny corners will virtually vanish. As shown in figure G, the base of the triangle is not $v \times \Delta t$, but $v_\perp \times \Delta t$, where $v_\perp$ is the portion or component of v perpendicular to the sun-planet line. Thus the area swept out during Δt can be expressed as

$$\tfrac{1}{2} v_\perp \times \Delta t \times R$$

This same derivation for area swept out will hold for any part of the orbit over a short time interval Δt. By Kepler's law of areas, the areas swept out during equal time intervals would be equal, so we can write

$$\tfrac{1}{2} v_\perp \times \Delta t \times R = \tfrac{1}{2} v_\perp \times \Delta t \times R' = \tfrac{1}{2} v''_\perp \times \Delta t \times R'' \text{ etc.}$$

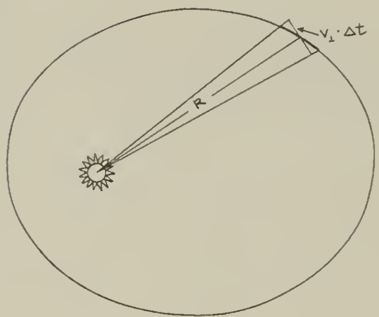
or, dividing through by $\frac{1}{2}\,\Delta t$,

$$v_\perp R = v'_\perp R' = v''_\perp R'' \text{ etc.}$$

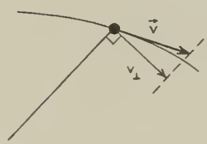
Therefore, we can express Kepler's law of areas as

$$v_\perp R = \text{constant.}$$

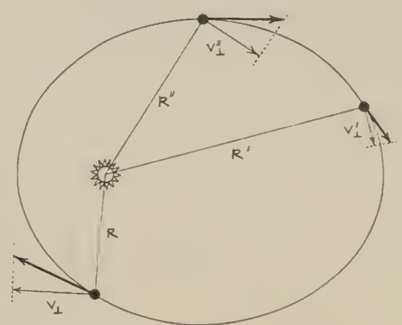
If the shape (eccentricity) of the orbit is known, together with the speed and distance at any one point, we can use this equation to calculate the speed at any other point in the orbit. (See SG 7.10.) Moreover, the law of areas, from which this relation is derived, is true for the motion of any body that experiences a force directed toward one of the foci of the ellipse — a so-called "central force." So the relation $v_\perp R =$ *constant* applies to double stars and to atoms as well as to the solar system.



(F)



(G)



(H)

7.4 Kepler's Law of Periods

Kepler's first two laws were published in 1609 in his book *Astronomia Nova*, but he was still dissatisfied because he had not yet found any relation among the motions of the different planets. Each planet seemed to have its own elliptical orbit and speeds, but there appeared to be no overall pattern relating all planets to one another. Kepler had begun his career by trying to explain the number of planets and their spacing. He was convinced that the observed orbits and speeds could not be accidental, but that there must be some regularity linking all the motions in the solar system. His conviction was so strong, that he spent years examining many possible combinations of factors to find, by trial and error, a third law that would relate all the planetary orbits. His long search, almost an obsession, illustrates a belief that has run through the whole history of science: that despite apparent difficulties in getting a quick solution, underneath it all, nature's laws are understandable. This belief is to this day a chief source of inspiration in science, often sustaining one's spirit in periods of seemingly fruitless labor. For Kepler it made endurable a life of poverty, illness, and other personal misfortunes, so that in 1619 he could write triumphantly in his *Harmony of the World*:

As Einstein later put it: "The Lord is subtle, but He is not malicious."

> . . . after I had by unceasing toil through a long period of time, using the observations of Brahe, discovered the true relation . . . overcame by storm the shadows of my mind, with such fullness of agreement between my seventeen years' labor on the observations of Brahe and this present study of mine that I at first believed that I was dreaming . . . .

Kepler's law of periods, also called the "harmonic law," relates the periods of the planets to their average distances from the sun. The period is the time taken to go once completely around the orbit. The law states that *the squares of the periods of the planets are proportional to the cubes of their average distances from the sun.* In the short form of algebra, calling the period T and the average distance R_{av}, this law can be expressed as

For the earth, T is one year. The average distance R_{av} of the earth from the sun is one astronomical unit, 1AU. So one way to express the value of the constant k is $k = 1$ year^2/AU^3.

$$T^2 \propto R_{av}^3 \quad \text{or} \quad T^2 = kR_{av}^3 \quad \text{or} \quad \frac{T^2}{R_{av}^3} = k$$

where k is a constant. Because this relation applies to all the planets and even to comets in orbit around the sun, we can use it to find the period of any planet once we know its average distance from the sun, and *vice versa*.

Kepler's three laws are so simple that their great power may be overlooked. When they are combined with his discovery that each planet moves in a plane passing through the sun, they let us derive the past and future history of each planet from only six quantities. Two of these quantities are the size and eccentricity of the orbit, three others are angles that relate the plane of the orbit to that of the earth's orbit, while the sixth tells where in its orbit

SG 7.11–7.14

the planet was on any one certain date. These quantities are explained more fully in the Activities section of *Handbook* 2 for Chapters 7 and 8.

	Copernicus' Values			Modern Values		
PLANET	PERIOD T, (YEARS)	AVERAGE DISTANCE R_{av}(AU)	$\frac{T^2}{R_{av}^3}$	PERIOD T, (YEARS)	AVERAGE DISTANCE R_{av}(AU)	$\frac{T^2}{R_{av}^3}$
Mercury	0.241	0.38	1.06	0.241	0.387	1.00
Venus	0.614	0.72	1.01	0.615	0.723	1.00
Mars	1.881	1.52	1.01	1.881	1.523	1.00
Jupiter	11.8	5.2	0.99	11.862	5.20	1.00
Saturn	29.5	9.2	1.12	29.458	9.54	1.00

The value of R_{av} for an ellipse is just half the major axis.

It is astonishing that in this manner the past and future positions of each planet (and, as we now know, also each comet) can be derived in a simpler and more precise way than through the multitude of geometrical devices on which all other planetary theories depended, whether those of Ptolemy, Copernicus, or Tycho. With different assumptions and procedures Kepler had at last solved the astronomical problem on which so many great men had worked over the centuries. Although he had to abandon the geometrical devices of the Copernican system, Kepler did depend on the Copernican viewpoint of a sun-centered universe. None of the earth-centered models could have led to Kepler's three laws.

In 1627, after many troubles with his publishers and Tycho's heirs, Kepler finally published a set of astronomical tables. In these tables Kepler combined Tycho's observations and the three laws in a way that permitted accurate calculations of planetary positions for any time, whether in the past or future. These tables remained useful for a century, until telescopic observations of greater precision replaced Tycho's observations.

The tables, named for Tycho's and Kepler's patron, Emperor Rudolph II, were called the *Rudolphine Tables.* They were also important for a quite different reason. In them Kepler pioneered in the use of logarithms for making rapid calculations and included a long section, practically a textbook, on the nature and use of logarithms (first described in 1614 by Napier in Scotland). His tables spread the use of this computational aid, widely needed for nearly three centuries, until modern computing machines came into use.

Kepler's scientific interest was not confined to the planetary problem alone. Like Tycho, who was much impressed by the new star of 1572, Kepler observed and wrote about new stars that appeared in 1600 and 1604. His observations and interpretations added to the impact of Tycho's earlier observations that changes did occur in the starry sky. As soon as Kepler learned of the development of the telescope, he spent most of a year making careful studies of how the images were formed. These he published in a book titled *Dioptrice* (1611), which became the standard work on optics for many years. In addition to a number of important books on mathematical and astronomical problems, Kepler wrote a popular and widely read description of the Copernican system as modified by his own discoveries. This added to the growing interest in and acceptance of the sun-centered model of the planetary system.

Q8 State Kepler's law of periods.

7.5 The new concept of physical law

One general feature of Kepler's life-long work has had a far-reaching effect on the way in which all the physical sciences developed. When Kepler began his studies, he still accepted Plato's assumptions about the importance of geometric models and Aristotle's emphasis on natural place to explain motion. But later he came to concentrate on algebraic laws describing how planets moved. His successful statement of empirical laws in mathematical form helped to establish the use of the *equation* as the normal form of stating laws in physical science.

More than anyone before him, Kepler expected an acceptable theory to agree with precise and quantitative observation. From Tycho's observations he learned to respect the power of precision measurement. Models and theories can be modified by human ingenuity, but good data endure regardless of changes in assumptions or viewpoints.

Kepler went beyond observation and mathematical description, and attempted to explain motion in the heavens by the action of physical forces. In Kepler's system the planets no longer were thought to revolve in their orbits because they had some divine nature or influence, or because this motion was "natural," or because their spherical shapes were self-evident explanation for circular motion. Rather, Kepler was the first to look for a physical law based on observed phenomena to describe the whole universe in a detailed quantitative manner. In an early letter he expressed his guiding thought:

> I am much occupied with the investigation of the physical causes. My aim in this is to show that the celestial machine is to be likened not to a divine organism but rather to a clockwork . . . insofar as nearly all the manifold movements are carried out by means of a single; quite simple magnetic force, as in the case of a clockwork, all motions are caused by a simple weight. Moreover, I show how this physical conception is to be presented through calculation and geometry. [Letter to Herwart, 1605]

To show the celestial machine to be like a clockwork propelled by a single force—this was a prophetic goal indeed. Stimulated by William Gilbert's work on magnetism published a few years earlier, Kepler could imagine magnetic forces from the sun driving the planets along their orbits. This was a reasonable and promising hypothesis. As it developed, the basic idea that a single kind of force controls the motions of all the planets was correct; but the force is not magnetism, and it is needed not to keep the planets moving forward, but to deflect their paths to form closed orbits.

Kepler's statement of empirical laws reminds us of Galileo's suggestion, made at about the same time, that we deal first with the how of motion in free fall and then with the why. A half century later Newton used the concept of *gravitational* force to tie together Kepler's three planetary laws with laws of terrestrial mechanics to provide a magnificent synthesis. (See Chapter 8.)

Q9 In what ways did Kepler's work exemplify a "new" concept of physical law?

7.6 Galileo and Kepler

One of the scientists with whom Kepler corresponded about scientific developments was Galileo. While Kepler's contributions to planetary theory were mainly his empirical laws based on the observations of Tycho, Galileo contributed to both theory and observation. As was reported in Chapters 2 and 3, Galileo's theory of motion was based on observations of bodies moving on the earth's surface. His development of the new science of mechanics contradicted the assumptions on which Aristotle's physics and interpretation of the heavens had been based. Through his books and speeches Galileo triggered wide discussion about the differences or similarities of earth and heaven. Outside of scientific circles, as far away as England, the poet John Milton wrote, some years after his visit to Galileo in 1638:

> . . . What if earth
> Be but the shadow of Heaven, and things therein
> Each to the other like, more than on earth is thought?

(*Paradise Lost*, Book V, line 574, published 1667.)

Galileo challenged the ancient interpretations of experience. As we saw earlier, he focused attention on new concepts: time and distance, velocity and acceleration, forces and matter, in contrast to the Aristotelian qualities of essences, final causes, and fixed geometric models. In Galileo's study of falling bodies he insisted on fitting the concepts to the observed facts. By seeking results that could be expressed in concise algebraic form, Galileo paralleled the new style being used by Kepler.

The sharp break between Galileo and most other scientists of the time arose from the kind of questions he asked. To his opponents, many of Galileo's problems seemed trivial. What was important about watching pendulums swing or rolling balls down inclines, when philosophical problems needed clarification? His procedures for studying the world seemed peculiar, even fantastic.

Although Kepler and Galileo lived at the same time, their lives were quite different. Kepler lived a hand-to-mouth existence under stingy patrons and was driven from city to city by the religious wars of the time. Few people, other than a handful of friends and correspondents, knew of or cared about his studies and results. He wrote lengthy, tortuous books which demanded expert knowledge to read.

Galileo, on the other hand, wrote his numerous essays and books in Italian, in a language and a style which could be understood by his contemporaries who did not read scholarly Latin.

In recent times, similar receptions were initially given to such artists as the painter Picasso, and the sculptor Giacometti, and the composers Stravinski and Schönberg. The same has often been true in most fields, whether literature or mathematics, economics or politics. But while great creative novelty is often attacked at the start, it does not follow that, conversely, everything that is attacked must be creative.

Galileo was a master at publicizing his work. He wanted as many as possible among the reading public to know of his studies and to accept the Copernican theory. He took the argument far beyond a small group of scholars out to the nobles, civic leaders, and religious dignitaries. His arguments included satire on individuals or ideas. In return, his efforts to inform and persuade on a topic as "dangerous" as cosmological theory stirred up the ridicule and even violence often poured upon those who have a truly new point of view.

Q10 Which of the following would you associate more with Galileo's work than with that of his predecessors: qualities and essences, popular language, concise mathematical expression, final causes?

7.7 The telescopic evidence

Two of Galileo's telescopes, displayed at the Museum of Science in Florence.

Like Kepler, Galileo was surrounded by colleagues who were convinced the heavens were eternal and could not change. Hence, Galileo was especially interested in the sudden appearance in 1604 of a new star, one of those observed by Kepler. Where there had been nothing visible in the sky, there was now a brilliant star. Galileo, like Tycho and Kepler, realized that such changes in the starry sky conflicted with the old idea that the stars could not change. Furthermore, this new star awakened in Galileo an interest in astronomy which lasted throughout his life.

Consequently, Galileo was ready to react to the news he received four or five years later: that a Dutchman "had constructed a spy glass by means of which visible objects, though very distant from the eye of the observer, were distinctly seen as if nearby." Galileo (as he tells it) quickly worked out some of the optical principles involved and set to work to grind the lenses and build such an instrument himself. His first telescope made objects appear three times closer than when seen with the naked eye. Reporting on his third telescope in his book *The Starry Messenger*:

> Finally, sparing neither labor nor expense, I succeeded in constructing for myself so excellent an instrument that objects seen by means of it appeared nearly one thousand times larger and over thirty times closer than when regarded with our natural vision.

Galileo meant that the *area* of the object was nearly 1000 times greater. The area is proportional to the *square* of the magnification (or "power") as we define it now.

What would you do if you were handed "so excellent an instrument"? Like the men of Galileo's time, you probably would put it to practical uses. "It would be superfluous," Galileo agreed,

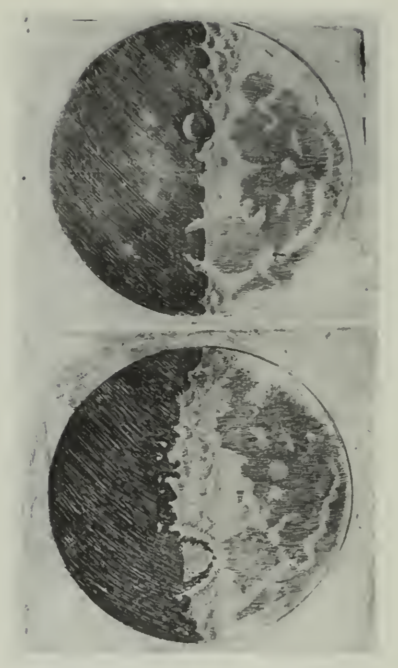
> to enumerate the number and importance of the advantages of such an instrument at sea as well as on land. But forsaking terrestrial observations, I turned to celestial ones, and first I saw the moon from as near at

> hand as if it were scarcely two terrestrial radii away. After that I observed often with wondering delight both the planets and the fixed stars . . . .

In the period of a few short weeks in 1609 and 1610, Galileo used his telescope to make several discoveries, each of which is of first rank.

First, Galileo pointed his telescope at the moon. What he saw led him to the conviction that

> . . . the surface of the moon is not smooth, uniform, and precisely spherical as a great number of philosophers believe it (and other heavenly bodies) to be, but is uneven, rough, and full of cavities and prominences, being not unlike the face of the earth, relieved by chains of mountains, and deep valleys.



Two of Galileo's early drawings of the moon (from Galileo's *Siderius Nuncius*).

Galileo did not stop with that simple observation, so contrary to the Aristotelian idea of heavenly perfection. He supported his conclusions with several kinds of evidence, including careful measurement. For instance, he worked out a method for determining the height of a mountain on the moon from the shadow it is seen to cast there. (His value of about four miles for the height of some lunar mountains is not far from modern results such as those obtainable in the experiment, The Height of Piton: A Mountain on the Moon.

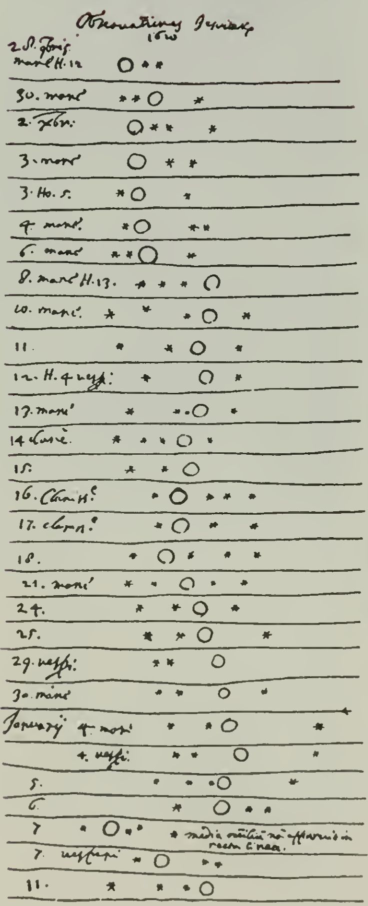
Next he looked at the stars. To the naked eye the Milky Way had seemed to be a continuous blotchy band of light; through the telescope it was seen to consist of thousands of faint stars. Wherever Galileo pointed his telescope in the sky, he saw many more stars than could be seen with the unaided eye. This observation was contrary to the old argument that the stars were created to provide light so men could see at night. If that were the explanation, there should not be stars invisible to the naked eye—but Galileo found thousands.

After his observations of the moon and the fixed stars, Galileo turned his attention to the discovery which in his opinion ". . . deserves to be considered the most important of all—the disclosure of four *Planets* never seen from the creation of the world up to our own time." He is here referring to his discovery of four of the satellites which orbit about Jupiter. Here before his eyes was a miniature solar system with its own center of revolution. Today, as to Galileo so long ago, it is a sharp thrill to see the moons of Jupiter through a telescope for the first time. One is immediately struck by this evidence so directly opposed to the Aristotelian notion that the earth was at the center of the universe and the chief center of revolution.

The manner in which Galileo discovered Jupiter's "planets" is a tribute to his ability as an observer. Each clear evening during this period he was discovering dozens if not hundreds of new stars never before seen by man. When looking in the vicinity of Jupiter

Telescopic photograph of Jupiter and its four bright satellites. This is approximately what Galileo saw and what you see through the simple telescope described in the *Handbook*.

As of 1970, 12 satellites of Jupiter have been observed.



These sketches of Galileo's are from the first edition of *The Starry Messenger*.

on the evening of January 7, 1610, he noticed ". . . that beside the planet there were three starlets, small indeed, but very bright. Though I believe them to be among the host of fixed stars, they aroused my curiosity somewhat by appearing to lie in an exact straight line . . . ." (The first page of the notebook in which he recorded his observations is reproduced on p. 81 at the end of this chapter.) When he saw them again the following night, he saw that they had changed position with reference to Jupiter. Each clear evening for weeks he observed that planet and its roving "starlets" and recorded their positions in drawings. Within days he had concluded that there were four "starlets" and that they were indeed satellites of Jupiter. He continued his observations until he was able to estimate the periods of their revolutions around Jupiter.

Of all of Galileo's discoveries, that of the satellites of Jupiter caused the most stir. His book, *The Starry Messenger*, was an immediate success, and copies were sold as fast as they could be printed. For Galileo the result was a great demand for telescopes and great public fame.

Galileo continued to use his telescope with remarkable results. By projecting an image of the sun on a screen, he observed sunspots. This was additional evidence that the sun, like the moon, was not perfect in the Aristotelian sense: it was disfigured rather than even and smooth. From his observation that the sunspots moved across the face of the sun in a regular pattern, he concluded that the sun rotated with a period of about 27 days.

Photographs of Venus at various phases with a constant magnification.

He also found that Venus showed all phases, just as the moon does (see photos above). Therefore, Venus must move completely around the sun as Copernicus and Tycho had believed, rather than be always between the earth and sun as the Ptolemaic astronomers assumed. Saturn seemed to carry bulges around its equator, as indicated in the drawings on the next page, but Galileo's telescopes were not strong enough to show that they were rings. With his telescopes he collected an impressive array of new information about the heavens—all of it seemed to contradict the basic assumptions of the Ptolemaic world scheme.

Q11 Could Galileo's observations of all phases of Venus support the heliocentric theory, the Tychonic system, or Ptolemy's system?

Q12 In what way did telescopic observation of the moon and sun weaken the earth-centered view of the universe?

Q13 What significance did observations of Jupiter have in weakening the Ptolemaic view of the world?

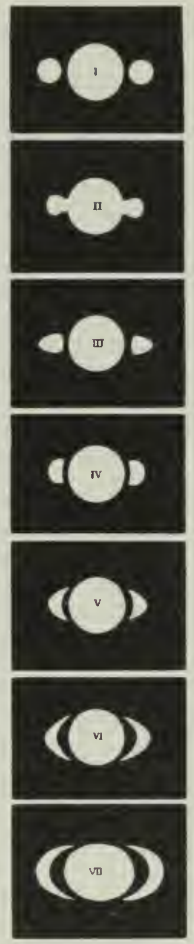
7.8 Galileo focuses the controversy

Galileo's observations supported his belief in the heliocentric Copernican system, but they were not the cause of his belief. In his great work, *Dialogue Concerning the Two Chief World Systems* (1632), his arguments were based more on assumptions that seemed self-evident to him than on observations. Galileo recognized, as Ptolemy and Copernicus had, that the observed motions of planets alone do not decide uniquely between a heliocentric and a geocentric hypothesis. With proper modifications of the systems, says Galileo, "The same phenomena would result from either hypothesis." But Galileo accepted the earth's motion as real because the heliocentric system seemed to him simpler and more pleasing. Elsewhere in this course you will find other cases where a scientist accepted or rejected an idea for reasons arising from a strong belief or feeling that frankly could not, at the time, be verified by experiment.

In the *Dialogue Concerning the Two Chief World Systems,* Galileo presents his arguments in a systematic and lively way. Like his later book, *Discourses Concerning Two New Sciences*, mentioned in Chapter 2, it is in the form of a discussion between three learned men. Salviati, the voice of Galileo, wins most of the arguments. His antagonist is Simplicio, an Aristotelian who speaks for and defends the Ptolemaic system. The third member, Sagredo, represents the objective and intelligent citizen not yet committed to either system. However, Sagredo's role is written so that he usually accepts most of Galileo's arguments in the end.

Galileo's arguments in favor of the Copernican system as set forth in *Two Chief World Systems* were mostly those given by Copernicus. Oddly enough, Galileo made no use of Kepler's laws. However, Galileo's observations did provide new evidence for Kepler's laws. After determining the periods of Jupiter's four moons, Galileo found that the larger the orbit of the satellite, the longer was its period of revolution. Copernicus had already found that the periods of the planets increased with their average distances from the sun. (Kepler's law of periods stated the relation for the planets in detailed quantitative form.) Now Jupiter's satellite system showed a similar pattern. These new patterns of regularities would soon replace the old assumptions of Plato, Aristotle, and Ptolemy.

Two Chief World Systems relies upon Copernican arguments,



Drawings of Saturn made in the seventeenth century.

Galilean observations, and arguments of plausibility to attack the basic assumptions of the geocentric model. In response, Simplicio, seemingly in desperation, tries to dismiss all of Galileo's arguments with a characteristic counter argument:

> . . . with respect to the power of the Mover, which is infinite, it is just as easy to move the universe as the earth, or for that matter a straw.

But to this Galileo makes a very interesting reply; notice how he quotes Aristotle against the Aristotelians:

> . . . what I have been saying was with regard not to the Mover, but only the movables . . . Giving our attention, then, to the movable bodies, and not questioning that it is a shorter and readier operation to move the earth than the universe, and paying attention to the many other simplifications and conveniences that follow from merely this one, it is much more probable that the diurnal motion belongs to the earth alone than to the rest of the universe excepting the earth. This is supported by a very true maxim of Aristotle's which teaches that . . . 'it is pointless to use many to accomplish what may be done with fewer.'

With characteristic enthusiasm, Galileo thought his telescopic discoveries would soon cause everyone to realize how absurd the assumptions were that prevented wide acceptance of the Copernican theory. But men cannot believe what they are not ready to believe. In their fight against the new Copernicans, the followers of Aristotle were convinced that they were adhering to the facts, that the heliocentric theory was obviously false and in contradiction to observation and also to common sense. The evidences of the telescope could be due to distortions; after all, glass lenses change the path of light rays. And even if telescopes seemed to work for terrestrial observation, nobody could be sure they worked equally well when pointed at these vastly more distant celestial objects.

Furthermore, the Aristotelians could not even consider the Copernican system as a possible theory without giving up many of their basic assumptions, as we saw in Chapter 6. This would have required them to do what is nearly humanly impossible: give up many of their common-sense ideas and find new bases for their theological and moral doctrines. They would have to admit that the earth is not at the center of creation. Then perhaps the universe was not created especially for mankind. Is it any wonder that Galileo's arguments stirred up a storm of opposition?

Galileo's observations intrigued many, but they were unacceptable to Aristotelian scholars. Most of these had reasons one can respect. But a few were driven to positions that must have seemed silly at that time, too. For example, the Florentine astronomer Francesco Sizzi argued in 1611 why there could not, indeed must not, be any satellites around Jupiter:

> There are seven windows in the head, two nostrils, two ears, two eyes and a mouth; so in the heavens there are two favorable stars, two unpropitious, two luminaries, and Mercury alone undecided and indifferent. From which and many other similar phenomena of nature such as the seven metals, etc., which it were tedious to enumerate, we gather that the number of planets is necessarily seven [including the sun and moon]. . . . Besides, the Jews and other ancient nations, as well as modern Europeans, have adopted the division of the week into seven days, and have named them from the seven planets; now if we increase the number of planets, this whole system falls to the ground. . . . Moreover, the satellites are invisible to the naked eye and therefore can have no influence on the earth, and therefore would be useless, and therefore do not exist.

A year after his discoveries, Galileo wrote to Kepler:

> You are the first and almost the only person who, even after a but cursory investigation, has . . . given entire credit to my statements. . . . What do you say of the leading philosophers here to whom I have offered a thousand times of my own accord to show my studies, but who with the lazy obstinacy of a serpent who has eaten his fill have never consented to look at the planets, or moon, or telescope?

Some of the arguments that were brought forward against the new discoveries were so silly that it is hard for the modern mind to take them seriously. . . . One of his [Galileo's] opponents, who admitted that the surface of the moon looked rugged, maintained that it was actually quite smooth and spherical as Aristotle had said, reconciling the two ideas by saying that the moon was covered with a smooth transparent material through which mountains and craters inside it could be discerned. Galileo, sarcastically applauding the ingenuity of this contribution, offered to accept it gladly—provided that his opponent would do him the equal courtesy of allowing him then to assert that the moon was even more rugged than he had thought before, its surface being covered with mountains and craters of this invisible substance ten times as high as any he had seen. [Quoted from *Discoveries and Opinions of Galileo*, translated by Stillman Drake.]

SG 7.16

Q14 Did Galileo's telescopic observations cause him to believe in the Copernican viewpoint?

Q15 What reasons did Galileo's opponents give for ignoring telescopic observations?

7.9 Science and freedom

The political and personal tragedy that befell Galileo is described at length in many books. Here we shall only mention briefly some of the major events. Galileo was warned in 1616 by the Inquisition to cease teaching the Copernican theory as true (rather than as just one of several possible methods to compute the planetary motions) for that theory was held contrary to Holy Scripture. At the same time Copernicus' book was placed on the *Index of Forbidden Books* and suspended "until corrected." As we saw before, Copernicus had, whenever possible, used Aristotelian doctrine to make his theory plausible. But Galileo had reached a new point of view: he urged that the heliocentric system be accepted on its merits alone. While he was himself a devoutly religious man, he deliberately ruled out questions of religious faith from scientific discussions. This was a fundamental break with the past.

When Cardinal Barberini, formerly a close friend of Galileo, was elected in 1623 to be Pope Urban VIII, Galileo talked with him regarding the decree against the Copernican ideas. As a result of the discussion, Galileo considered it safe enough to write again on the controversial topic. In 1632, having made some required changes, Galileo obtained the necessary papal consent to publish *Two Chief World Systems*. This book presented very persuasively the Ptolemaic and Copernican viewpoints and their relative merits. After the book's publication, his opponents argued that Galileo seemed to have tried to get around the warning of 1616. Furthermore, Galileo's forthright and sometimes tactless behavior and the Inquisition's need to demonstrate its power over suspected heretics combined to mark him for punishment.

Among the many factors in this complex story, we must remember that Galileo, though a suspect of the Inquisition, considered himself religiously faithful. In letters of 1613 and 1615 Galileo wrote that God's mind contains all the natural laws; consequently he held that the occasional glimpses of these laws which the human investigator may gain were direct revelations of God, just as valid and grand as those in the Bible: "From the Divine Word, the Sacred Scripture and Nature did both alike proceed. . . . Nor does God less admirably discover himself to us in Nature's action than in the Scripture's sacred dictions." These opinions are held by many today whether they are scientists or not, and are no longer regarded as being in conflict with theological doctrines. But in Galileo's time they could be regarded as symptoms of pantheism. This was one of the heresies for which Galileo's contemporary, Giordano Bruno, was burned at the stake. The Inquisition was alarmed by Galileo's contention that the Bible was not a certain source of knowledge for the teaching of natural science. In reply, arrogant as Galileo often was, he quoted Cardinal Baronius: "The Holy Spirit intended to teach us how to go to heaven, not how the heavens go."

Pantheism refers to the idea that God is no more (and no less) than the forces and laws of nature.

Though he was old and ailing, Galileo was called to Rome and confined for a few months. From the proceedings of Galileo's trial, of which parts are still secret, we learn that he was tried, threatened with torture, forced to make a formal confession of holding and teaching forbidden ideas and to make a denial of the Copernican theory. In return for his confessions and denial, he was sentenced only to perpetual house arrest. Galileo's friends in Italy did not dare to defend him publicly. His book was placed on the *Index* where it remained, along with that of Copernicus and one of Kepler's, until 1835. Thus, he was used as a warning to all men that the demand for spiritual conformity also required intellectual conformity.

According to a well-known, but probably aprocryphal story, at the end of these proceedings Galileo muttered, "*E pur se muove*—but it does move."

But without intellectual freedom, science cannot flourish for long. Perhaps it is not a coincidence that for two centuries after Galileo, Italy, which had been the mother of many outstanding men, produced hardly a single great scientist, while elsewhere in Europe they appeared in great numbers. Today scientists are acutely aware of this famous part of the story of the development of planetary theories. Teachers and scientists in our time have had to face strong enemies of open-minded inquiry and of unrestricted teaching. Today, as in Galileo's time, men and women who create or publicize new thoughts must be ready to stand up before those who fear and wish to suppress the open discussion of new ideas and new evidence.

Plato knew that an authoritarian state is threatened by intellectual nonconformists and recommended for them the now well-known treatment: re-education, prison, or death. Not long ago, Soviet geneticists were required to discard well-established theories, not on the basis of compelling new scientific evidence, but because of conflicts with political doctrines. Similarly, discussion of the theory of relativity was banned from textbooks in Nazi Germany because Einstein's Jewish parentage was said to invalidate his work. Another example of intolerance was the condition that led to the "Monkey Trial" held during 1925 in Tennessee, where the teaching of Darwin's theory of biological evolution was attacked because it conflicted with certain types of biblical interpretation.

Over 200 years after his confinement in Rome, opinions had changed so that Galileo was honored as in the fresco "Galileo presenting his telescope to the Venetian Senate" by Luigi Sabatelli (1772-1850).

On two points, one must be cautious not to romanticize the lesson of this episode. While a Galileo sometimes still may be persecuted or ridiculed, not everyone who is persecuted is, therefore, a Galileo. He may in fact be just wrong, or a crank. Secondly, it has turned out that, at least for a time, science in some form can continue to live in the most hostile surroundings. When political philosophers decide what may be thought and what may not, science will suffer (like everything else), but it will not necessarily be extinguished. Scientists can take comfort from the judgment of history. Less than 50 years after Galileo's trial, Newton's great book, the *Principia*, brilliantly united the work of Copernicus, Kepler, and Galileo with Newton's new statement of the principles of mechanics. Without Kepler and Galileo, there probably could have been no Newton. As it was, the work of these three, together with the work of many contemporaries working in the same spirit, marked the triumphant beginning of modern science. Thus, the hard-won new laws of science and new views of man's place in the world were established. What followed has been termed by historians The Age of Enlightenment.

SG 7.17

Q16 Which of the following appears to have contributed to Galileo's being tried by the Inquisition?

(a) He did not believe in God.
(b) He was arrogant.
(c) He separated religious and scientific questions.
(d) He wrote in Italian.

Palomar Observatory houses the 200-inch Hale reflecting telescope. It is located on Palomar Mountain in southern California.

7.1 The Project Physics learning materials particularly appropriate for Chapter 7 include the following:

Experiments
- The Orbit of Mars
- The Orbit of Mercury

Activities
- Three-dimensional Model of Two Orbits
- Inclination of Mars Orbit
- Demonstrating Satellite Orbits
- Galileo
- Conic-Section Models
- Challenging Problems: Finding Earth-Sun Distance
- Measuring Irregular Areas

Reader Articles
- Kepler
- Kepler on Mars
- Kepler's Celestial Music
- The Starry Messenger
- Galileo

Film Loop
- Jupiter Satellite Orbit

Transparency
- Orbit Parameters

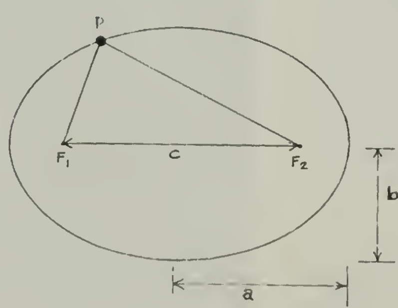
7.2 How large was an error of 8 minutes of arc in degrees? How far right or left do you think a dot over an i on this page would have to be before you would notice it was off-center? What angle would this shift be as seen from a reading distance of 10 inches?

7.3 Summarize the steps Kepler used to determine the orbit of the earth.

7.4 For the orbit positions nearest and furthest from the sun, a planet's speeds are inversely proportional to the distances from the sun. What is the percentage change between the earth's slowest speed in July when it is 1.02 AU from the sun, and its greatest speed in January when it is 0.98 AU from the sun?

7.5 Summarize the steps Kepler used to determine the orbit of Mars.

7.6 In any ellipse the sum or the distances from the two foci to a point on the curve equals the length of the major axis, or $(F_1P + F_2P) = a$. This property of ellipses allows us to draw them by using a loop of string around two tacks at the foci. What should the length of the string be?



7.7 In describing orbits around the sun, the point nearest the sun is called the *perihelion point* and the point farthest from the sun is called the *aphelion point*. The distances of these two points from the sun are called the *perihelion distance* and the *aphelion distance* respectively. The terms perihelion and aphelion come from the Greek, in which *helios* is the sun, *peri* means near, and *apo* means away from.

(a) List some other words in which the prefixes *peri* and *apo* or *ap* have similar meanings.
(b) In describing earth satellite orbits, the terms *apogee* and *perigee* are often used. What do they mean?
(c) What would such points for satellites orbiting the moon be called?

7.8 For the planet Mercury the perihelion distance (closest approach to the sun) has been found to be about 45.8×10^6 kilometers, and the aphelion distance (greatest distance from the sun) is about 70.0×10^6 kilometers. What is the eccentricity of the orbit of Mercury?

7.9 The eccentricity of Pluto's orbit is 0.254. What will be the ratio of the minimum orbital speed to the maximum orbital speed of Pluto?

7.10 The rule $v_\perp R = $ const. makes it easy to find $v_\perp$ for any point on an orbit if the speed and distance at any other point are known. Make a sketch to show how you would find v once you know $v_\perp$.

7.11 Halley's comet has a period of 76 years, and its orbit has an eccentricity of .97.

(a) What is its average distance from the sun?
(b) What is its greatest distance from the sun?
(c) What is its least distance from the sun?
(d) How does its greatest speed compare with its least speed?

7.12 The mean distance of the planet Pluto from the sun is 39.6 AU. What is the orbital period of Pluto?

7.13 Three new major planets have been discovered since Kepler's time. Their orbital periods and mean distances from the sun are given in the table below. Determine whether Kepler's law of periods holds for these planets also.

	Discovery Date	Orbital Period	Average Distance From Sun	Eccentricity of Orbit
Uranus	1781	84.013 yr	19.19 AU	0.047
Neptune	1846	164.783	30.07	0.009
Pluto	1930	248.420	39.52	0.249

7.14 Considering the data available to him, do you think Kepler was justified in concluding that the ratio T^2/R_{av}^3 is a constant?

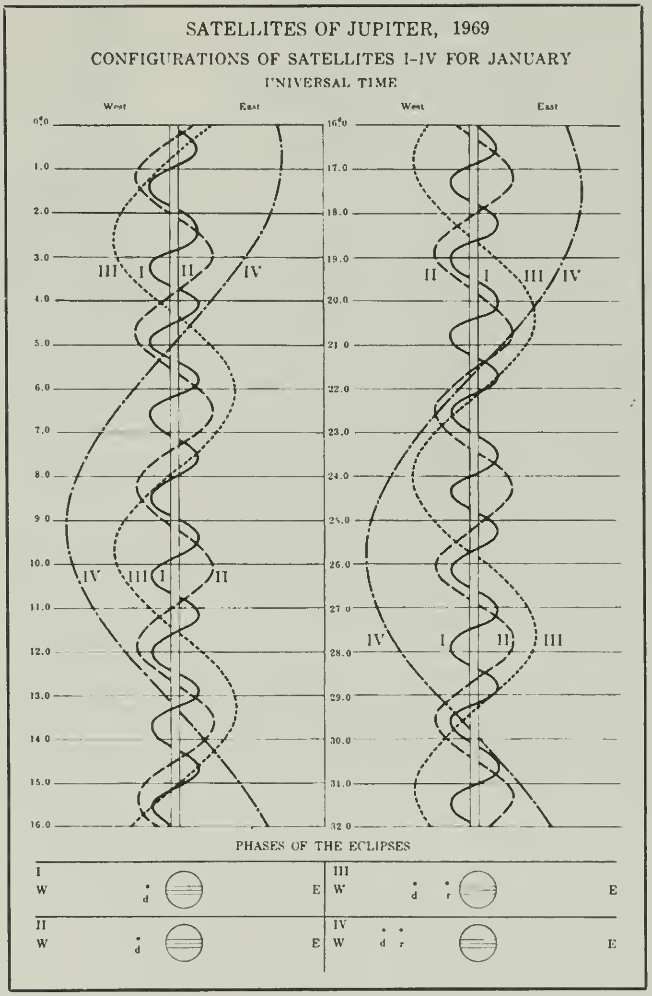
7.15 The chart on p. 79 is reproduced from the January, 1969, issue of *Sky and Telescope*.

(a) Make a sketch of how Jupiter and its satellites appeared at one week intervals, beginning with day "0."

JUPITER'S SATELLITES

The four curving lines represent Jupiter's four bright (Galilean) satellites: I, Io; II, Europa; III, Ganymede; IV, Callisto. The location of the planet's disk is indicated by the pairs of vertical lines. If a moon is invisible because it is behind the disk (that is, occulted by Jupiter), the curve is broken.

For successive dates, the horizontal lines mark 0h Universal time, or 7 p.m. Eastern standard time (or 4 p.m. Pacific standard time) on the preceding date. Along the vertical scale, 1/16 inch is almost seven hours. In this chart, west is to the left, as in an inverting telescope for a Northern Hemisphere observer. At the bottom, "d" is the point of disappearance of a satellite in the shadow of Jupiter; "r" is the point of reappearance. From the *American Ephemeris and Nautical Almanac.*



(b) Make measurements of the chart to find the R_{av} and T for each satellite. (For this problem, R_{av} can be to any convenient scale, such as cm on the diagram.)

(c) Does Kepler's law of periods T^2/R_{av}^3 = constant hold for Jupiter's satellites?

7.16 Below are two passages from Galileo's "Letters on Sunspots." On the basis of these quotations, comment on Galileo's characteristics as an observer and as a scientist.

(May 4th, 1612)

I have resolved not to put anything around Saturn except what I have already observed and revealed – that is, two small stars which touch it, one to the east and one to the west, in which no alteration has ever yet been seen to take place and in which none is to be expected in the future, barring some very strange event remote from every other motion known to or even imagined by us. But as to the supposition of Apelles that Saturn sometimes is oblong and sometimes accompanied by two stars on its flanks. Your Excellency may rest assured that this results either from the imperfection of the telescope or the eye of the observer, for the shape of Saturn is thus: oOo , as shown by perfect vision and perfect instruments, but appears thus: , where perfection is lacking, the shape and distinction of the three stars being imperfectly seen. I, who have observed it a thousand times at different periods with an excellent instrument, can assure you that no change whatever is to be seen in it. And reason, based upon our experiences of all other stellar motions, renders us certain that none ever will be seen, for if these stars had any motion similar to those of other stars, they would long since have been separated from or conjoined with the body of Saturn, even if that movement were a thousand times slower than that of any other star which goes wandering through the heavens.

(December 1, 1612)

About three years ago I wrote that to my great surprise I had discovered Saturn to be three-bodied; that is, it was an aggregate of three stars arranged in a straight line parallel to the ecliptic, the central star being much larger than the others. I believed them to be mutually motionless, for when I first saw them they seemed almost to touch, and they remained so for almost two years without the least change. It was reasonable to believe them to be fixed with respect to each other, since a single second of arc (a movement incomparably smaller than any other in even the largest orbs) would have become sensible in that time, either by separating or by completely uniting these stars. Hence I stopped observing Saturn for more than two years. But in the past few days I returned to it and found it to be solitary, without its customary supporting stars, and as perfectly round and sharply bounded as Jupiter. Now what can be said of this strange metamorphosis? That the two lesser stars have been consumed in the manner of the sunspots? Has Saturn devoured his children? Or was it indeed an illusion and a fraud with which the lenses of my telescope deceived me for so long—and not only me, but many others who have observed it with me? Perhaps the day has arrived when languishing hope may be revived in those who, led by the most profound reflections, once plumbed the fallacies of all my new observations and found them to be incapable of existing!

I need not say anything definite upon so strange an event; it is too recent, too unparalleled, and I am restrained by my own inadequacy and the fear of error. But for once I shall risk a little temerity; may this be pardoned by Your Excellency since I confess it to be rash, and protest that I mean not to register here as a prediction. but only as a probable conclusion. I say, then, that I believe that after the winter solstice of 1614 they may once more be observed.

(*Discoveries and Opinions of Galileo,* translated by Stillman Drake, Doubleday, 1957, pp. 101-102, 143-144.)

7.17 What are the current procedures by which the public is informed of new scientific theories? Do you think they are adequate? To what extent do news media emphasize clashes of points of view? Bring in some examples from news magazines.

7.18 Recently the Roman Catholic Church decided to reconsider its condemnation of Galileo. The article reproduced opposite, which appeared in *The New York Times,* July 1968, quotes passages from an Austrian Cardinal's view of the question.

(a) In the quoted remarks Cardinal Konig lists three forms of knowledge: "divine revelations," "philosophical constructions," and "spontaneously naive views of reality." Under which of these do you think he would classify Galileo's claims? Would Galileo agree?

(b) What seems to be the basis for the reconsideration? Is it doubt about the *conclusions* of the trial, or about the *appropriateness* of trying scientific ideas at all? Is it being reconsidered because of a change in Church philosophy, or because Galileo turned out to be right?

To Rehabilitate Galileo

The following are excerpts from a speech entitled "Religion and Natural Sciences" by Franz Cardinal Konig of Vienna at a meeting of Nobel Prize winners in Germany last week.

Neither the Christian churches nor modern science have managed to date to control that component of human nature which mirrors visibly a like phenomenon in the animal kingdom: aggressiveness. I hold that the neutralization of this instinct, which now is creating more dangers than ever before, ought to be a prime goal of objective cooperation between theologians and scientists. This work should try to bridge the incongruity between man's complete and perfected power of destruction and his psychic condition which remains unbridled and prey to atavism.

Removing Barriers

To enable such cooperation it is first of all necessary to remove the barriers of the past. Perhaps the biggest obstacle, blocking for centuries cooperation between religion and science, was the trial of Galileo.

For the church after the second Vatican Council, turning as it is to the world as an advocate of legitimate rights and the freedom of the human mind, the time appears to have come to terminate as thoroughly as possible the era of unpleasantness and distrust which began with Galileo's censure in 1633. For over 300 years the scientific world has rightly regarded as a painful, unhealing wound the church's unjust verdict on one of those men who prepared the path for modern science. Galileo's judgment is felt all the more painful today since all intelligent people inside and outside the church have come to the conclusion that the scientist Galileo was right and that his work particularly gave modern mechanics and physics a first, firm basis. His insights enabled the human mind to develop a new understanding of nature and universe, thus replacing concepts and notions inherited from antiquity.

An open and honest clarification of the Galileo case appears all the more necessary today if the church's claim to speak for truth, justice and freedom is not to suffer in credibility and if those people are not to lose faith in the church who in past and present have defended freedom and the right to independent thought against various forms of totalitariansim and the so-called *raison d'état.*

I am in a position to announce before this meeting that competent authorities have already initiated steps to bring the Galileo case a clear and open solution.

The Catholic Church is undoubtedly ready today to subject the judgment in the Galileo trial to a revision. Clarification of the questions which at Galileo's time were still clouded allow the church today to resume the case with full confidence in itself and without prejudice. Faithful minds have struggled for truth under pain and gradually found the right way through experience and discussions conducted with passion.

The church has learned to treat science with frankness and respect. It now knows that harmony is possible between modern man's scientific thinking and religion. The seeming contradiction between the Copernican system or, more precisely, the initial mechanics of modern physics and the Biblical story of creation has gradually disappeared. Theology now differentiates more sharply between essentially divine revelations, philosophical constructions and spontaneously naive views of reality.

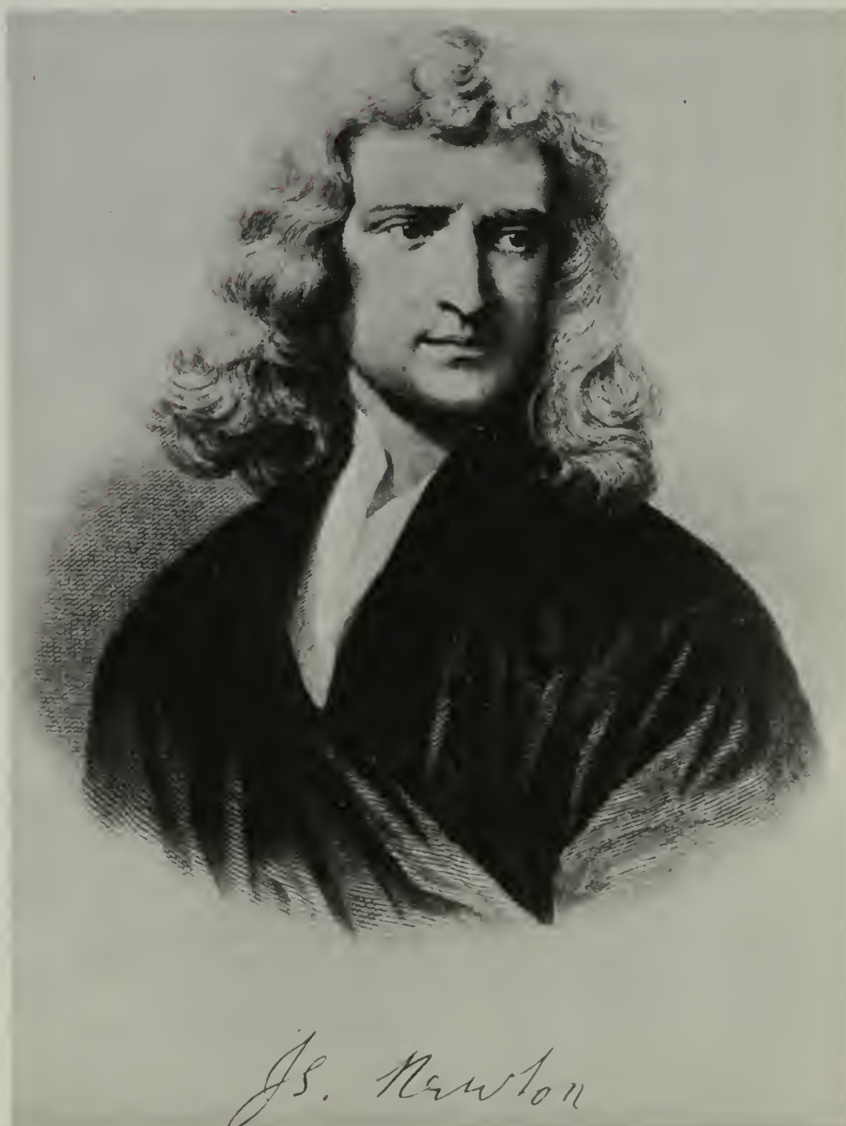
What used to be insurmountable obstacles for Galileo's contemporaries have stopped long ago to irritate today's educated faithful. From their perspective Galileo no longer appears as a mere founder of a new science but also as a prominent proponent of religious thinking. In this field, too, Galileo was in many respects a model pioneer.

Trial and Error

In Galileo's wake and in the spirit of his endeavors the Catholic church has through trial and error come to recognize the possibility of harmonious cooperation between free research and free thinking on the one hand and absolute loyalty to God's word on the other. Today's task is to draw the consequences from this recognition. Without fixing borders, God has opened his creation—the universe—to man's inquiring mind.

The church has no reason whatsoever to shun a revision of the disputed Galileo verdict. To the contrary, the case provides the church with an opportunity to explain its claim to infallibility in its realm and to define its limits. However, it will also be a chance to prove that the church values justice higher than prestige.

Excerpt from The New York Times, July 1968.



Isaac Newton (1642-1727)

CHAPTER EIGHT

The Unity of Earth and Sky —The Work of Newton

8.1 Newton and seventeenth-century science

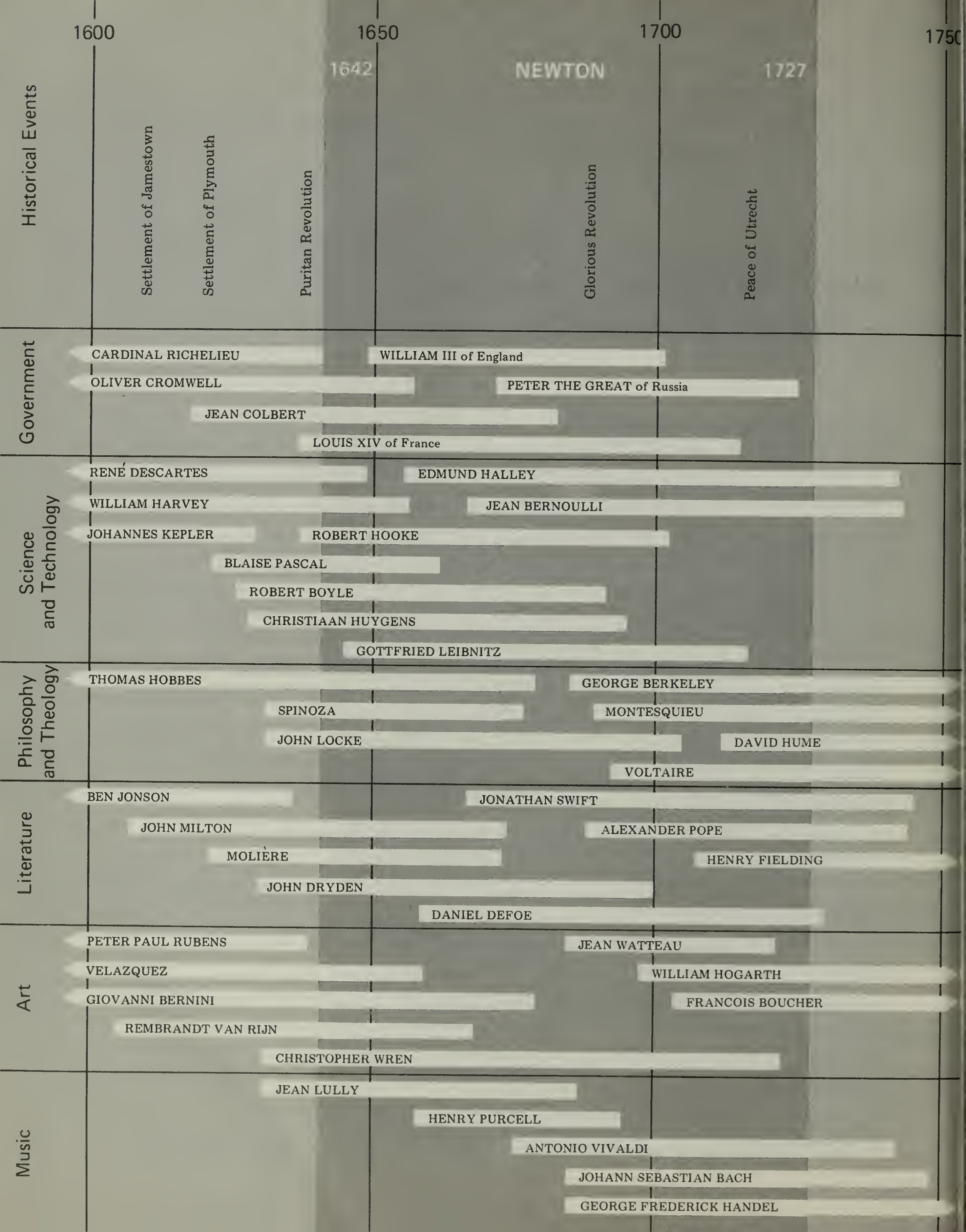
In the forty-five years between the death of Galileo in 1642 and the publication of Newton's *Principia* in 1687, major changes occurred in the social organization of scientific studies. The new philosophy of experimental science, applied by enthusiastic and imaginative men, was giving a wealth of new results. These men were beginning to work together and organize scientific societies in Italy, France, and England. One of the most famous is the Royal Society of London for Improving Natural Knowledge, which was founded in 1662. Through these societies the scientific experimenters exchanged information, debated new ideas, argued against the opponents of the new experimental activities, published technical papers, and sometimes quarreled heatedly. Each society sought public support for its work and published studies in widely read scientific journals. Through the societies, scientific activities were becoming well-defined, strong, and international.

SG8.1

This development of scientific activities was part of the general cultural, political, and economic changes occurring in the 1500's and 1600's (see the time chart on p. 84). Craftsmen and men of wealth and leisure became involved in scientific studies. Some sought the improvement of technological methods and products. Others found the study of nature through experiment a new and exciting hobby. But the availability of money and time, the growing interest in science, and the creation of organizations are not enough to explain the growing success of scientific studies. Historians agree that this rapid growth of science depended upon able men, well-formulated problems, and good experimental and mathematical tools.

The forms "1500's" and "16th century" are used interchangeably in referring to the time period roughly between 1500 and 1600.

Some of the important scientists who lived between 1600 and 1750 are shown in the time chart for the Age of Newton. The list includes amateurs as well as university professors.


1600
1650
1700
1750
1642
NEWTON
1727
Historical Events
Settlement of Jamestown
Settlement of Plymouth
Puritan Revolution
Glorious Revolution
Peace of Utrecht
Government
CARDINAL RICHELIEU
WILLIAM III of England
OLIVER CROMWELL
PETER THE GREAT of Russia
JEAN COLBERT
LOUIS XIV of France
Science and Technology
RENÉ DESCARTES
EDMUND HALLEY
WILLIAM HARVEY
JEAN BERNOULLI
JOHANNES KEPLER
ROBERT HOOKE
BLAISE PASCAL
ROBERT BOYLE
CHRISTIAAN HUYGENS
GOTTFRIED LEIBNITZ
Philosophy and Theology
THOMAS HOBBES
GEORGE BERKELEY
SPINOZA
MONTESQUIEU
JOHN LOCKE
DAVID HUME
VOLTAIRE
Literature
BEN JONSON
JONATHAN SWIFT
JOHN MILTON
ALEXANDER POPE
MOLIÈRE
HENRY FIELDING
JOHN DRYDEN
DANIEL DEFOE
Art
PETER PAUL RUBENS
JEAN WATTEAU
VELAZQUEZ
WILLIAM HOGARTH
GIOVANNI BERNINI
FRANCOIS BOUCHER
REMBRANDT VAN RIJN
CHRISTOPHER WREN
Music
JEAN LULLY
HENRY PURCELL
ANTONIO VIVALDI
JOHANN SEBASTIAN BACH
GEORGE FREDERICK HANDEL

Well-formulated problems were numerous in the writings of Galileo and Kepler. Their studies showed how useful mathematics could be when used together with experimental observation. Furthermore, their works raised exciting new questions. For example, what forces act on the planets to explain the paths actually observed? And why do objects fall as they do at the earth's surface?

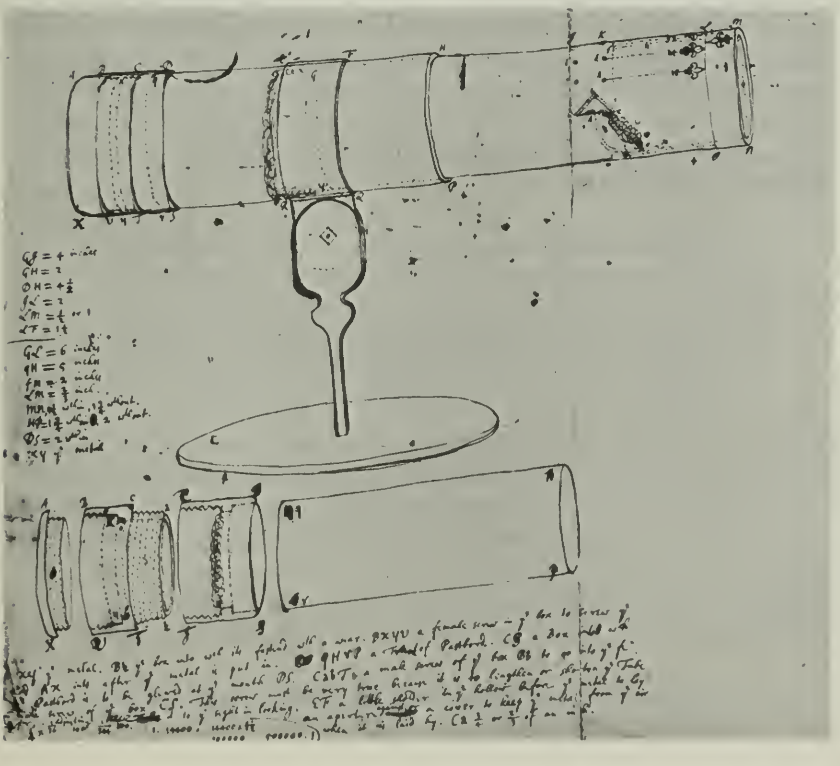
Good experimental and mathematical tools were being created. With mathematics being applied to physics, studies in each field stimulated developments in the other. Similarly, the instrument-maker and the scientist aided each other.

Another factor of great importance was the rapid accumulation of scientific knowledge itself. From the time of Galileo, repeatable experiments reported in books and journals were woven into testable theories and were available for study, modification, and application. Each study could build on those done previously.

Newton, who lived in this new scientific age, is the central person in this chapter. However, before we follow Newton's work, we must recall that in science, as in any other field, many men made useful contributions. The whole structure of science depends not only upon those whom we recognize as geniuses, but also upon many lesser-known men. As Lord Rutherford, one of the founders of modern atomic theory, said:

> It is not in the nature of things for any one man to make a sudden violent discovery; science goes step by step, and

Newton entered Trinity College, Cambridge University, in 1661 at the age of eighteen. He was doing experiments and teaching while still a student. This early engraving shows the quiet student wearing a wig and heavy academic robes.



This drawing of the reflecting telescope he invented was done by Newton while he was still a student.

> every man depends upon the work of his predecessors. . . . Scientists are not dependent on the ideas of a single man, but on the combined wisdom of thousands of men.

To tell the story properly, we should trace fully each man's dependence upon those who worked before him, the influences of his contemporaries, and his influence upon his successors. While this would be interesting and rewarding, within the space available to us we can only briefly hint at these relationships.

Isaac Newton was born on Christmas Day, 1642, in the small English village of Woolsthorpe in Lincolnshire. He was a quiet farm boy, who, like young Galileo, loved to build mechanical gadgets and seemed to have a liking for mathematics. With financial help from an uncle he went to Trinity College of Cambridge University in 1661. There he enrolled in the study of mathematics (perhaps as applied to astrology) and was an enthusiastic and successful student. In 1665 the Black Plague, which swept through England, caused the college to be closed, and Newton went home to Woolsthorpe. There, by the time he was twenty-four, he had made spectacular discoveries in mathematics (the binomial theorem and differential calculus), in optics (theory of colors), and in mechanics. During his isolation, Newton had formulated a clear concept of the first two laws of motion, the law of gravitational attraction, and the equation for centripetal acceleration. However, he did not announce the centripetal acceleration equation until many years after Huygens' equivalent statement.

This must have been the time of the famous and disputed fall of the apple. One of the records of the apple story is in a biography of Newton written in 1752 by his friend, William Stukeley. In it we read that on a particular occasion Stukeley was having tea with Newton; they were sitting under some apple trees in a garden, and Newton recalled that:

> he was just in the same situation, as when formerly, the notion of gravitation came into his mind. It was occasion'd by the fall of an apple, as he sat in a contemplative mood. Why should that apple always descend perpendicularly to the ground, thought he to himself. Why should it not go sideways or upwards, but constantly to the earth's centre?

The main emphasis in this story should probably be placed on the contemplative mood and not on the apple. Moreover, it fits again the pattern we have seen before: a great puzzle (here, that of the forces acting on planets) begins to be solved when a clear-thinking person contemplates a long-known phenomenon (such as the fall of objects on earth). Where others had seen no relationship, Newton did. Refering to the plague years Newton once wrote,

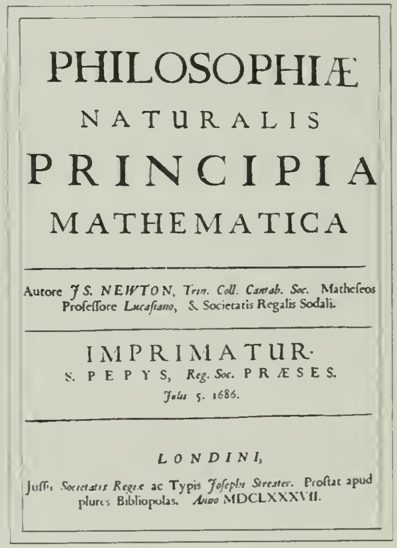
> I began to think of gravity extending to the orb of the moon, and . . . from Kepler's rule [third law, law of

periods] . . . I deduced that the forces which keep the Planets in their orbs must be reciprocally as the squares of their distances from the centers about which they revolve: and thereby compared the force requisite to keep the moon in her orb with the force of gravity at the surface of the earth, and found them to answer pretty nearly. All this was in the two plague years of 1665 and 1666, for in those days I was in the prime of my age for invention, and minded mathematics and philosophy more than at any time since.

Soon after Newton's return to Cambridge, he was chosen to follow his former teacher as professor of mathematics. He taught at the university and contributed papers to the Royal Society. At first, his contributions were mainly on optics. His *Theory of Light and Colors*, finally published in 1672, was the occasion of so long and bitter a controversy with certain other scientists that the introspective and complex man resolved never to publish anything more.

In 1684 Newton's devoted friend Halley, a noted astronomer, came to ask his advice in a controversy with Wren and Hooke about the force that would have to act on a body to cause it to move along an ellipse in accord with Kepler's laws. Halley was pleasantly surprised to learn that Newton had already derived the exact solution to this problem ("and much other matter"). Halley then persuaded his friend to publish these studies that solved one of the most debated and interesting scientific problems of the time. To encourage Newton, Halley became responsible for all the costs of publication. Less than two years later, after incredible labors, Newton had the *Principia* ready for the printer. Publication of the *Principia* in 1687 quickly established Newton as one of the greatest thinkers in history.

Several years afterward, Newton had a nervous breakdown. He recovered, but from then until his death, thirty-five years later, he made no major scientific discoveries. He rounded out earlier studies on heat and optics and turned more and more to writing on theology. During those years he received many honors. In 1699 he was appointed Warden of the Mint and subsequently its Master, partly because of his great interest in and knowledge about the chemistry of metals. In that office he helped to re-establish the value of British coins, in which lead and copper were being included in place of silver and gold. In 1689 and 1701 he represented Cambridge University in Parliament, and he was knighted in 1705 by Queen Anne. He was president of the Royal Society from 1703 to his death in 1727. He was buried in Westminster Abbey.


PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA

Autore *J S. NEWTON*, *Trin. Coll. Cantab. Soc.* Matheseos Professore *Lucasiano*, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, *Reg. Soc.* PRÆSES.
Julii 5. 1686.

LONDINI,
Jussu *Societatis Regiæ* ac Typis *Josephi Streater*. Prostat apud plures Bibliopolas. *Anno* MDCLXXXVII.

Title page of Newton's *Principia mathematica*. Because the Royal Society sponsored the book, the title page includes the name of the Society's president, Samuel Pepys, famous for his diary, which describes life during the seventeenth century.

8.2 Newton's *Principia*

In the original preface to Newton's *Principia* we find a clear outline of the book:

> Since the ancients (as we are told by Pappus) esteemed the science of mechanics of greatest importance in the investigation of natural things, and the moderns, rejecting substantial forms and occult qualities, have endeavored to subject the phenomena of nature to the laws of mathematics, I have in this treatise cultivated mathematics as far as it relates to philosophy [we would say 'physical science'] . . . for the whole burden of philosophy seems to consist in this—from the phenomena of motions to investigate [induce] the forces of nature, and then from these forces to demonstrate [deduce] the other phenomena, and to this end the general propositions in the first and second Books are directed. In the third Book I give an example of this in the explication of the system of the World; for by the propositions mathematically demonstrated in the former Books, in the third I derive from the celestial phenomena the forces of gravity with which bodies tend to the sun and the several planets. Then from these forces, by other propositions which are also mathematical, I deduce the motions of the planets, the comets, the moon, and the sea [tides] . . . .

The work begins with definitions—mass, momentum, inertia, force. Next come the three laws of motion and the principles of addition for forces and velocities (discussed in Unit 1). Newton also included an equally important and remarkable passage on "Rules of Reasoning in Philosophy." The four rules, or assumptions, reflect his profound faith in the uniformity of all nature. They were intended to guide scientists in making hypotheses, and also, we might say, to lay his philosophical cards on the table. These rules which had their roots in ancient Greece, are still useful. The first has been called a Principle of Parsimony, the second and third, Principles of Unity. The fourth expresses a faith needed for us to use the process of logic.

These rules are stated by Newton at the beginning of Book III of the *Principia.*

In a brief form, and using some modern language, Newton's rules are:

> 1. "Nature does nothing . . . in vain, and more is in vain when less will serve." Nature is essentially simple; therefore we ought not to introduce more hypotheses than are sufficient and necessary for the explanation of observed facts. This fundamental faith of all scientists is nearly a paraphrase of Galileo's "Nature . . . does not that by many things, which may be done by few." Galileo in turn was reflecting an opinion of Aristotle. Thus, the belief in simplicity has a long history.
> 2. "Therefore to the same natural effects we must, as far as possible, assign the same causes. As to respiration in a man and in a beast; the descent of stones in Europe and in America; . . . the reflection of light in the earth, and in the planets."
> 3. Properties common to all those bodies within reach of our experiments are to be assumed (even if only tentatively) to apply to all bodies in general. For example,

since all physical objects known to experimenters had always been found to have mass, this rule would guide Newton to propose that *every* object has mass (even those beyond our reach, in the celestial region).

4. In "experimental philosophy," hypotheses or generalizations which are based on experience are to be accepted as "accurately or very nearly true, notwithstanding any contrary hypotheses that may be imagined" until we have additional evidence by which our hypotheses may be made more accurate or revised.

Notice that Newton's assumption denies the distinction between terrestrial and celestial matter.

You should restate these rules in your own words before going on to the next section. (A good topic for an essay would be whether Newton s rules of reasoning are applicable outside of science.)

The *Principia* was an extraordinary document. Its three main sections contained a wealth of mathematical and physical discoveries. But overshadowing everything else in the book is the theory of universal gravitation, with the proofs and arguments leading to it. Newton uses a form of argument patterned after that of Euclid—the type of proofs you encountered in studying geometry. Because the style of detailed mathematical steps used in the *Principia* is no longer so familiar, many of the steps given above have been restated in modern terms.

The central idea of universal gravitation can be simply stated: *every object in the universe attracts every other object.* Moreover, the amount of attraction depends in a simple way on the masses of the objects and the distance between them.

This was Newton's great synthesis, boldly bringing together the terrestrial laws of force and motion and the astronomical laws of motion. Gravitation is a universal force that applies to the earth and apples, to the sun and the planets, and to all other bodies (such as comets, moving in the solar system). Heaven and Earth were united in one grand system dominated by the Law of Universal Gravitation. The general astonishment and awe were reflected in the words of the English poet Alexander Pope:

Nature and Nature's laws lay hid in night:
God said, Let Newton be! and all was light.

As you will find by inspection, the *Principia*, written in Latin, was filled with long, geometrical arguments and was difficult to read. Happily, gifted popularizers wrote summaries that allowed a wide circle of readers to learn of Newton's arguments and conclusions. One of the most widely read of these popular books was published in 1736 by the French philosopher and reformer Voltaire.

Readers of these books must have been excited and perhaps puzzled by the new approach and assumptions. From ancient Greece until well after Copernicus, the ideas of natural place and natural motion had been used to explain the general position and movements of the planets. From the time of the Greeks it was widely believed that the planets moved in their orbits because that was their "natural motion." However, to Newton the natural motion of a body was at a uniform rate along a straight line. Motion in a curve was evidence that a net force was continuously accelerating the planets away from their natural motion along straight lines. Yet the force acting on the planets was entirely natural, and acted between all bodies in heaven and on the earth. Furthermore, it was the same force that caused bodies on the earth to fall. What a reversal of the assumptions about what was "natural"!

8.3 The inverse-square law of planetary force

Newton believed that the natural path of a planet was a straight line and that it was forced into a curved path by the influence of the sun. He was able to show that Kepler's law of areas could be true if, and only if, forces exerted on the planets were always directed toward a single point. (Details of his argument for this "central" force are given on the special pages, Motion under a central force.) He showed also that the single point was the location of the sun. The law of areas will be satisfied no matter what the *magnitude* of the force is, as long as it is always directed to the same point. So it was still necessary to show that a central gravitational force would cause the precise relationship observed between orbital radius and period. But how great was the gravitational force, and how did it differ for different planets?

Newton proved that the centripetal accelerations of the six known planets toward the sun decreased inversely as the square of the planets' average distances from the sun. The proof for circular orbits is very short. The expression for centripetal acceleration a_c of a body moving uniformly in a circular path, in terms of the radius R and the period T, is

$$a_c = \frac{4\pi^2 R}{T^2}$$

(We derived this expression in Chapter 4.) As Kepler claimed in his law of periods, there is a definite relation between the orbital periods of the planets and their average distances from the sun:

$$T^2/R_{av}^3 = \text{constant}$$

If we use the symbol k for the constant, we can write

$$T^2 = kR_{av}^3$$

For circular orbits, R_{av} is just R. Substituting kR^3 for T^2 in the centripetal force equation gives

$$a_c = \frac{4\pi^2 R}{kR^3} = \frac{4\pi^2}{kR^2}$$

Since $4\pi^2/k$ is a constant, we can write simply

$$a_c \propto \frac{1}{R^2}$$

This conclusion follows necessarily from Kepler's law of periods and the definition of acceleration. If Newton's second law $F \propto a$

holds for planets as well as for bodies on earth, then there must be a centripetal force F_c acting on a planet, and it must decrease in proportion to the square of the distance of the planet from the sun: $F \propto a$ holds for planets as well as for bodies on earth, then there must be a centripetal force F_c acting on a planet, and it must decrease in proportion to the square of the distance of the planet from the sun:

$$F_c \propto \frac{1}{R^2}$$

Newton showed that the same result holds for ellipses—indeed that any object moving in an orbit, that is a conic section (circle, ellipse, parabola, or hyperbola), around a center of force is being acted upon by a centripetal force that varies inversely with the square of the distance from the center of force.

Newton had still more evidence from the telescopic observations of Jupiter's satellites and Saturn's satellites. The satellites of Jupiter obeyed Kepler's law of areas around Jupiter as a center, and the satellites of Saturn obeyed it around Saturn as a center. For Jupiter's satellites, Kepler's law of periods T^2/R^3 = constant held, but the value of the constant was different from that for the planets around the sun. It held also for Saturn's satellites, but with still a different constant. Therefore, Jupiter's satellites were acted on by a central force directed toward Jupiter, and the force decreased with the square of the distance from Jupiter and similarly for Saturn's satellites and Saturn. So Newton was able to show that the observed interactions of astronomical bodies could be accounted for by a "$1/R^2$" attractive central force.

In Newton's time, four of Jupiter's satellites and four of Saturn's satellites had been observed.

SG 8.2

Q1 What can be proved from the fact that the planets sweep out equal areas with respect to the sun in equal times?

Q2 With what relationship can T^2/R_{av}^3 = constant be combined to prove that the gravitational attraction varies as $1/R^2$?

Q3 What simplifying assumption was made in the derivation given in this section?

Q4 Did Newton limit his own derivation by the same assumption?

8.4 Law of universal gravitation

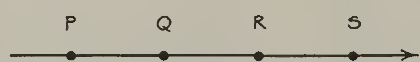
Subject to further evidences, we shall now accept that a central force is holding the planets in their orbits. Furthermore, the strength of this central force changes inversely with the square of the distance from the sun. This strongly suggests that the sun is the source of the force—but it does not necessarily require this conclusion. Newton's results so far include no physical mechanism.

The French philosopher Descartes (1596-1650) had proposed a theory in which all space was filled with a subtle, invisible fluid

Motion under a central force

How will a moving body respond to a central force? In order to follow Newton's analysis, we shall need to remember that the area of a triangle equals $\frac{1}{2}$ base $\times$ altitude. Any of the three sides can be chosen as the base, and the altitude is the perpendicular distance to the opposite corner.

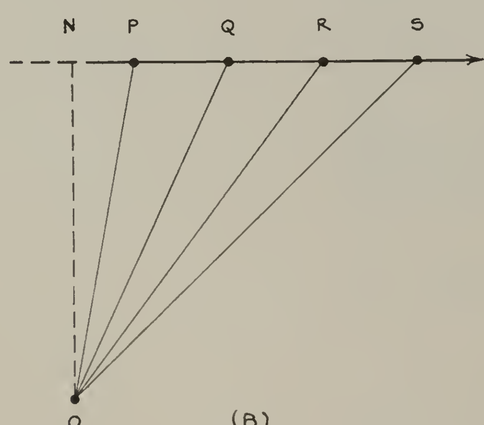
Suppose that a body was initially passing some point P, already moving at uniform speed v along the straight line through PQ. (See Fig. A below.) If it goes on



(A)

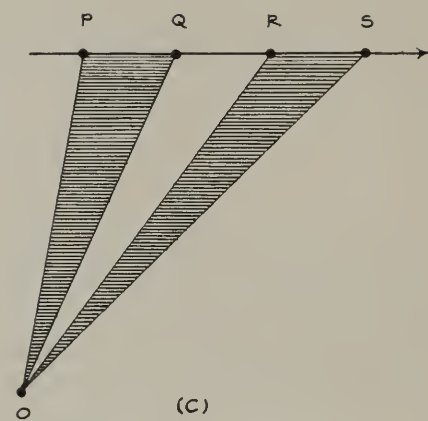
with no force acting, then in equal intervals of time Δt it will continue to move equal distances, PQ, QR, RS, etc.

How will its motion appear to an observer at some point O? Consider the triangles OPQ and OQR in Fig. B below.



(B)

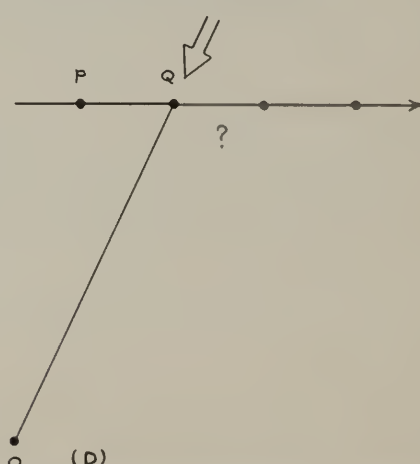
The triangles have equal bases, PQ = QR = RS, and also equal altitudes, ON for all three. Therefore the triangles OPQ and OQR have equal areas. And therefore the line drawn from an observer at point O to the body moving at a uniform speed in a straight line PQR will sweep over equal areas in equal times.



(C)

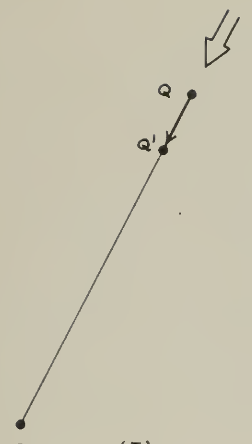
So, strange as it may seem at first, Kepler's law of areas applies even to a body on which there is no net force, and which therefore is moving uniformly along a straight line.

How will the motion of the object we discussed in Fig. A be changed if, while passing through point Q, it is exposed to a brief force, such as a blow, directed toward point O? (Refer to Fig. D below.)



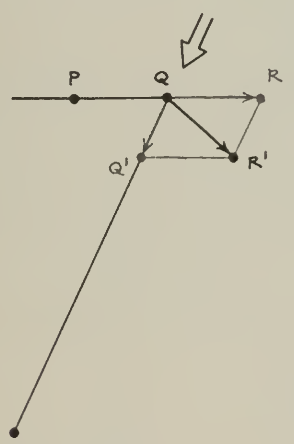
(D)

First consider what happens if a body initially at rest at point Q were exposed to the same blow. The body would be accelerated during the blow toward O. It would then continue to move toward O at constant speed, and after some definite time interval Δt, it will have moved a definite distance to a new point Q′. (See Fig. E on the next page.)



(E)

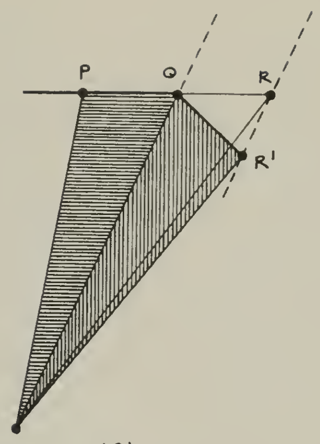
Now consider the effect of the blow on the object that was initially moving toward point R. The resultant motion is the combination of these two components—to point R′. (See Fig. F below.)



(F)

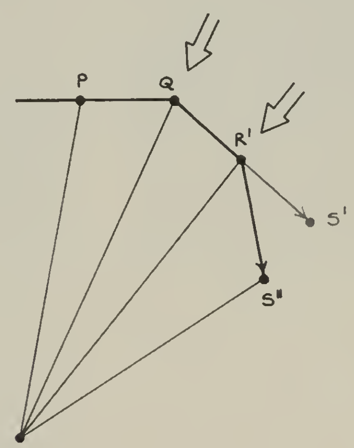
Earlier we found that the areas of the triangles OPQ and OQR were equal. Is the area of the triangle OQR′ the same? Both triangles OQR and OQR′ have a common base, OQ. Also, the altitudes of both triangles are the perpendicular distance from line OQ to line RR′. (See Fig. G.) Therefore, the areas of triangles OQR and OQR′ are equal.

If now another blow directed toward O were given at point R′, the body would move



(G)

to some point S″, as indicated in Fig. H below. By a similar analysis you can find that the areas of triangles OR′S″ and OR′S′ are equal. Their areas also equal the area of triangle OPQ.



(H)

In this geometrical argument we have always applied the force toward the same point, O. A force always directed toward a single point is called a *central* force. (Notice that the proof has nothing to do with the *magnitude* of the force, or how it changes with distance from O.) Also, we have applied the force at equal intervals Δt. If each time interval Δt were made vanishingly small, so that the force would appear to be applied continuously, the argument would still hold. We then have an important conclusion: *If a body is acted upon by any central force, it will move in accordance with Kepler's law of areas.*

which carried the planets around the sun in a huge whirlpool-like motion. This was a useful idea, and at the time it was widely accepted. However, Newton was able to prove by an elaborate and precise argument that this mechanism could not account for the details of planetary motion summarized in Kepler's laws.

SG 8.3

Kepler had made a different suggestion some years earlier. He proposed that some magnetic force reached out from the sun to keep the planets moving. His model was inadequate, but at least he was the first to regard the sun as the controlling mechanical agent behind planetary motion. And so the problem remained: was the sun actually the source of the force? If so, on what characteristics of the sun or the planets did the amount of the force depend?

As you read in Sec. 8.1, Newton had begun to think about the planetary force during the years at home in the time of the Black Plague. The idea came to him—perhaps while watching an apple fall and perhaps not—that the planetary force was the same kind of force that pulled objects down to the earth near its surface. He first tried this idea on the earth's attraction for the moon. From the data available to him, Newton knew that the distance between the center of the earth and the center of the moon was nearly sixty times the radius of the earth. If the attractive force varied as $1/R^2$, the gravitational acceleration the earth exerts on matter at the distance of the moon should be only $1/60^2$ (or 1/3600) of that exerted upon matter (say, an apple) at the surface of the earth. From observations of falling bodies it was long known that the gravitational acceleration at the earth's surface was about 9.80 meters per second per second. Therefore, the moon should fall at 1/3600 of that acceleration value: (9.80/3600) meters per second per second, or 2.72×10^{-3} m/sec^2. Does it?

To us, who have heard about gravity from our early school years, this may not seem to have been a particularly clever idea. But in Newton's time, after centuries of believing celestial events to be completely different from earthly events, it was the mental leap of a genius. Newton had already assumed the planets to be subject to the earth's laws of motion when he derived a $1/R^2$ force law using the formula for a_c. But it was a still greater step to guess that the force on planets was not some special celestial force, but nothing other than familiar old weight of everyday objects.

Newton started from the knowledge that the orbital period of the moon was very nearly $27\frac{1}{3}$ days. The centripetal acceleration a_c of a body moving uniformly with period T in a circle or radius R (developed in Sec. 4.6 of Unit 1) is $a_c = 4\pi^2R/T^2$. When we put in values for the known quantities R and T (in meters and seconds) for the moon, and do the arithmetic, we find that the *observed* acceleration is:

$$a_c = 2.74 \times 10^{-3} \text{ m/sec}^2$$

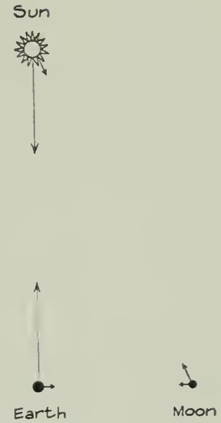
This is a very good agreement. From the values available to Newton, which were close to these, he concluded that he had

> . . . compared the force requisite to keep the moon in her orbit with the force of gravity at the surface of the earth, and found them to answer pretty nearly.
>
> Therefore, the force by which the moon is retained in its orbit becomes, at the very surface of the earth, equal to the force of gravity which we observe in heavy bodies there. And, therefore, (by Rules of Reasoning 1 and 2) the force by which the moon is retained in its orbit is that very same force which we commonly call gravity . . . .

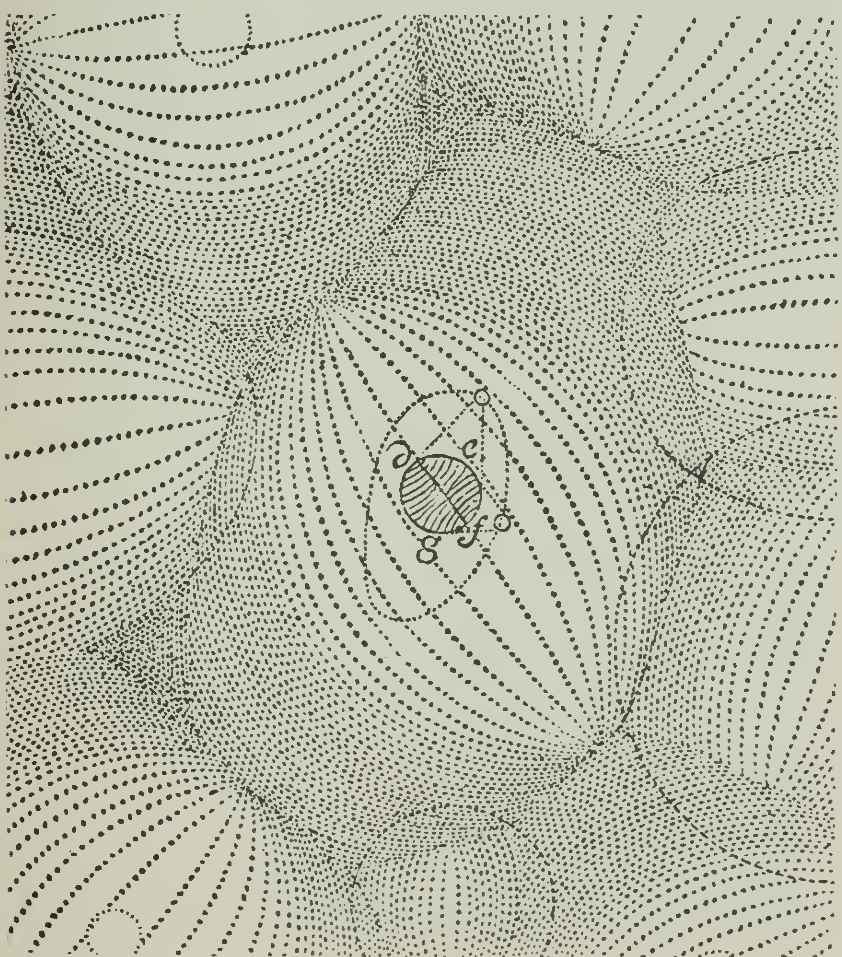
This was really a triumph: the same gravity that brings little apples down from the tree also holds the moon in its orbit. This assertion is the first portion of what is known as the Law of Universal Gravitation and says: *every object in the universe attracts every other object with a gravitational force.* If this is so, there must be gravitational forces not only between a rock and the earth, but also between the earth and the moon, between Jupiter and its satellites—and between the sun and each of the planets.

But Newton did not stop by saying only that there is a gravitational force between the planets and the sun. He further claimed that the force is just exactly the right size to account *completely* for the motion of every planet. No other mechanism is needed—no whirlpools in invisible fluids, no magnetic forces. Gravitation, and gravitation alone, underlies the dynamics of the heavens.

Because this concept is so commonplace to us, we are in danger of passing it by without really understanding what it was that Newton was claiming. First, he proposed a truly universal physical law. Guided by his Rules of Reasoning, which allowed him to extend to the whole universe what he found true for its observable



The sun, moon, and earth each pull on the other. The forces are in matched pairs, in agreement with Newton's third law of motion. As the moon moves through space, the gravitational attraction of the earth causes the moon to "fall" toward the earth. The continuous combination of its straight line inertial motion and its "fall" produce the curved orbit.



A drawing by which Descartes (1596-1650) illustrated his theory of space being filled with whirlpools of matter that drive the planets along their orbits.

parts, he excluded no object in the universe from the effect of gravity.

Less than a century before, it would have been foolish and even dangerous to suggest that terrestrial laws and forces were the same as those that regulated the whole universe. But Kepler and Galileo had begun the unification of the physics of the heavens and earth which Newton was able to carry to its conclusion. This extension of the mechanics of terrestrial objects to explain also the motion of celestial bodies is called the *Newtonian synthesis*.

A second feature of Newton's claim, that the orbit of a planet is determined by the gravitational attraction between it and the sun, was to move physics away from geometrical explanations and toward physical ones. Most philosophers and scientists before Newton had been occupied mainly with the question "What are the motions?" Newton shifted this to ask, "What force explains the motions?" In both the Ptolemaic system and Copernicus' system the planets moved about *points* in space rather than about *objects*, and they moved as they did owing to their "nature" or geometrical shape, not because forces acted on them. Newton, on the other hand, spoke not of points, but of things, of objects, of physical bodies. Without the gravitational attraction to the sun to deflect them continuously from straight-line paths, the planets would fly out into the darkness of space. Thus, it was the physical sun which was important, rather than the point at which the sun happened to be located.

Newton's synthesis centered on the idea of gravitational force. In calling it a force of gravity, Newton knew, however, that he was not explaining *why* it should exist. When you hold a stone above the surface of the earth and release it, it will accelerate to the ground. Our laws of motion tell us that there must be a force acting on the stone accelerating it toward the earth. We know the direction of the force, and we can find the magnitude of the force by multiplying the mass of the stone by the acceleration. We can give it a name: weight, or gravitational attraction to the earth. But why there is such an interaction between bodies remains a puzzle. It is still an important problem in physics today.

Q5 What idea came to Newton while he was thinking about falling objects and the moon's acceleration?

Q6 Kepler, too, believed that the sun exerted forces on the planets. How did his view differ from Newton's?

Q7 The central idea of Chapter 8 is the "Newtonian synthesis." What did Newton bring together?

8.5 Newton and hypotheses

Newton's claim that there is a mutual force (gravitational interaction) between a planet and the sun raised a new question.

How can a planet and the sun act upon each other at enormous distances without any visible connections between them? On earth you can exert a force on an object by pushing it or pulling it. We are not troubled when we see a cloud or a balloon drifting across the sky, even though nothing seems to be touching it; although air is invisible, we know that it is actually a material substance which we can feel when it blows against us. Objects falling to the earth and iron objects being attracted to a magnet are more troublesome examples, but the distances are small. However, the earth is over 90 million miles, and Saturn more than 2 billion miles, from the sun. How could there possibly be any physical contact between such distant objects? How can we account for such "action at a distance"?

There were in Newton's time, and for a long time afterward, a series of suggestions to explain how mechanical forces could be exerted at such distances. Most of these involved imagining space to be filled with some invisible substance that would transmit the force. But, at least in public, Newton refused to speculate on possible mechanisms, because he could find no way to devise an experiment to test his private guess that an ether was involved. As he said in a famous passage in the General Scholium which he added in his second edition of the *Principia* (1713):

> . . . Hitherto I have not been able to discover the cause of those properties of gravity from phenomena, and I frame no hypotheses; for whatever is not deduced from the phenomena is to be called an hypothesis; and hypotheses, whether metaphysical or physical, whether of occult qualities or mechanical, have no place in experimental philosophy . . . . And to us it is enough that gravity does really exist, and act according to the laws which we have explained, and abundantly serves to account for all the motions of the celestial bodies, and of our sea.

We quoted Newton at length because one particular phrase is frequently misquoted and misinterpreted. The original Latin reads: *hypotheses non fingo*. This means "I frame no hypotheses" or "I do not feign hypotheses," in the sense of "I do not make *false* hypotheses." We know that Newton did make numerous hypotheses in his many publications, and his letters to friends contain many other speculations which he did not publish. So his stern disavowal of hypotheses in the General Scholium must be properly interpreted. The lesson to be drawn (and it is equally useful today) is that there are two main kinds of hypotheses or assumptions:

(1) The most frequently encountered kind is a proposal of some hidden mechanism to explain observations. For example, after looking at the moving hands of a watch, we can quickly invent or imagine some arrangement of gears and springs that causes the motion. This would be a *hypothesis that is directly or indirectly testable, at least in principle, by reference to phenomena.* Our hypothesis about the watch, for example, can be tested by opening the watch or by x-raying it. Newton felt that the hypothesis of an

invisible fluid that transmitted gravitational force, the so-called "ether," has directly testable consequences quite apart from what it was first invented to account for. Many experiments had been tried to "catch" the ether; to see, for example, if any wind, or pressure, or friction due to the ether remained in a bottle from which air had been evacuated. Nothing of this sort worked (nor has it since). So Newton wisely refrained from making a hypothesis that he felt should be testable, but that was not at that time.

(2) A quite different type of assumption, often made in published scientific work, both of Newton and of scientists to this day, is a hypothesis of the sort which everyone knows is not directly testable, but which is necessary nevertheless *just to get started on one's work*. An example is such a statement as "nature is simple" or any other of Newton's Four Rules of Reasoning. The commitment to either the heliocentric system or the geocentric system was of the same kind, since all "the phenomena" could be equally accommodated in either. In choosing the heliocentric system over its rival, in making the hypothesis that the sun is at the center of the universe, Copernicus, Kepler, and Galileo were not proposing a directly testable hypothesis; rather, they were adopting a point of view which seemed to them more convincing, more simple, and as Copernicus put it, more "pleasing to the mind." It was this kind of hypothesis that Newton used without apology in his published work.

Every scientist's work involves both kinds of hypothesis. In addition—and quite contrary to the commonly held stereotype of a scientist—as a person who uses only deliberate, logical, objective thoughts—the scientist feels quite free to consider any guess, speculation, or hunch, whether it is yet provable or not, in the hope it might be fruitful. (Sometimes these are dignified by the phrase "working hypotheses.") But, like Newton, most scientists today do not like to *publish* something which is still only an unproven hunch.

Q8 Did Newton explain the gravitational attraction of all bodies?

Q9 What was the popular type of explanation for "action at a distance"?

Q10 Why didn't Newton use this type of explanation?

Q11 What are two main types of hypotheses used in science?

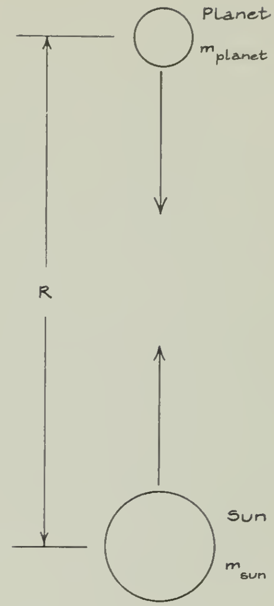
8.6 The magnitude of planetary force

The general statement that gravitational forces exist universally must now be turned into a quantitative law that gives an expression for both the *magnitude* and *direction* of the forces any two objects exert on each other. It was not enough to assert that a mutual gravitational attraction exists between the sun and say, Jupiter. To be convincing, Newton had to specify what quantitative factors determine the magnitudes of those mutual forces, and how they could be measured, either directly or indirectly.

The first problem was defining precisely the distance R – should it, for example, be taken as the distance between the surface of the earth and the surface of the moon? For many astronomical problems, the sizes of the interacting bodies are extremely small compared to the distances between them; for these cases the distance between the surfaces is practically the same as the distance between the centers, or any other part of the bodies. (For Newton's original case of the earth and the moon, the distance between centers was about 2% greater than the distance between surfaces.) Some historians believe that Newton's uncertainty about a rigorous answer to this problem led him to drop the study for many years.

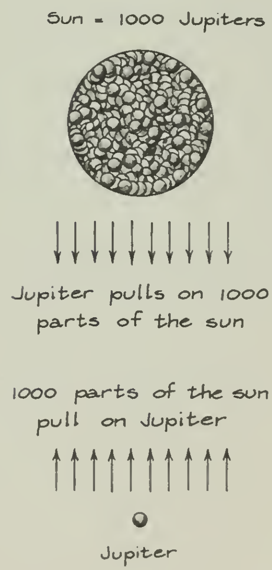
Eventually Newton was able to prove that the gravitational force exerted by a spherical body was the same as if all its mass were concentrated at its center. Conversely, the gravitational force exerted on a spherical body by another body is the same as would be exerted on it if all its mass were concentrated at its center. Therefore, the distance R in the Law of Gravitation is the distance between centers.

This was a very critical discovery. It allows us to consider the gravitational attraction between spherical bodies as though their masses were concentrated at single points; in thought we can replace the objects by *mass-points*.



The gravitational force on a planet owing to the sun's pull is equal and opposite to the gravitational force on the sun owing to the planet.

If we believe that Newton's third law (action equals reaction) is applicable universally, we must conclude that the amount of force the sun exerts on the planet is exactly equal to the amount of force the planet exerts on the sun. The claim that the forces are equal and opposite, even between a very large mass and a small mass, may seem contrary to common sense. But the equality is easy to prove if we assume only that Newton's third law holds between small chunks of matter: for example, that a 1-kg chunk of Jupiter pulls *on* a 1-kg chunk of the sun as much as it is pulled *by* it. Consider for example, the attraction between Jupiter and the sun, whose mass is about 1000 times greater than Jupiter's. As the figure in the right margin indicates, we could consider the sun as a globe containing 1000 Jupiters. Let us call one unit of force the force that two Jupiter-sized masses exert on each other when separated by the radius of Jupiter's orbit. Then Jupiter pulls on the *sun* (a globe of 1000 Jupiters) with a total force of 1000 units. Because each of the 1000 parts of the sun pulls on the planet Jupiter with one unit, the total pull of the sun on *Jupiter* is also 1000 units. Each part of the massive sun not only pulls *on* the planet, but is also pulled upon *by* the planet. The more mass there is to *attract*, the more there is to *be attracted*. (But although the mutual attractive forces are equal in magnitude, the resulting *accelerations* are not. Jupiter pulls on the sun as hard as the sun pulls on Jupiter, but the sun *responds* to the pull with only 1/1000 of the acceleration – its inertia, remember, is 1000 times Jupiter's.)



In Sec. 3.8 of Unit 1, we developed an explanation for why bodies of different mass fall with the same acceleration near the

earth's surface: the greater the inertia of a body, the more strongly it is acted upon by gravity. Or, more precisely: near the earth's surface, the gravitational force on a body is directly proportional to its mass. Like Newton, let us generalize this earthly effect to all gravitation and assume that the gravitational force exerted on a planet by the *sun* is proportional to the mass of the planet. Similarly, the gravitational force exerted on the sun by the *planet* is proportional to the mass of the sun. Since the forces the sun and a planet exert on each other are equal in magnitude, it follows that the magnitude of the gravitational force is proportional to the mass of the sun *and* to the mass of the planet. That is, the gravitational attraction between two bodies is proportional to the *product* of their
SG 8.4 masses. If the mass of either body is tripled, the force is tripled. If the masses of both bodies are tripled, the force is increased by a factor of 9. If we use the symbol F_{grav} for the magnitude of the forces we can write $F_{grav} \propto m_{planet}m_{sun}$.

Thus far we have concluded that the amount of attraction between the sun and a planet will be proportional to the product of
SG 8.5 the masses. Earlier we concluded that the attraction also depends universally on the square of the distance between the bodies. Once again we combine the two proportionalities to find one force law that now includes masses as well as distance:

$$F_{grav} \propto \frac{m_{planet}m_{sun}}{R^2}$$

Such an expression of proportionality can be written as an equation by introducing a constant to allow for the units of measurement used. Using G for the proportionality constant, we can write the law of planetary forces as:

$$F = G\,\frac{m_{planet}m_{sun}}{R^2}$$

This equation is a bold assertion that the force between the sun and any planet depends only upon the masses of the sun and planet
SG 8.6 and the distance between them. This equation seems unbelievably simple when we remember the observed complexity of the planetary motions. Yet every one of Kepler's empirical Laws of Planetary Motion, as we shall see, is consistent with this relation. More than that, Kepler's empirical laws can be derived from this force law together with Newton's second law of motion. But more important still, the force law allowed the calculation of details of planetary motion that were not possible using only Kepler's laws.

Newton's proposal that such a simple equation describes completely the forces between the sun and planets was not the final step. He believed that there was nothing unique or special about the mutual force between the sun and planets, or the earth and apples: so that an identical relation should apply universally to *any two bodies* separated by a distance that is large compared to the dimensions of the two bodies—even to two atoms or two stars. That is, he proposed that we can write a *general Law of Universal Gravitation*:

$$F_{grav} = G\frac{m_1 m_2}{R^2}$$

where m_1 and m_2 are the masses of the bodies and R is the distance between their centers. The numerical constant G, called the *constant of universal gravitation*, Newton assumed to be the same for all gravitational interaction, whether between two grains of sand, two members of a solar system, or two stars in different parts of the sky. As we shall see, the successes made possible by this simple relationship have been so great that we have come to assume that this equation applies everywhere and at all times, past, present, and future.

SG 8.7
SG 8.8

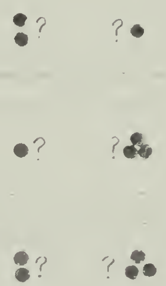
Even before we gather more supporting evidence, the sweeping majesty of Newton's theory of universal gravitation commands our wonder and admiration. It also leads to the question of how such a bold universal theory can be proved. There is no complete proof, of course, for that would mean examining every interaction between all bodies in the universe! But the greater the variety of single tests we make, the greater will be our belief in the correctness of the theory.

Q12 According to Newton's law of action and reaction, the earth should experience a force and accelerate toward a falling stone.

(a) How does the force on the earth compare with the force on the stone?

(b) How does the earth's acceleration compare with the stone's acceleration?

Q13 The top diagram at the right represents two bodies of equal mass which exert gravitational forces of magnitude F on one another. What will be the magnitude of the gravitational attractions in each case?



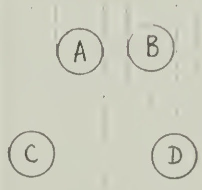
Q14 A, B, C, and D are bodies with equal masses. How do the forces of attraction that A and B exert on each other compare with the force that C and D exert on each other?

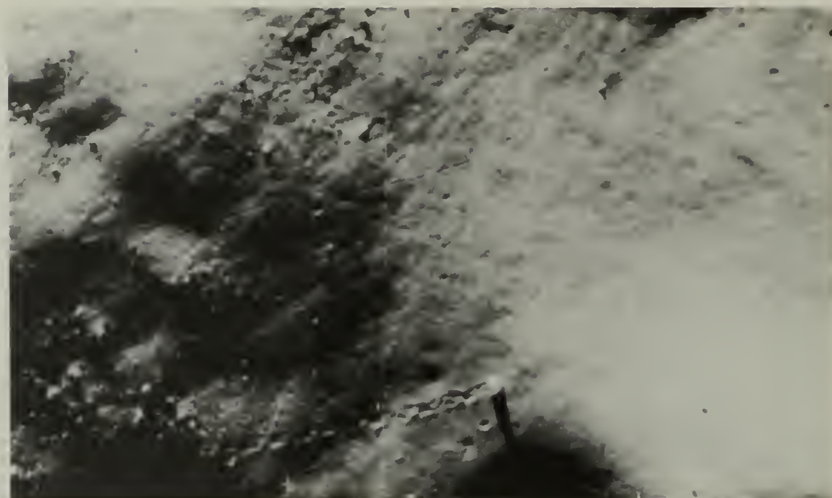
(a) $F_{AB} = 3 \times F_{CD}$

(b) $F_{AB} = 4 \times F_{CD}$

(c) $F_{AB} = 9 \times F_{CD}$

(d) $F_{AB} = 16 \times F_{CD}$





This photograph, taken from an unmanned capsule orbiting the moon, shows some latter-day evidence that the laws of mechanics for heavenly bodies are the same as for the earth: the trails of two huge boulders that rolled 1000 ft down a lunar slope.

8.7 Planetary motion and the gravitational constant

According to Newton's mechanics, if a planet of mass m_p is moving along an orbit of radius R and period T, there is continually a centripetal acceleration $a_c = 4\pi^2 R/T^2$. Therefore, there must continually be a central force $F_c = m_p a_c = 4\pi^2 R m_p/T^2$. If we identify gravity as the central force, then

$$F_{grav} = F_c$$

$$\text{or} \qquad G\frac{m_p m_{sun}}{R^2} = \frac{4\pi^2 R m_p}{T^2}$$

By simplifying this equation and rearranging some terms, we can get an expression for G:

$$G = \frac{4\pi^2}{m_{sun}}\left(\frac{R^3}{T^2}\right)$$

We know from Kepler that for the planets' motion around the sun, the ratio R^3/T^2 is a constant; $4\pi^2$ is a constant also. If we assume that the mass of the sun is constant, then all the factors on the right of the equation for G are constant. So G must be a constant for the gravitational effect of the sun on the planets. By similar reasoning the value of G must be a constant for the effect of Jupiter on its moons—and for the effect of Saturn on its moons—and for an apple and the moon above the earth. But is it the same value of G for all these cases?

It is impossible to *prove* that G is the same for the gravitational interaction of *all* bodies. If, however, we *assume* that G is a universal constant, we can get some remarkable new information—the relative masses of the sun and the planets!

We begin by again equating the centripetal force on the planets with the gravitational attraction to the sun, but this time we solve the equation for m_{sun}.

$$F_{grav} = F_c$$

$$\frac{G\, m_p\, m_{sun}}{R^2} = \frac{4\pi^2 R m_p}{T^2}$$

$$m_{sun} = \frac{4\pi^2 R^3}{G\, T^2}$$

If we write k_{sun} for the constant ratio T^2/R^3, we have

$$m_{sun} = \frac{4\pi^2}{G k_{sun}}$$

By similar derivation, $m_{Jupiter} = \dfrac{4\pi^2}{G k_{Jupiter}}$

$$m_{Saturn} = \frac{4\pi^2}{G k_{Saturn}}$$

$$m_{earth} = \frac{4\pi^2}{G k_{earth}}$$

where $k_{Jupiter}$, k_{Saturn}, and k_{earth} are the known values of the constant ratios T^2/R^3 for the satellites of Jupiter, Saturn, and the earth.

To compare Jupiter's mass to the mass of the sun, we have only to divide the formula for $m_{Jupiter}$ by the formula for m_{sun}:

$$\frac{m_{Jupiter}}{m_{Sun}} = \frac{\dfrac{4\pi^2}{G k_{Jupiter}}}{\dfrac{4\pi^2}{G k_{Sun}}}$$

or $$\frac{m_{Jupiter}}{m_{sun}} = \frac{k_{sun}}{k_{Jupiter}}$$

SG 8.9

Similarly, the mass of any two planets can be compared if the values of T^2/R^3 are known for them both—that is, if they both have satellites whose motion has been carefully observed.

Masses Compared to Earth

Earth	1
Saturn	95
Jupiter	318
Sun	333,000

These comparisons are based on the *assumption* that G is a universal constant. Calculations based on this assumption have led to *consistent* results on a wide variety of astronomical data, including the behavior of a space ship orbiting and landing on the moon. Results consistent with this assumption were also found when more difficult calculations were made for the small disturbing effects that the planets have on each other. There is still no way of proving G is the same everywhere and always, but it is a reasonable working assumption until evidence to the contrary appears.

If the numerical value of G were known, the actual masses of the earth, Jupiter, Saturn, and the sun could be calculated. G is defined by the equation $F_{grav} = G\, m_1 m_2/R^2$. To find the value of G it is necessary to know values for all the other variables—that is, to measure the force F_{grav} between two measured masses m_1 and m_2 when they are separated by a measured distance R. Newton knew this, but in his time there were no instruments sensitive enough to measure the very tiny force expected between any masses small enough for experimental use.

Q15 What information can be used to compare the masses of two planets?

Q16 What additional information is necessary for calculation of the actual masses?

8.8 The value of G and the actual masses of the planets

The masses of small solid objects can be found easily enough from their weights. Measuring the distance between solid objects of spherical shape is also not a problem. But how is one to measure the small mutual gravitational force between relatively small objects in a laboratory, particularly when each is also experiencing separately a huge gravitational force toward the tremendously massive earth?

This serious technical problem of measurement was eventually solved by the English scientist, Henry Cavendish (1731-1810). As a device for measuring gravitational forces, he employed a torsion balance in which the gravitational force of attraction between two pairs of lead spheres twisted the wire holding up one of the pairs. The twist of the wire could be calibrated by measuring the twist produced by small known forces. A typical result for a 100 kg sphere and a 1 kg sphere at a center-to-center distance of 0.1m would be a force of about one-millionth of a newton! As the calculations in the margin show, these data lead to a value for G of about 10^{-10} ($N{\cdot}m^2/kg^2$). This experiment has been progressively improved, and the accepted value of G is now:

Calculation of G from approximate experimental values:

$F \quad G\frac{Mm}{R}$

so $G \frac{F \;\; R}{Mm}$

$\frac{(10 \;\; N)(0.1\ m)}{(100\ kg)(1\ kg)}$

$\frac{10 \times 10}{10}$ N m /kg

10 N m /kg

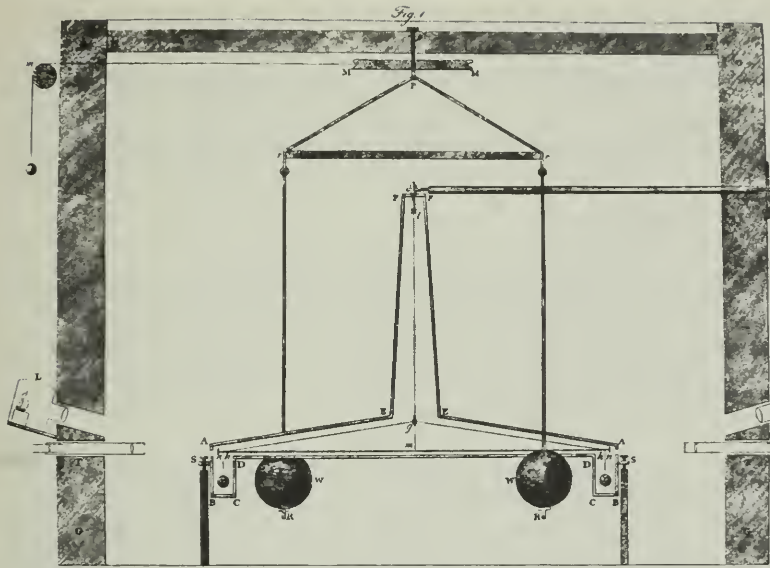
$$G = 6.67 \times 10^{-11}\ N{\cdot}m^2/kg^2$$

Evidently gravitation is a weak force which becomes important only when at least one of the masses is very great. The gravitational force on a 1-kg mass at the surface of the earth is 9.8 newtons. (which we know because, if released, it will fall with an acceleration of 9.8 m/sec.) Substituting 9.8 newtons for F_{grav} and substituting the radius of the earth for R, you can calculate the mass of the earth! (See SG 8.11)

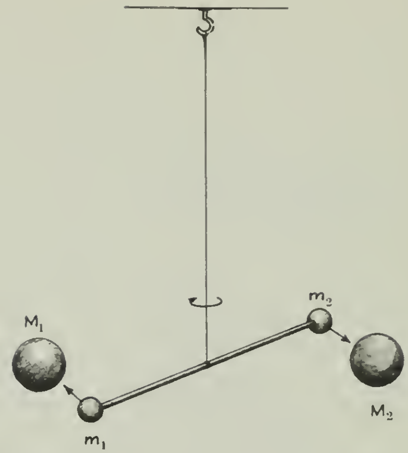
N m /kg can be expressed as m /kg sec

SG 8.10–8.13

By assuming that the same value for G applies to all gravitational interaction, we can calculate values for the masses of bodies from the known values of T^2/R^3 for their satellites. Since Newton's time, satellites have been discovered around all of the outer planets except Pluto. The values of their masses, calculated from $m = 4\pi^2/G \times R^3/T^2$, are given in the table on the next page. Venus and Mercury have no satellites, but values for their masses



Cavendish's original drawing of his apparatus for determining the value of G. To prevent disturbance from air currents, he inclosed it in a sealed case. He observed the deflection of the balance rod from outside with telescopes.



Schematic diagram of the device used by Cavendish for determining the value of the gravitational constant G. Large lead balls of masses M_1 and M_2 were brought close to small lead balls of masses m_1 and m_2. The mutual gravitational attraction between M_1 and m_1 and between M_2 and m_2, caused the vertical wire to be twisted by a measurable amount.

are found by analyzing the slight disturbing effects each has on other planets. Modern values for the actual masses of the planets are listed in the margin. Notice that the planets taken together add up to not much more than 1/1000th part of the mass of the solar system. By far, most of the mass is in the sun, and this of course accounts for the fact that the sun dominates the motion of the planets, acting like an almost infinitely massive and fixed object.

In all justice to the facts, we should modify this picture a little. Newton's third law tells us that for every pull the sun exerts on a planet the sun itself experiences an equally strong pull in the opposite direction. Of course the very much greater mass of the sun keeps its acceleration to a correspondingly smaller value. But this acceleration is, after all, not exactly zero. Hence, the sun cannot be really fixed in space even in the heliocentric system, if we accept Newtonian dynamics, but rather it moves a little about the point that forms the common center of mass of the sun and the moving planets that pull on it. This is true for every one of the 9 planets; and since these generally do not happen to be moving all in one line, the sun's motion is actually a complex superposition of 9 small ellipses. Still, unless we are thinking of a solar system in which the planets are very heavy compared to their sun, such motion of the sun is not large enough to be of interest to us for most purposes.

Actual Masses

Sun	1,980,000	$\times 10^{24}$ kg
Mercury	.328	
Venus	4.83	
Earth	5.98	
Mars	.637	
Jupiter	1,900	
Saturn	567	
Uranus	88.0	
Neptune	103.	
Pluto	1.1	

SG 8.14–8.17

Q17 Which of the quantities in the equation $F_{grav} = G\, m_1 m_2/R^2$ did Cavendish *measure*?

Q18 Knowing a value for G, what other information can be used to find the mass of the earth?

Q19 Knowing a value for G, what other information can be used to find the mass of Saturn?

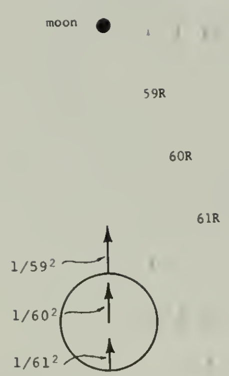
Q20 The mass of the sun is about 1000 times the mass of Jupiter. How does the sun's acceleration due to Jupiter's attraction compare with Jupiter's acceleration due to the sun's attraction?

8.9 Further successes

Newton did not stop with the fairly direct demonstrations we have described so far. In the *Principia* he showed that his law of universal gravitation could account also for more complicated gravitational interactions, such as the tides of the sea and the perverse drift of comets across the sky.

The tides: The flooding and ebbing of the tides, so important to navigators, tradesmen, and explorers through the ages, had remained a mystery despite the studies of such men as Galileo. Newton, however, through the application of the law of gravitation, was able to explain the main features of the ocean tides. These he found to result from the attraction of the moon and sun upon the waters of the earth. Each day two high tides normally occur. Also, twice each month, when the moon, the sun, and the earth are in line, the tides are significantly higher than average.

Two questions about tidal phenomena demand special attention. First, why do high tides occur on both sides of the earth, including the side away from the moon? Second, why does the time of high tide occur at a location some hours after the location has passed directly under the moon?



Tidal Forces.
The earth-moon distance indicated in the figure is greatly reduced because of the space limitations.

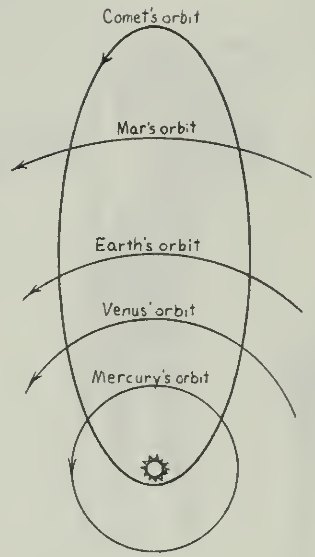
Newton realized that the tides result from the *difference* between the acceleration (due to the moon and sun) of the whole solid earth and the acceleration of the fluid waters at the earth's surface. The moon's distance from the earth's center is 60 earth radii. On the side of the earth nearer the moon, the distance of the water from the moon is only 59 radii. On the side of the earth, away from the moon, the water is 61 earth radii from the moon. The accelerations are shown in the figure at the left. On the side of the earth nearer the moon, the acceleration of the water toward the moon is greater than the acceleration of the earth as a whole—the net effect is that the water is accelerated away from the earth. On the side of the earth away from the moon, the acceleration of the water toward the moon is less than that of the earth as a whole—the net result is that the earth is accelerated away from the water.

If you have been to the seashore or examined tide tables, you know that high tide does not occur when the moon is highest in the sky, but some hours later. To explain this even qualitatively, we must remember that on the whole the oceans are not very deep. As a result, the waters moving in the oceans in response to the moon's attraction are slowed by friction with the ocean floors, especially in shallow water, and consequently the time of high tide is delayed.

In any particular place, the amount of the delay and the height of the tides depends greatly upon the ease with which the waters can flow. No general theory can be expected to account for all the particular details of the tides. Most of the local predictions in the tide tables are based on empirical rules using the cyclic variations recorded in the past.

Since there are tides in the seas, you may wonder if there are tides in the fluid atmosphere and in the earth itself. There are. The earth is not completely rigid, but bends somewhat like steel. The tide in the earth is about a foot high. The atmospheric tides are generally masked by other weather changes. However, at heights of about a hundred miles, where satellites have been placed in orbit, the thin atmosphere rises and falls considerably.

Comets: Comets, whose unexpected appearances throughout antiquity and the Middle Ages had been interpreted as omens of disaster, were shown by Halley and Newton to be nothing more than a sort of shiny, cloudy mass that moved around the sun according to Kepler's laws, just as planets do. They found that most comets were visible only when closer to the sun than the distance of Jupiter. Several of the very bright comets were found to have orbits that took them well inside the orbit of Mercury, to within a few million miles of the sun, as the figure at the right indicates. Many of the orbits have eccentricities near 1.0 and are almost parabolas; these comets have periods of thousands or even millions of years. Some other faint comets have periods of only five to ten years.



Schematic diagram of the orbit of a comet projected onto the ecliptic plane; comet orbits are tilted at all angles.

Unlike the planets, whose orbits are nearly in a single plane, the planes of comet orbits are tilted at all angles. Yet, like all members of the solar system, they obey all the laws of dynamics, including that of universal gravitation.

Edmund Halley applied Newton's concepts of celestial motions to the motion of bright comets. Among the comets he studied were those seen in 1531, 1607, and 1682, whose orbits he found to be very nearly the same. Halley suspected that these might be the same comet, seen at intervals of about seventy-five years and moving in a closed orbit. He predicted that it would return in about 1757 – which it did, although Halley did not live to see it. Halley's comet appeared in 1833 and 1909 and is due to be near the sun and bright again in 1985.

With the period of this bright comet known, its approximate dates of appearance could be traced back in history. In the records found in ancient Indian, Chinese, and Japanese documents, this comet has been identified at all expected appearances except one since 240 B.C. That the records of such a celestial event are incomplete in Europe is a sad commentary upon the level of interests and culture of Europe during the so-called Dark Ages. One of the few European records of this comet is the famous Bayeux tapestry, embroidered with seventy-two scenes of the Norman Conquest of England in 1066; it shows the comet overhead and the

SG 8.18

A scene from the Bayeux tapestry, which was embroidered about 1070. The bright comet of 1066 can be seen at the top of the figure. This comet was later identified as being Halley's comet. At the right, Harold, pretender to the throne of England, is warned that the comet is an ill omen. Later that year at the Battle of Hastings, Harold was defeated by William the Conqueror.

frightened ruler and court cowering below. A major triumph of Newtonian science was its use to explain that comets, which for centuries had been fearful events, were regular members of the solar system.

The scope of the principle of universal gravitation: Newton made numerous additional applications of his law of universal gravitation which we cannot consider in detail here. He investigated the causes of the somewhat irregular motion of the moon and showed that these causes are explainable by the gravitational forces acting on the moon. As the moon moves around the earth, the moon's distance from the sun changes continually. This changes the resultant force of the earth and sun on the orbiting moon. Newton also showed that other changes in the moon's motion occur because the earth is not a perfect sphere, but has an equatorial diameter twenty-seven miles greater than the diameter through the poles. Newton commented on the problem of the moon's motion that "the calculation of this motion is difficult." Even so, he obtained predicted values in reasonable agreement with the

SG 8.19

observed values available at that time, and even predicted some details of the motion which had not been noticed before.

Newton investigated the variations of gravity at different latitudes on the spinning and bulging earth. From the differences in the rates at which pendulums swung at different latitudes, he was able to derive an approximate shape for the earth.

In short, Newton had created a whole new quantitative approach to the study of astronomical motion. Because some of his predicted variations had not been observed, improved instruments were built. These were needed anyway to improve the observations which could now be fitted together under the grand theory. Numerous new theoretical problems also clamored for attention. For example, what were the predicted and observed influences among the planets themselves upon their motions? Although the planets are small compared to the sun and are very far apart, their interactions are observable. As precise data have accumulated, the Newtonian theory has permitted calculations about the past and future states of the planetary system. For past or future intervals up to some hundreds of millions of years (beyond which the extrapolations become too uncertain) the Newtonian theory tells us that the planetary system has been and will be about as it is now.

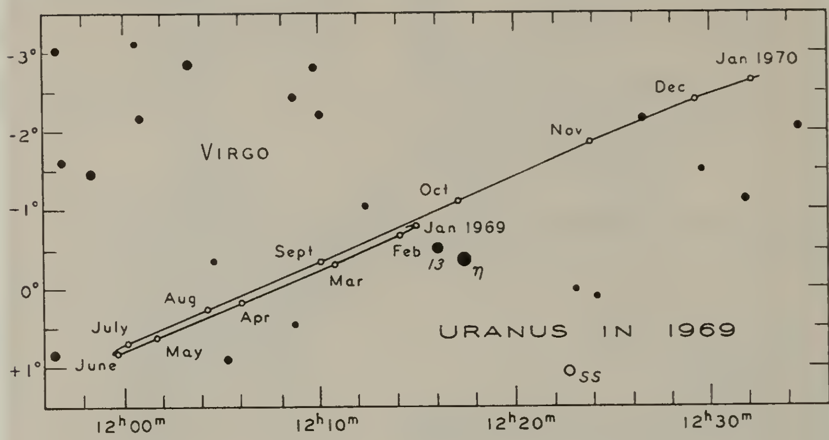
What astonished Newton's contemporaries and increases our own esteem for him was not only the scope and genius of his work in mechanics, not only the great originality and the elegance of his proofs, but the detail in which he developed the implications of each of his ideas. Having satisfied himself of the correctness of his principle of universal gravitation, he applied it to a wide range of terrestrial and celestial problems, with the result that the theory became more and more widely accepted. Nor has it failed us since for any of the new problems concerning motion in the solar system; for example, the motion of every artificial satellite and space probe has been reliably calculated on the assumption that at every instant a gravitational force is acting on it according to Newton's law of universal gravitation. We can well agree with the reply given to ground control as Apollo 8 returned from man's first trip to the moon—ground control: "Who's driving up there?" Apollo 8: "I think Isaac Newton is doing most of the driving right now."

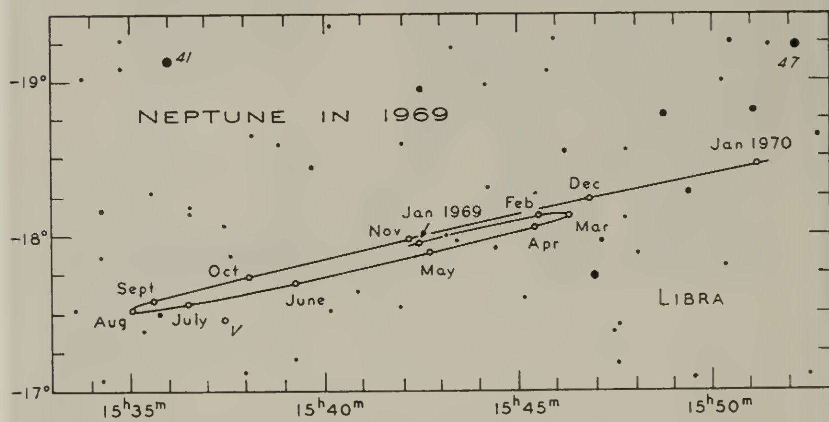
Tiny variations from a $1/R^2$ centripetal acceleration of satellites in orbit around the moon have led to a mapping of "mascons" on the moon—usually dense concentrations of mass under the surface.

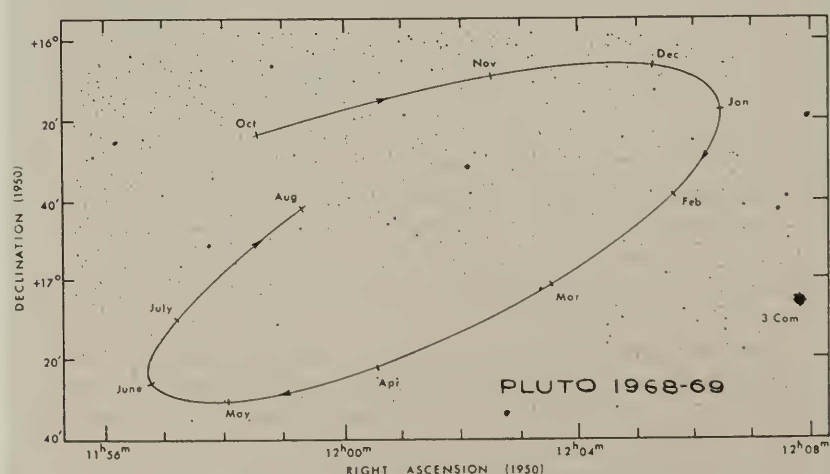
Beyond the solar system: We have seen how Newton's laws have been applied to explain the motions and other observables about the earth and the entire solar system. But now we turn to a new and even more grandiose question. Do Newton's laws, which are so useful within the solar system, also apply at greater distances, for example among the stars?

Over the years following publication of the *Principia*, several sets of observations provided an answer to this important question. Toward the end of the 1700's, William Herschel, a British musician turned amateur astronomer, was, with the help of his sister

Paths of the outer three planets during 1969 (Diagrams reproduced from *Sky and Telescope* magazine.)

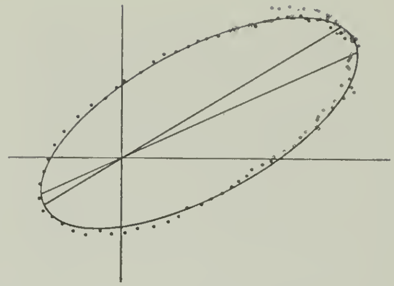






The planet Uranus was discovered in 1781 with a reflecting telescope. Disturbances in Uranus' orbit observed over many years led astronomers to seek another planet beyond Uranus: Neptune was observed in 1846 just where it was expected to be from analysis of Uranus' orbit disturbance (by Newtonian mechanics). A detailed account of the Neptune story appears in the Project Physics Supplementary Unit, *Discoveries in Physics*. Disturbances observed in Neptune's orbit over many years led astronomers to seek still another planet. Again the predictions from Newtonian mechanics were successful, and Pluto (too faint to be seen by eye even in the best telescopes) was discovered in 1930 with a long time-exposure photograph.

Caroline, making a remarkable series of observations of the sky through his homemade, but high quality telescopes. While planning how to measure the parallax of stars due to the earth's motion around the sun, he noted that sometimes one star had another star quite close. He suspected that some of these pairs might actually be double stars held together by their mutual gravitational attractions rather than being just two stars in nearly the same line of sight. Continued observations of the directions and distances from one star to the other of the pair showed that in some cases one star moved during a few years in a small arc of a curved path around the other (the figure shows the motion of one of the two stars in a system.) When enough observations had been gathered, astronomers found that these double stars, far removed from the sun and planets, also moved around each other in accordance with Kepler's laws, and therefore, in agreement with Newton's law of universal gravitation. Using the same equation as we used for planets (see p. 103), astronomers have calculated that the masses of these stars range from about 0.1 to 50 times the sun's mass.



The motion over many years for one of the two components of a binary star system. Each circle indicates the average of observations made over an entire year.

A theory can never be completely proven; but it becomes increasingly acceptable as it is found useful over a wider and wider range of problems. No theory has stood this test better than Newton's theory of universal gravitation as applied to the planetary system. After Newton, it took nearly a century for physicists and astronomers to comprehend, verify, and extend his work as applied to the problems of planetary motion. After two centuries (in the late 1800's), it was still reasonable for leading scientists and philosophers to claim that most of what had been accomplished in the science of mechanics since Newton's day was but a development or application of his work.

Q21 Why does the moon cause the water level to rise on *both* sides of the earth?

Q22 In which of the following does the moon produce tides?
(a) the seas (b) the atmosphere (c) the solid earth

Q23 Why is the precise calculation of the moon's motion so difficult?

Q24 How are the orbits of comets different from the orbits of the planets?

Q25 Do these differences affect the validity of Newton's law of universal gravitation for comets?

8.10 Some effects and limitations of Newton's work

Today we honor Newton and his system of mechanics for many valid reasons. The content of the *Principia* historically formed the basis for the development of much of our physics and technology. Also, the success of Newton's approach to his problems became the method used in all the physical sciences for the subsequent two centuries.

Throughout Newton's work, we find his basic belief that celestial phenomena can be explained by applying quantitative earthly laws. He felt that his laws had physical meaning and were not just mathematical conveniences behind which unknowable laws were hiding, but rather just the opposite: the natural physical laws governing the universe were accessible to man, and the simple mathematical forms of the laws were evidence of their reality.

Newton combined the skills and approaches of both the experimental and the theoretical scientist. He made ingenious pieces of equipment, such as the first reflecting telescope, and performed skillful experiments, especially in optics. Yet he also applied his great mathematical and logical powers to the creation of explicit, testable predictions.

Many of the concepts which Newton used came from his scientific predecessors and contemporaries. For example, Galileo and Descartes had contributed the first steps to a proper idea of inertia, which became Newton's First Law of Motion; Kepler's planetary laws were central in Newton's consideration of planetary motions; Huygens, Hooke, and others clarified the concepts of force and acceleration, ideas which had been evolving for centuries.

In addition to his own experiments, Newton selected and used data from a large number of sources. Tycho Brahe was only one of several astronomers whose observations of the motion of the moon he used. When he could not complete his own measurements, he knew whom he could ask.

Lastly, we must recall how exhaustively and how fruitfully he used and expanded his own specific contributions. For instance, in developing his theory of universal gravitation, he used his laws of motion and his various mathematical inventions again and again. Yet Newton was modest about his achievements, and he once said that if he had seen further than others "it was by standing upon the shoulders of Giants."

We recognize today that Newton's mechanics holds only within a well-defined region of our science. For example, although the forces within each galaxy appear to be Newtonian, this may not be true for forces acting between one galaxy and another. At the other end of the scale, among atoms and subatomic particles, we shall see that an entirely non-Newtonian set of concepts had to be developed to account for the observations.

Even within the solar system, there are several small discrepancies between the predictions and the observations. The most famous is the angular motion of the axis of Mercury's orbit, which is greater than the value predicted from Newton's laws by about 1/80° per century. For a while, it was thought that the error might arise because gravitational force does not vary inversely exactly with the square of the distance—perhaps, for example, the law was $F_{grav} = 1/R^{2.000001}$.

Such difficulties should not be hastily assigned to some minor imperfection in the Law of Gravitation, which applies so well with unquestionable accuracy to all the other planetary motions. It may be that the whole idea behind the theory is mistaken, as was the idea behind the Ptolemaic system of epicycles. Out of many studies has come the conclusion that there is no way that details of Newtonian mechanics can be modified to explain certain observations. Instead, these observations can be accounted for only by constructing *new* theories based on some very different assumptions. The predictions from these theories are almost identical to those from Newton's laws for phenomena familiar to us, but they are accurate in some extremes where the Newtonian predictions begin to show inaccuracies. Newtonian science is linked at one end with *relativity theory*, which is important for bodies with very great mass or moving at very high speeds. At the other end Newtonian science approaches *quantum mechanics*, which is important for particles of extremely small mass and size—atoms, molecules, and nuclear particles. For a vast range of problems between these extremes, the Newtonian theory gives accurate results and is far simpler to use. Moreover, it was in Newtonian mechanics that relativity theory and quantum mechanics took root.

Newtonian mechanics refers to the science of the motion of bodies, based on Newton's work. It includes his laws of motion and of gravitation as applied to a range of bodies from microscopic size to stars, and incorporates developments of mechanics for over two centuries after Newton's own work.

SG 8.20—8.22

EPILOGUE In this unit we started at the beginnings of recorded history and followed the attempts of men to explain the cyclic motions observed in the heavens. We had several purposes. The first was to examine with some care the difficulties of changing from an earth-centered view of the heavens to the modern one in which the earth came to be seen as just another planet moving around the sun. We also wanted to put into perspective Newton's synthesis of earthly and heavenly motions. From time to time we have also suggested that there was an interaction of these new world views with the general culture. We stressed that each contributor was a creature of his times, limited in the degree to which he could abandon the teachings on which he was reared. Gradually, through the successive work of many, a new way of looking at heavenly motions arose. This in turn opened new possibilities for even further new ideas, and the end is not in sight.

Still another purpose was to see how theories are made and tested, what is the place of assumption and experiment, of mechanical models and mathematical description. In later parts of the course, we will come back to the same questions in more recent context, and we shall find that the attitudes developed toward theory-making during the seventeenth-century scientific revolution are still immensely fruitful today.

In our study we have referred to scientists in Greece, Egypt, Poland, Denmark, Austria, Italy, England, and other countries. Each, as Newton said, stood on the shoulders of others. And for each major success there are many lesser advances or, indeed, failures. We see science as a cumulative intellectual activity not restricted by national boundaries or by time. It is not inevitably and relentlessly successful, but it grows more as a forest grows, new growth replacing and drawing nourishment from the old, sometimes with unexpected changes in its different parts. It is not a cold, calculated pursuit, for it may involve passionate controversy, religious convictions, esthetic judgments of what beauty is, and sometimes wild private speculation.

It is also clear that the Newtonian synthesis did not put an end to the study of science by solving all problems. In many ways it opened whole new lines of investigations, both theoretical and observational. In fact, much of our present science and also our technology had their effective beginnings with the work of Newton. New models, new mathematical tools, and new self-confidence (sometimes misplaced, as the study of the nature of light will show) encouraged those who followed, to attack the new problems. A never-ending series of questions, answers, and more questions was well launched. The modern view of science is that it is a continuing quest into ever more interesting fields.

Among the many problems remaining after Newton's work was the study of objects interacting not by gravitational forces, but by friction and collision. This led, as the next unit shows, to the concepts of momentum and energy, and then to a much broader view of the connection between different parts of science—physics, chemistry, and biology. Eventually, from this line of study, emerged other statements as grand as Newton's law of universal gravitation: the conservation laws on which so much of modern science – and technology – is based, especially the part having to do with many interacting bodies making up a system. That account will be the main subject of Unit 3.

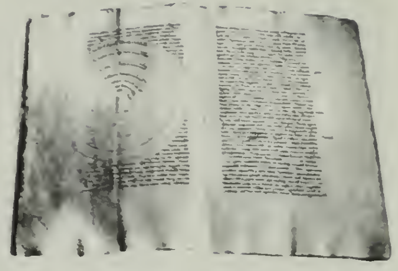
Newton's influence was not, however, limited to science alone. The century following the death of Newton in 1727 was a period of consolidation and further application of Newton's discoveries and methods, whose effects were felt especially in philosophy and literature, but also in may other fields outside science. Let us round our view of Newton by considering some of these effects.

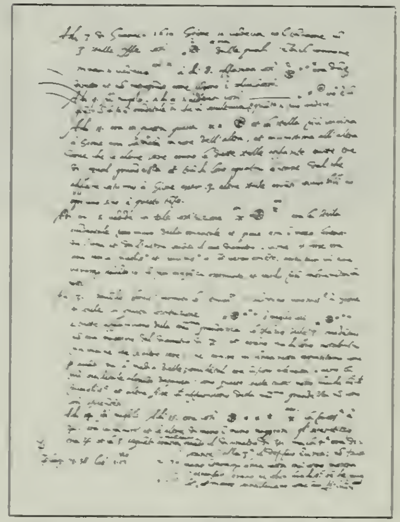
During the 1700's, the so-called Age of Reason or Century of Enlightenment, the viewpoint called the Newtonian cosmology became firmly entrenched in European science and philosophy. The impact of Newton's achievements may be summarized thus: he had shown that man, by observing and reasoning, by considering mechanical models and deducing mathematical laws, could uncover the workings of the physical universe. Therefore, it was argued, man should attempt by the same method to understand not only nature but also society and the human mind. As the French writer Fontenelle (1657-1757) expressed it:

> The geometric spirit is not so bound up with geometry that it cannot be disentangled and carried into other fields. A work of morals, or politics, of criticism, perhaps even of eloquence, will be the finer, other things being equal, if it is written by the hand of a geometer.

The English philosopher John Locke (1632-1704) was greatly influenced by Newton's work and in turn reinforced Newton's influence on others; he argued that the goal of philosophy should be to solve problems, including those that affect our daily life, by observation and reasoning. "Reason must be our best judge and guide in all things," he said. Locke thought that the concept of "natural law" could be used in religion as well as in physics; and indeed the notion of a religion "based on reason" appealed to many Europeans who hoped to avoid a revival of the bitter religious wars of the 1600's.

Locke advanced the theory that the mind of the new-born child contains no "innate ideas"; it is like a blank piece of paper on which anything may be written. If this were true, it would be futile to search within oneself for a God-given sense of what is true or morally right. Instead, one must look at nature and society to discover whether there are any "natural laws" that may exist.





The engraving of the French Academy by Sebastian LeClerc (1698) reflects the activity of learned societies at that time. The picture does not depict an actual scene, of course, but in allegory shows the excitement of communication that grew in an informal atmosphere. The dress is symbolic of the Greek heritage of the sciences. Although all the sciences are represented, the artist has put anatomy, botany, and zoology, symbolized by skeletons and dried leaves, toward the edges, along with alchemy and theology. Mathematics and the physical sciences, including astronomy, occupy the center stage.

Conversely, if one wants to improve the quality of man's mind, one must improve the society in which he lives.

Locke's view also implied an "atomistic" structure of society: each person is separate from other individuals in the sense that he has no "organic" relation to them. Previously, political theories had been based on the idea of society as an organism in which each person has a perscribed place, function and obligation. Later theories, based on Locke's ideas, asserted that government should have no function except to protect the freedom and property of the individual person.

Although "reason" was the catchword of the eighteenth-century philosophers, we do not have to accept their own judgment that their theories about improving religion and society were necessarily the most reasonable. Like most others, these men would not give up a doctrine such as the equal rights of all men merely because they could not find a strictly mathematical or scientific proof for it. Newtonian physics, religious toleration, and republican government were all advanced by the same movement; but this does not mean there was really a logical connection among them. Nor, for that matter, did many of the eighteenth-century thinkers in any field or nation seem much bothered by another gap in logic and feeling; they believed that "all men are created equal," and yet they did little to remove the chains of black slaves, the ghetto walls imprisoning Jews, or the laws that denied voting rights to women.

Still, compared with the previous century, the dominant theme of the 1700's was *moderation*—the happy medium, based on toleration of different opinions, restraint of excess in any direction, and balance of opposing forces. Even reason was not allowed to ride roughshod over religious faith; atheism, which some philosophers thought to be the logical consequence of unlimited rationality, was still regarded with horror by most Europeans.

The Constitution of the United States of America, with its ingenious system of "checks and balances" to prevent any faction from getting too much power, is one of the most enduring achievements of this period. It attempts to establish in politics a stable equilibrium of opposing trends similar to the balance between the sun's gravitational pull and the tendency of a planet to fly off in a straight line. If the gravitational attraction increased without a corresponding increase in planetary speed, the planet would fall into the sun; if its speed increased without a corresponding increase in gravitational attraction, the planet would escape from the solar system.

Just as the Newtonian laws of motion kept the earth at its proper distance from the sun, so the political philosophers, some of whom used Newtonian physics as a model of thought, hoped to devise a system of government which would avoid the extremes of dictatorship and anarchy. According to James Wilson (1742-1798), who played a major role in drafting the American Constitution:

> In government, the perfection of the whole depends on the balance of the parts, and the balance of the parts consists in the independent exercise of their separate powers, and, when their powers are separately exercised, then in their mutual influence and operation on one another. Each part acts and is acted upon, supports and is supported, regulates and is regulated by the rest. It might be supposed, that these powers, thus mutually checked and controlled, would remain in a state of inaction. But there is a necessity for movement in human affairs; and these powers are forced to move, though still to move in concert. They move, indeed, in a line of direction somewhat different from that, which each acting by itself would have taken; but, at the same time, in a line partaking of the natural directions of the whole—the true line of public liberty and happiness.

A related effect of Newton's work in physics on other fields was the impetus Newton as a person and Newton's writing gave to the idea of political democracy. A former farm boy had penetrated to the outermost reaches of the human imagination, and what he found there meant, first of all, that God had made only one set of laws for heaven and earth. This smashed the old hierarchy and raised what was once thought base to the level of the noble. It was an extension of a new democracy throughout the universe: Newton had shown that all matter, whether of sun or of ordinary stone, was created equal, was of the same order in "the Laws of Nature and of

Nature's God," to cite the phrase used at the beginning of the Declaration of Independence to justify the elevation of the colonists to an independent people. The whole political ideology was heavily influenced by Newtonian ideas. The *Principia*, many thought, gave an analogy and extension direct support to the proposition being formulated also from other sides that all men, like all natural objects, are created equal before nature's creator. Some of these important trends are discussed in articles in *Reader* 2.

In literature, too, many welcomed the new scientific viewpoint as a source of metaphors, allusions, and concepts which they used in their poems and essays. Newton's discovery that white light is composed of colors was referred to in many poems of the 1700's (see Unit 4). Samuel Johnson advocated that words drawn from the natural sciences be used in literary works, defining such words in his *Dictionary* and illustrating their application in his "Rambler" essays.

Other writers distrusted the new cosmology and so used it for purposes of satire. In his epic poem *The Rape of the Lock*, Alexander Pope exaggerated the new scientific vocabulary for comic effect. Jonathan Swift, sending Gulliver on his travels to Laputa, described an academy of scientists and mathematicians whose experiments and theories were as absurd as those of the Fellows of the Royal Society must have seemed to the layman of the 1700's.

The first really powerful reaction against Newtonian cosmology was the Romantic movement, begun in Germany about 1780 by young writers inspired by Johann Wolfgang von Goethe. The most familiar examples of Romanticism in English literature are the poems and novels of Blake, Coleridge, Wordsworth, Shelley, Byron, and Scott.

The Romantics scorned the mathematical view of nature, and emphasized the importance of quality rather than quantity. They preferred to study the unique element of an individual person or experience, rather than make abstractions. They exalted emotion and feeling at the expense of reason and calculation. In particular, they abhorred the theory that the universe is in any way like a clockwork, made of inert matter set into motion by a God who never afterwards shows His presence. Reflecting this attitude, the historian and philosopher of science, E. A. Burtt, has written scathingly that:

> . . . the great Newton's authority was squarely behind that view of the cosmos which saw in man a puny, irrelevant spectator (so far as being wholly imprisoned in a dark room can be called such) of the vast mathematical system whose regular motions according to mechanical principles constituted the world of nature. The gloriously romantic universe of Dante and Milton, that set no bounds to the imagination of man as it played over space and time, had now been swept away. Space was identified with the realm of geometry, time with the continuity of number. The world that people had thought

This is, of course, a distortion of what scientists themselves believe —one of the wrong images of science discussed in "The Seven Images of Science" in *Reader 3*.

> themselves living in—a world rich with color and sound, redolent with fragrance, filled with gladness, love and beauty, speaking everywhere of purposive harmony and creative ideals—was crowded now into minute corners in the brains of scattered organic beings. The really important world outside was a world hard, cold, colorless, silent, and dead; a world of quantity, a world of mathematically computable motions in mechanical regularity. The world of qualities as immediately perceived by man became just a curious and quite minor effect of that infinite machine beyond.

Because in their view, the whole (whether it be a single human being or the entire universe) is pervaded by a spirit that cannot be rationally explained but can only be intuitively felt, the Romantics insisted that phenomena cannot meaningfully be analyzed and reduced to their separate parts by mechanistic explanations.

Continental leaders of the Romantic movement, such as the German philosopher Friedrich Schelling (1775-1854) proposed a new way of doing scientific research, a new type of science called "Nature Philosophy." (This term is not to be confused with the older "natural philosophy," meaning mainly physics.) The Nature Philosopher does not analyze phenomena such as a beam of white light into separate parts or factors which he can measure quantitatively in his laboratory—or at least that is not his primary purpose. Instead, he tries to understand the phenomenon as a whole, and looks for underlying basic principles that govern all phenomena. The Romantic philosophers in Germany regarded Goethe as their greatest scientist as well as their greatest poet, and they pointed in particular to his theory of color, which flatly contradicted Newton's theory of light. Goethe held that white light does not consist of a mixture of colors but rather that the colors are produced by the prism acting on and changing the light which was itself pure.

In the judgment of all modern physicists, Newton was right and Goethe wrong. Yet, in retrospect, Nature Philosophy was not simply an aberration. The general tendency of Nature Philosophy did encourage speculation about ideas which could never be tested by experiment; hence, Nature Philosophy was condemned by most scientists. But it is now generally agreed by historians of science that Nature Philosophy played an important role in the historical origins of some scientific discoveries. Among these was the general principle of conservation of energy, which is described in Chapter 10. The recognition of the principle of conservation of energy came in part out of the viewpoint of Nature Philosophy, for it asserted that all the "forces of nature"—the phenomena of heat, gravity, electricity, magnetism, and so forth—are manifestations of one underlying "force" (which we now call energy).

Much of the dislike which Romantics (like some modern artists and intellectuals) expressed for science was based on the mistaken notion that scientists claimed to be able to find a mechanistic explanation for *everything*, including the human mind. If everything is explained by Newtonian science, then everything would also be *determined* in the way the motions of different parts of a machine are determined by its construction. Most modern scientists no longer believe this, but some scientists in the past have made statements of this kind. For example, the French mathematical physicist Laplace (1749-1827) said:

> We ought then to regard the present state of the universe as the effect of its previous state and as the cause of the one which is to follow. Given for one instant a mind which could comprehend all the forces by which nature is animated and the respective situation of the beings who compose it—a mind sufficiently vast to submit these data to analysis—it would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes.

Even the ancient Roman philosopher Lucretius (100-55 B.C.), who supported the atomic theory in his poem *On the Nature of Things*, did not go as far as this. In order to preserve some vestige of "free will" in the universe, Lucretius suggested that the atoms might swerve randomly in their paths. This was still unsatisfactory to the Romantics and also to some scientists such as Erasmus Darwin (grandfather of evolutionist Charles Darwin), who asked:

> Dull atheist, could a giddy dance
> Of atoms lawless hurl'd
> Construct so wonderful, so wise,
> So harmonised a world?

The Nature Philosophers thought they could discredit the Newtonian scientists by forcing them to answer this question; to say "yes," they argued, would be absurd, and to say "no" would be disloyal to their own supposed beliefs. We shall see how successful the Newtonians were in explaining the physical world without committing themselves to any definite answer to Erasmus Darwin's question. Instead, they were led to the discovery of immensely powerful and fruitful laws of nature, discussed in the next units.

SG 8.23, 8.24

8.1 The Project Physics learning materials particularly appropriate for Chapter 8 include:

Experiment
Stepwise Approximation to an Orbit

Activities
Model of the orbit of Halley's comet
Other comet orbits
Forces on a Pendulum
Haiku
Trial of Copernicus
Discovery of Neptune and Pluto

Reader Articles
Newton and the *Principia*
The Laws of Motion and Proposition I
Universal Gravitation
An Appreciation of the Earth
The Great Comet of 1965
Gravity Experiments
Space the Unconquerable
The Life Story of a Galaxy
Expansion of the Universe
Negative Mass
The Dyson Sphere

Flim Loops
Jupiter Satellite Orbit
Program Orbit I
Program Orbit II
Central forces—iterated blows
Kepler's Laws
Unusual Orbits

Transparency
Motion under central force

8.2 In the table below are the periods and distances from Jupiter of the four large satellites, as measured by telescopic observations. Does Kepler's law of periods apply to the Jupiter system?

SATELLITE	PERIOD	DISTANCE FROM JUPITER'S CENTER (in terms of Jupiter's radius, *r*)
I	1.77 days	6.04 r
II	3.55	9.62
III	7.15	15.3
IV	16.7	27.0

8.3 Give some reasons why Descartes' theory of planetary motion might have been "a useful idea."

8.4 On p. 100 it was claimed that the dependence of the gravitational force on the masses of both interacting bodies could be expressed as $m_{sun}\, m_{planet}$.

(a) Using a diagram similar to that for Q 13 on p. 101, show that this is correct.
(b) To test alternatives to using the product, consider the possibilities that the force could depend upon the masses in either of two ways:
(1) total force depends on $(m_{sun} + m_{planet})$, or
(2) total force depends on (m_{sun}/m_{planet}).

What would these relationships imply would happen to the force if either mass were reduced to zero? Would there still be a force even though there were only one mass left? Could you speak of a gravitational force when there was no body to be accelerated?

8.5 Use the values for the mass and size of the moon (See table on page 123) to show that the "surface gravity" (acceleration due to gravity near the moon's surface) is only about $\frac{1}{6}$ of what it is near the earth's.

8.6 Complicated mathematics is necessary to find the exact force exerted by a spherical body, but it is not difficult to prove that the *direction* of the net force is toward the center. Newton's argument involved symmetry and considering tiny pieces of the whole body. Develop such an argument.

8.7 Use the equation for centripetal force and the equation for gravitational force to derive an expression for the period of a Satallite orbiting around a planet in terms of the radius of the orbit and mass of the planet.

8.8 The sun's mass is about 27,000,000 times greater than the moon's mass; the sun is about 400 times further from the earth than the moon is. How does the gravitational force exerted on the earth by the sun compare with that exerted by the moon?

8.9 By Newton's time, telescopic observations of Jupiter led to values for the orbital periods and radii of Jupiter's four large satellites. For example, the one named Callisto was found to have a period of 16.7 days and the radius of its orbit was calculated as 1/80 AU.

(a) From these data calculate the value of $k_{Jupiter}$. (First convert days to years.)
(b) Show that Jupiter's mass is about 1/1000 the mass of the sun.
(c) How was it possible to have a value for the orbital radius of a satellite of Jupiter?

8.10 What orbit radius must an earth satellite be given to keep it always above the same place on the earth—that is, in order to have a 24-hour period? (Hint: See SG 8.7)

8.11 Calculate the mass of the earth from the fact that a 1kg object at the earth's surface is attracted to the earth with a force of 9.8 newtons. The distance from the earth's center to its surface is 6.4×10^6 meters. How many times greater is this than the greatest masses which you have had some experience in accelerating (for example, cars)?

8.12 The mass of the earth can be calculated also from the distance and period of the moon. Show that the value obtained in this way agrees with the value calculated from measurements at the earth's surface. (See table on page 192.)

8.13 Cavendish's value for G made it possible to calculate the mass of the earth, and therefore its average density. The "density" of water is 1000 kg per cubic meter. That is, for any sample of water, dividing the mass of the sample by its volume gives 100 kg/m³.

(a) What is the earth's average density?
(b) The densest kind of rock known has a density of about 5000 kg/m³. Most rock we find has a density of about 3000 kg/m³.

What do you conclude from this about the structure of the earth?

8.14 The manned Apollo 8 capsule (1968) was put into a nearly circular orbit 112 km above the moon's surface. The period of the orbit was 120.5 minutes. From these data calculate the mass of the moon. (The radius of the moon is 1740 km. Don't forget to use a consistent set of units.)

8.15 Why do you suppose there is no reliable value for the mass of Pluto?

8.16 Mars has two satellites. Phobos and Deimos – Fear and Panic. A science-fiction story was once written in which the natives of Mars showed great respect for a groove in the ground. The groove turned out to be the path of Mars' closest moon, "Bottomos."

(a) If such an orbit were possible, what would the period be?
(b) What speed would it need to have in order to go into such an orbit?
(c) What difficulties do you see for such an orbit?

8.17 Using the values given in the table on p. 105 make a table of relative masses compared to the mass of the earth.

8.18 The period of Halley's comet is about 75 years. What is its average distance from the sun? The eccentricity of its orbit is 0.967. How far from the sun does it go? How close?

8.19 Accepting the validity of $F_{grav} = Gm_1m_2/R^2$, and recognizing that G is a universal constant, we are able to derive, and therefore to understand better, many particulars that previously seemed separate. For example, we can conclude:

(a) That a_g for a body of any mass m_0 should be constant at a particular place on earth.
(b) That a_g might be different at places on earth at different distances from the earth's center.
(c) That at the earth's surface the weight of a body be related to its mass.
(d) That the ratio R^3/T^2 is a constant for all the satellites of a body.
(e) That high tides occur about six hours apart.

Describe briefly how each of these conclusions can be derived from the equation.

8.20 The making of theories to account for observations is a major purpose of scientific study. Therefore some reflection upon the theories encountered thus far in this course will be useful. Comment in a paragraph or more, with examples from Units 1 and 2, on some of the statements below. Look at all the statements and select at least six, in any order you wish.

(1) A good theory should summarize and not conflict with a body of tested observations. (For example, Kepler's unwillingness to explain away the difference of eight minutes of arc between his predictions and Tycho's observations.)
(2) There is nothing more practical than a good theory.
(3) A good theory should permit predictions of new observations which sooner or later can be made.
(4) A good new theory should give almost the same predictions as older theories for the range of phenomena where they worked well.
(5) Every theory involves assumptions. Some involve also esthetic preferences of the scientist.
(6) A new theory relates some previously unrelated observations.
(7) Theories often involve abstract concepts derived from observation.
(8) Empirical laws or "rules" organize many observations and reveal how changes in one quantity vary with changes in another but such laws provide no explanation of the causes or mechanisms.
(9) A theory never fits all data exactly.
(10) Predictions from theories may lead to the observation of new effects.
(11) Theories that later had to be discarded may have been useful because they encouraged new observations.
(12) Theories that permit quantitative predictions are preferred to qualitative theories.
(13) An "unwritten text" lies behind the statement of every law of nature.
(14) Communication between scientists is an essential part of the way science grows.
(15) Some theories seem initially so strange that they are rejected completely or accepted only very slowly.
(16) Models are often used in the making of a theory or in describing a theory to people.
(17) The power of theories comes from their generality.

8.21 What happened to Plato's problem? Was it solved?

8.22 Why do we believe today in a heliocentric system? Is it the same as either Copernicus' or Kepler's? What is the experimental evidence? Is the geocentric system disproved?

8.23 Is Newton's work only of historical interest, or is it useful today? Explain.

8.24 What were some of the major consequences of Newton's work on scientists' view of the world?

SATELLITES OF THE PLANETS

		DISCOVERY	AVERAGE RADIUS OF ORBIT	PERIOD OF REVOLUTION			DIAMETER
EARTH:	Moon		238,857 miles	27d	7h	43m	2160 miles
MARS:	Phobos	1877, Hall	5,800	0	7	39	10?
	Deimos	1877, Hall	14,600	1	6	18	5?
JUPITER:	V	1892, Barnard	113,000	0	11	53	150?
	1 (Io)	1610, Galileo	262,000	1	18	28	2000
	II (Europa)	1610, Galileo	417,000	3	13	14	1800
	III (Ganymede)	1610, Galileo	666,000	7	3	43	3100
	IV (Callisto)	1610, Galileo	1,170,000	16	16	32	2800
	VI	1904, Perrine	7,120,000	250	14		100?
	VII	1905, Perrine	7,290,000	259	14		35?
	X	1938, Nicholson	7,300,000	260	12		15?
	XII	1951, Nicholson	13,000,000	625			14?
	XI	1938, Nicholson	14,000,000	700			19?
	VIII	1908, Melotte	14,600,000	739			35?
	IX	1914, Nicholson	14,700,000	758			17?
SATURN:	Mimas	1789, Herschel	115,000	0	22	37	300?
	Enceladus	1789, Herschel	148,000	1	8	53	350
	Tethys	1684, Cassini	183,000	1	21	18	500
	Dione	1684, Cassini	234,000	2	17	41	500
	Rhea	1672, Cassini	327,000	4	12	25	1000
	Titan	1655, Huygens	759,000	15	22	41	2850
	Hyperion	1848, Bond	920,000	21	6	38	300?
	Phoebe	1898, Pickering	8,034,000	550			200?
	Iapetus	1671, Cassini	2,210,000	79	7	56	800
URANUS:	Miranda	1948, Kuiper	81,000	1	9	56	—
	Ariel	1851, Lassell	119,000	2	12	29	600?
	Umbriel	1851, Lassell	166,000	4	3	28	400?
	Titania	1787, Herschel	272,000	8	16	56	1000?
	Oberon	1787, Herschel	364,000	13	11	7	900?
NEPTUNE:	Triton	1846, Lassell	220,000	5	21	3	2350
	Nereid	1949, Kuiper	3,440,000	359	10		200?

THE SOLAR SYSTEM

	RADIUS	MASS	AVERAGE RADIUS OF ORBIT	PERIOD OF REVOLUTION
Sun	6.95×10^{8} meters	1.98×10^{30} kilograms	—	—
Moon	1.74×10^{6}	7.34×10^{22}	3.8×10^{8} meters	2.36×10^{6} seconds
Mercury	2.57×10^{6}	3.28×10^{23}	5.79×10^{10}	7.60×10^{6}
Venus	6.31×10^{6}	4.83×10^{24}	1.08×10^{11}	1.94×10^{7}
Earth	6.38×10^{6}	5.98×10^{24}	1.49×10^{11}	3.16×10^{7}
Mars	3.43×10^{6}	6.37×10^{23}	2.28×10^{11}	5.94×10^{7}
Jupiter	7.18×10^{7}	1.90×10^{27}	7.78×10^{11}	3.74×10^{8}
Saturn	6.03×10^{7}	5.67×10^{26}	1.43×10^{12}	9.30×10^{8}
Uranus	2.67×10^{7}	8.80×10^{25}	2.87×10^{12}	2.66×10^{9}
Neptune	2.48×10^{7}	1.03×10^{26}	4.50×10^{12}	5.20×10^{9}
Pluto	?	?	5.9×10^{12}	7.28×10^{9}

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(continued on p. 198)

The Project Physics Course

Handbook 2

Motion in the Heavens

Acknowledgments, Handbook Section

P. 131 The table is reprinted from *Solar and Planetary Longitudes for Years* −2500 to +2500 prepared by William D. Stalman and Owen Gingerich (University of Wisconsin Press, 1963)

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Contents HANDBOOK SECTION

Chapter 5 Where is the Earth?—The Greek's Answers

EXPERIMENT 14 NAKED-EYE ASTRONOMY (Continued from Unit 1, Experiment 1)

Weather permitting, you have been watching events in the day and night sky since this course started. Perhaps you have followed the sun's path, or viewed the moon, planets, or stars.

From observations much like your own, scientists in the past developed a remarkable sequence of theories. The more aware you are of the motions in the sky and the more you interpret them yourself, the more easily you can follow the development of these theories. If you do not have your own data, you can use the results provided in the following sections.

A. One Day of Sun Observations

One student made the following observations of the sun's position on September 23.

Eastern Daylight Time (EDT)	Sun's Altitude	Sun's Azimuth
7:00 A.M.	------	------
8:00	08°	097°
9:00	19	107
10:00	29	119
11:00	38	133
12:00	45	150
1:00 P.M.	49	172
2:00	48	197
3:00	42	217
4:00	35	232
5:00	25	246
6:00	14	257
7:00	03	267

If you plot altitude (vertically) against azimuth (horizontally) on a graph and mark the hours for each point, it will help you to answer these questions.

1. What was the sun's greatest altitude during the day?
2. What was the latitude of the observer?
3. At what time (EDT) was the sun highest?
4. When during the day was the sun's direction (azimuth) changing fastest?

5. When during the day was the sun's altitude changing fastest?
6. At what time of day did the sun reach its greatest altitude? How do you explain the fact that it is not exactly at 12:00? (Remember that daylight time is an hour ahead.)

B. A Year of Sun Observations

One student made the following monthly observations of the sun through a full year. (He had remarkably clear weather!)

Dates	Sun's Noon Altitude	Sunset Azimuth	Time Between Noon and Sunset
Jan 1	20°	238°	$4_{h}25_{m}$*
Feb 1	26	245	4 50
Mar 1	35	259	5 27
Apr 1	47	276	6 15
May 1	58	291	6 55
Jun 1	65	300	7 30
Jul 1	66	303	7 40
Aug 1	61	295	7 13
Sep 1	52	282	6 35
Oct 1	40	267	5 50
Nov 1	31	250	5 00
Dec 1	21	239	4 30

*h = hours, m = minutes.

Use these data to make three plots (different colors or marks on the same sheet of graph

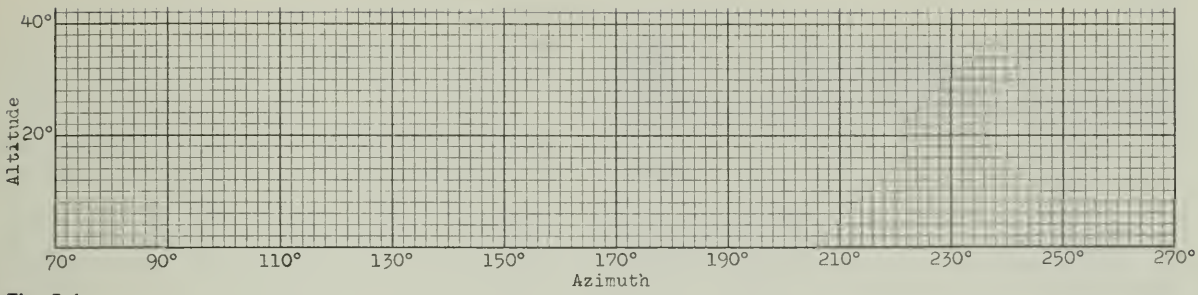


Fig. 5-1

paper) of the sun's noon altitude, direction at sunset, and time of sunset after noon. Place these data on the vertical axis and the dates on the horizontal axis.

1. What was the sun's noon altitude at the equinoxes (March 21 and September 23)?
2. What was the observer's latitude?
3. If the observer's longitude was 71°W, what city was he near?
4. Through what range (in degrees) did his sunset point change during the year?
5. By how much did the observer's time of sunset change during the year?
6. If the time from sunrise to noon was always the same as the time between noon and sunset, how long was the sun above the horizon on the shortest day? on the longest day?

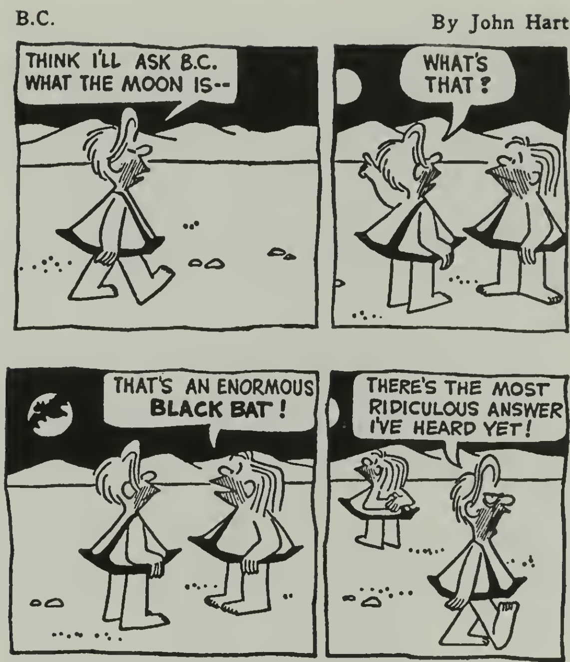
C. Moon Observations

During October 1966 a student in Las Vegas, Nevada made the following observations of the moon at sunset when the sun had an azimuth of about 255°.

Date	Angle from Sun to Moon	Moon Altitude	Moon Azimuth
Oct.			
16	032°	17°	230°
18	057	25	205
20	081	28	180
22	104	30	157
24	126	25	130
26	147	16	106
28	169	05	083

1. Plot these positions of the moon on a chart such as in Fig. 5-1.
2. From the data and your plot, estimate the dates of new moon, first quarter moon, and full moon.
3. For each of the points you plotted, sketch the shape of the lighted area of the moon.

Phases of the moon: (1) 23 days, (2) 26 days, (3) 17 days, (4) 5 days, (5) 3 days after new moon.



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D. Locating the Planets

Table 1, Planetary Longitudes lists the position of each major planet along the ecliptic. The positions are given, accurate to the nearest degree, for every ten-day interval. By interpolation you can find a planet's position on any given day.

The column headed "J.D." shows the corresponding Julian Day calendar date for each entry. This calendar is simply a consecutive numbering of days that have passed since an arbitrary "Julian Day 1" in 4713 B.C.: September 30, 1970, for example, is the same as J.D. 2,440,860.

Julian dates are used by astronomers for convenience. For example, the number of days between March 8 and September 26 of this year is troublesome to figure out, but it is easy to find by simple subtraction if the Julian dates are used instead.

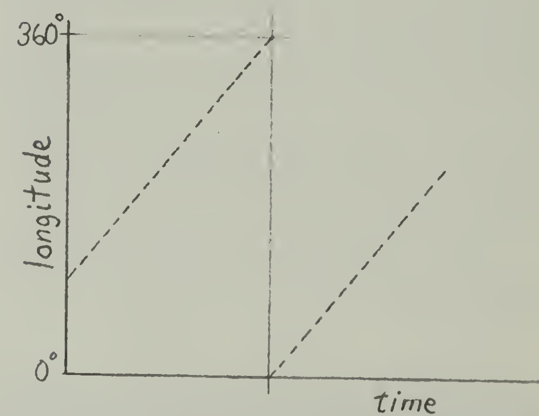
Look up the sun's present longitude in the table. Locate the sun on your SC-1 Constellation Chart: The sun's path, the ecliptic, is the curved line marked off in 360 degrees of longitude.

A planet that is just to the west of the sun's position (to the right on the chart) is "ahead of the sun," that is, it rises and sets just before the sun does. One that is 180° from the sun rises near sundown and is in the sky all night.

When you have decided which planets may be visible, locate them along the ecliptic shown on your sky map SC-1. Unlike the sun, they are not exactly on the ecliptic, but they are never more than 8° from it. Once located on the Constellation Chart you know where to look for a planet among the fixed stars.

E. Graphing the Position of the Planets

Here is a useful way to display the information in Table I, Planetary Longitudes. On ordinary graph paper, plot the sun's longitude versus time. Use Julian Day numbers along the horizontal axis, beginning close to the present date. The plotted points should fall on a nearly straight line, sloping up toward the right until they reach 360° and then starting again at zero.



How long will it be before the sun again has the same longitude it has today? Would the answer to that question be the same if it were asked three months from now? What is the sun's average angular speed (in degrees per day) over a whole year? When is its angular speed greatest?

Plot Mercury's longitudes on the same graph (use a different color or shape for the points). According to your plot, how far (in longitude) does Mercury get from the sun? (This is Mercury's "maximum elongation.") At what time interval does Mercury pass the sun?

Table 1 Planet Longitudes at 10-Day Intervals

Yr.	Date	J.D.	Sun	Mer	Ven	Mars	Jup	Sat
1969	Nov 24	0550	242	246	227	314	206	32
1969	Dec 4	0560	252	262	240	322	207	31
1969	Dec 14	0570	262	278	252	329	209	31
1969	Dec 24	0580	272	292	265	337	211	31
1970	Jan 3	0590	283	300	277	344	212	31
1970	Jan 13	0600	293	293	290	352	214	32
1970	Jan 23	0610	303	284	303	359	215	32
1970	Feb 2	0620	313	288	315	6	215	32
1970	Feb 12	0630	323	298	327	13	216	33
1970	Feb 22	0640	333	312	340	21	216	34
1970	Mar 4	0650	343	327	353	28	216	35
1970	Mar 14	0660	353	345	5	35	215	36
1970	Mar 24	0670	3	4	17	42	214	37
1970	Apr 3	0680	13	25	30	49	213	38
1970	Apr 13	0690	23	42	42	56	212	39
1970	Apr 23	0700	33	52	55	63	211	41
1970	May 3	0710	42	52	67	70	209	42
1970	May 13	0720	52	45	80	77	208	43
1970	May 23	0730	62	42	92	83	207	45
1970	Jun 2	0740	71	47	104	90	207	46
1970	Jun 12	0750	81	58	115	97	206	47
1970	Jun 22	0760	90	74	127	103	206	48
1970	Jul 2	0770	100	94	139	109	206	49
1970	Jul 12	0780	109	116	150	116	206	50
1970	Jul 22	0790	119	135	162	122	207	50
1970	Aug 1	0800	128	152	173	129	208	51
1970	Aug 11	0810	138	165	184	136	209	52
1970	Aug 21	0820	148	175	194	142	211	53
1970	Aug 31	0830	157	178	204	149	212	53
1970	Sep 10	0840	167	172	213	155	214	53
1970	Sep 20	0850	177	164	222	161	216	53
1970	Sep 30	0860	187	168	229	167	217	52
1970	Oct 10	0870	197	184	234	174	219	52
1970	Oct 20	0880	206	201	236	180	221	51
1970	Oct 30	0890	216	218	234	186	224	50
1970	Nov 9	0900	227	234	230	192	226	49
1970	Nov 19	0910	237	250	224	199	228	48
1970	Nov 29	0920	247	265	220	205	231	47
1970	Dec 9	0930	257	278	222	212	233	46
1970	Dec 19	0940	267	285	226	218	235	45
1970	Dec 29	0950	277	276	233	224	237	44
1971	Jan 8	0960	288	268	242	231	238	44
1971	Jan 18	0970	298	273	251	237	240	44
1971	Jan 28	0980	308	285	261	243	242	44
1971	Feb 7	0990	318	299	272	249	243	45
1971	Feb 17	1000	328	315	283	256	244	46
1971	Feb 27	1010	338	332	294	262	245	46
1971	Mar 9	1020	348	351	306	268	246	48
1971	Mar 19	1030	358	10	318	274	246	49
1971	Mar 29	1040	8	27	330	280	247	50
1971	Apr 8	1050	18	34	341	286	246	51
1971	Apr 18	1060	28	31	353	292	246	52
1971	Apr 28	1070	37	24	5	297	245	53
1971	May 8	1080	47	23	17	303	244	55
1971	May 18	1090	57	31	29	308	242	56
1971	May 28	1100	66	43	41	314	241	58
1971	Jun 7	1110	76	60	53	318	240	59
1971	Jun 17	1120	85	80	66	320	239	60
1971	Jun 27	1130	95	103	78	322	238	61
1971	Jul 7	1140	104	122	90	323	237	62
1971	Jul 17	1150	114	138	103	324	237	63
1971	Jul 27	1160	123	151	115	323	236	64
1971	Aug 6	1170	133	159	127	320	237	65
1971	Aug 16	1180	143	160	140	315	237	65
1971	Aug 26	1190	152	153	152	313	238	66
1971	Sep 5	1200	162	147	164	312	239	66
1971	Sep 15	1210	172	154	177	312	240	66
1971	Sep 25	1220	182	170	189	314	242	67
1971	Oct 5	1230	191	189	202	315	243	67
1971	Oct 15	1240	201	206	214	318	245	66
1971	Oct 25	1250	211	222	227	322	247	66
1971	Nov 4	1260	221	238	239	328	249	65
1971	Nov 14	1270	231	252	252	333	251	64
1971	Nov 24	1280	241	264	264	340	253	63
1971	Dec 4	1290	252	269	277	346	256	61
1971	Dec 14	1300	262	259	289	352	258	60
1971	Dec 24	1310	272	252	302	358	261	59
1972	Jan 3	1320	282	259	314	5	263	59
1972	Jan 13	1330	292	272	326	12	265	59

Yr.	Date	J.D.	Sun	Mer	Ven	Mars	Jup	Sat
1972	Jan 23	1340	303	286	338	18	267	58
1972	Feb 2	1350	313	302	350	24	269	58
1972	Feb 12	1360	323	319	2	31	271	59
1972	Feb 22	1370	333	337	14	38	273	59
1972	Mar 3	1380	343	356	26	44	274	60
1972	Mar 13	1390	353	12	37	51	276	61
1972	Mar 23	1400	3	16	48	57	277	61
1972	Apr 2	1410	13	10	59	64	278	62
1972	Apr 12	1420	23	4	69	70	278	64
1972	Apr 22	1430	32	6	78	77	279	65
1972	May 2	1440	42	15	86	84	279	66
1972	May 12	1450	52	29	91	90	278	67
1972	May 22	1460	61	46	95	97	278	69
1972	Jun 1	1470	71	66	94	103	277	70
1972	Jun 11	1480	80	89	89	109	276	72
1972	Jun 21	1490	90	108	83	115	274	73
1972	Jul 1	1500	99	124	77	122	273	74
1972	Jul 11	1510	109	136	76	128	271	75
1972	Jul 21	1520	119	142	80	134	270	76
1972	Jul 31	1530	128	140	85	141	269	77
1972	Aug 10	1540	138	133	93	147	269	78
1972	Aug 20	1550	147	130	101	153	268	79
1972	Aug 30	1560	157	139	111	160	268	79
1972	Sep 9	1570	167	157	121	166	269	80
1972	Sep 19	1580	176	176	132	173	269	80
1972	Sep 29	1590	186	194	143	179	270	80
1972	Oct 9	1600	196	210	154	186	271	80
1972	Oct 19	1610	206	225	166	192	273	80
1972	Oct 29	1620	216	239	178	198	274	81
1972	Nov 8	1630	226	249	190	205	276	80
1972	Nov 18	1640	236	253	202	212	278	79
1972	Nov 28	1650	246	241	214	218	280	78
1972	Dec 8	1660	257	237	227	225	282	77
1972	Dec 18	1670	267	246	239	232	284	76
1972	Dec 28	1680	277	259	252	238	287	75
1973	Jan 7	1690	287	274	264	245	289	74
1973	Jan 17	1700	297	290	277	252	291	73
1973	Jan 27	1710	307	306	289	259	294	73
1973	Feb 6	1720	317	324	302	266	296	73
1973	Feb 16	1730	328	342	315	273	298	73
1973	Feb 26	1740	338	356	327	280	300	73
1973	Mar 8	1750	348	358	340	287	303	74
1973	Mar 18	1760	358	349	352	294	305	74
1973	Mar 28	1770	8	345	4	301	306	75
1973	Apr 7	1780	17	350	17	308	308	76
1973	Apr 17	1790	27	0	29	315	309	77
1973	Apr 27	1800	37	14	41	322	310	78
1973	May 7	1810	47	32	54	329	311	79
1973	May 17	1820	56	52	66	336	312	81
1973	May 27	1830	66	75	79	343	312	82
1973	Jun 6	1840	75	94	91	350	312	83
1973	Jun 16	1850	85	109	103	357	312	85
1973	Jun 26	1860	94	119	116	4	312	86
1973	Jul 6	1870	104	123	128	10	311	87
1973	Jul 16	1880	114	120	140	16	310	89
1973	Jul 26	1890	123	113	152	22	308	89
1973	Aug 5	1900	133	114	164	28	306	90
1973	Aug 15	1910	142	125	176	33	305	91
1973	Aug 25	1920	152	143	188	37	304	92
1973	Sep 4	1930	162	163	200	39	303	93
1973	Sep 14	1940	171	181	211	40	302	94
1973	Sep 24	1950	181	198	223	40	302	94
1973	Oct 4	1960	191	213	235	88	302	94
1973	Oct 14	1970	201	226	246	36	302	95
1973	Oct 24	1980	211	235	257	32	303	94
1973	Nov 3	1990	221	236	268	27	304	94
1973	Nov 13	2000	231	224	278	25	305	94
1973	Nov 23	2010	241	222	288	24	306	93
1973	Dec 3	2020	251	232	297	25	308	93
1973	Dec 13	2030	261	246	304	26	310	92
1973	Dec 23	2040	272	262	309	28	312	91
1974	Jan 2	2050	282	278	313	32	314	90
1974	Jan 12	2060	292	294	311	37	317	89
1974	Jan 22	2070	302	311	306	41	319	88
1974	Feb 1	2080	312	328	300	46	321	87
1974	Feb 11	2090	322	341	296	51	324	87
1974	Feb 21	2100	332	340	297	57	326	87
1974	Mar 3	2110	342	330	301	63	329	87
1974	Mar 13	2120	352	327	308	68	331	88

Yr.	Date	J.D.	Sun	Mer	Ven	Mars	Jup	Sat
1974	Mar 23	2130	2	335	316	73	333	88
1974	Apr 2	2140	12	346	325	79	335	89
1974	Apr 12	2150	22	1	335	85	337	89
1974	Apr 22	2160	32	18	346	91	340	90
1974	May 2	2170	42	38	357	97	342	91
1974	May 12	2180	51	61	8	103	343	93
1974	May 22	2190	61	80	19	109	345	94
1974	Jun 1	2200	70	94	31	115	346	95
1974	Jun 11	2210	80	102	42	121	347	96
1974	Jun 21	2220	89	103	54	128	347	97
1974	Jul 1	2230	99	97	66	134	348	99
1974	Jul 11	2240	109	94	77	140	348	100
1974	Jul 21	2250	118	97	89	146	347	102
1974	Jul 31	2260	128	110	101	152	348	103
1974	Aug 10	2270	137	129	114	159	347	104
1974	Aug 20	2280	147	150	126	165	345	105
1974	Aug 30	2290	157	169	139	171	344	106
1974	Sep 9	2300	166	185	151	177	342	106
1974	Sep 19	2310	176	200	163	184	341	107
1974	Sep 29	2320	186	212	176	191	339	108
1974	Oct 9	2330	196	220	188	197	338	108
1974	Oct 19	2340	206	219	201	204	338	109
1974	Oct 29	2350	216	207	213	211	337	109
1974	Nov 8	2360	226	207	226	218	338	109
1974	Nov 18	2370	236	218	238	224	338	109
1974	Nov 28	2380	246	234	251	231	339	109
1974	Dec 8	2390	256	250	264	238	340	108
1974	Dec 18	2400	266	266	276	245	341	108
1974	Dec 28	2410	276	281	289	253	342	107
1975	Jan 7	2420	287	298	301	260	344	105
1975	Jan 17	2430	297	314	314	267	346	104
1975	Jan 27	2440	307	325	326	274	348	103
1975	Feb 6	2450	317	322	339	281	350	103
1975	Feb 16	2460	327	312	351	289	353	102
1975	Feb 26	2470	337	312	4	296	355	102
1975	Mar 8	2480	347	320	16	304	357	102
1975	Mar 18	2490	357	332	28	312	0	102
1975	Mar 28	2500	7	347	41	319	2	102
1975	Apr 7	2510	17	5	53	327	5	103
1975	Apr 17	2520	27	25	65	334	7	103
1975	Apr 27	2530	36	46	76	342	9	104
1975	May 7	2540	46	65	88	349	12	105
1975	May 17	2550	56	78	99	357	14	106
1975	May 27	2560	65	83	110	4	17	107
1975	Jun 6	2570	75	81	120	12	18	108
1975	Jun 16	2580	84	75	131	19	20	110
1975	Jun 26	2590	94	74	140	27	21	111
1975	Jul 6	2600	103	82	148	33	22	112
1975	Jul 16	2610	113	96	155	40	24	114
1975	Jul 26	2620	123	115	160	47	24	115
1975	Aug 5	2630	132	137	162	54	24	116
1975	Aug 15	2640	142	156	160	60	25	117
1975	Aug 25	2650	151	172	154	66	25	118
1975	Sep 4	2660	161	187	149	72	24	119
1975	Sep 14	2670	171	198	145	77	23	120
1975	Sep 24	2680	181	205	146	82	22	121
1975	Oct 4	2690	190	202	150	86	21	122
1975	Oct 14	2700	200	190	156	90	19	122
1975	Oct 24	2710	210	192	164	92	18	123
1975	Nov 3	2720	220	205	174	93	16	123
1975	Nov 13	2730	230	221	184	92	15	123
1975	Nov 23	2740	240	237	194	89	14	123

Courtesy of William D. Stahlman and Owen Gingerich, Solar and Planetary Longitudes for Years −2500 to +2500 by 10-day Intervals, the University of Wisconsin Press, Madison.

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Plot the positions of the other planets using a different color for each one. The data on the resulting chart is much like the data that puzzled the ancients. In fact, the table of longitudes is just an updated version of the tables that Ptolemy, Copernicus, and Tycho had made.

The graph contains a good deal of useful information. For example, when will Mercury and Venus next be close enough to each other so that you can use bright Venus to help you find Mercury? Where are the planets, relative to the sun, when they go through their retrograde motions?

A "full earth" photograph from 22,300 miles in space.

EXPERIMENT 15 SIZE OF THE EARTH

People have been telling you for many years that the earth has a diameter of about 8000 miles and a circumference of about 25,000 miles. You've believed what they told you. But suppose someone challenged you to prove it? How would you go about it?

The first recorded calculation of the size of the earth was made a long time ago—in the third century B.C., by Eratosthenes. He compared the lengths of shadows cast by the sun at two different points in Egypt. The points were rather far apart, but nearly on a north-south line on the earth's surface. The experiment you do here uses a similar method. Instead of measuring the length of a shadow, you will measure the angle between the vertical and the sight line to a star or to the sun.

You will need a colleague at least 200 miles away, due north or south of your position, to take simultaneous measurements. The two of you will need to agree in advance on the star, the date, and the time for your observations. See how close you can come to calculating the actual size of the earth.

Assumptions and Theory of the Experiment

The experiment is based on the assumptions that

1. the earth is a perfect sphere,
2. a plumb line points towards the center of the earth, and
3. the distance from the earth to the stars and sun is very great compared with the earth's diameter.

The two observers must be located at points nearly north and south of each other. Suppose they are at points A and B, separated by a distance s, as shown in Fig. 5-2. The observer at A and the observer at B both sight on the same star at the prearranged time, when the star is on or near their meridian, and measure the angle between the vertical of the plumb line and the sight line to the star.

Light rays from the star reaching locations A and B are parallel (this is implied by assumption 3).

You can therefore relate the angle θ_A at A to the angle θ_B at B, and to the angle ϕ between the two radii, as shown in Fig. 5-3.

In the triangle A′BO

$$\phi = (\theta_A - \theta_B) \qquad (1)$$

If C is the circumference of the earth, and s is an arc of the meridian, then you can make the proportion

$$\frac{s}{C} = \frac{\phi}{360°} \qquad (2)$$

Combining equations (1) and (2), you have

$$C = \frac{360°}{\theta_A - \theta_B} s,$$

where θ_A and θ_B are measured in degrees.

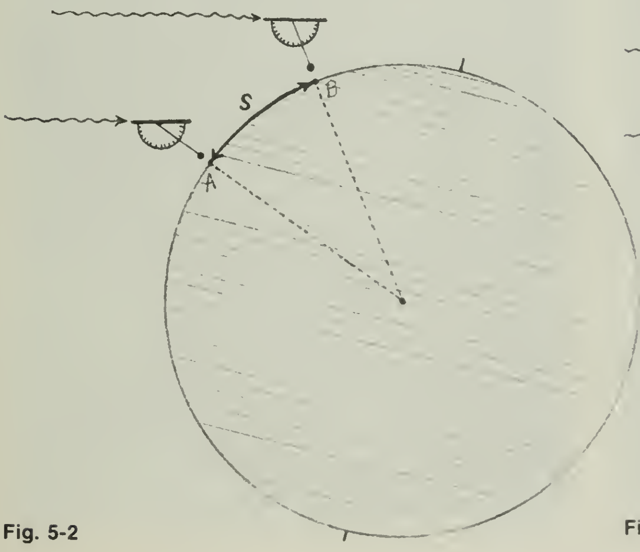

Fig. 5-2

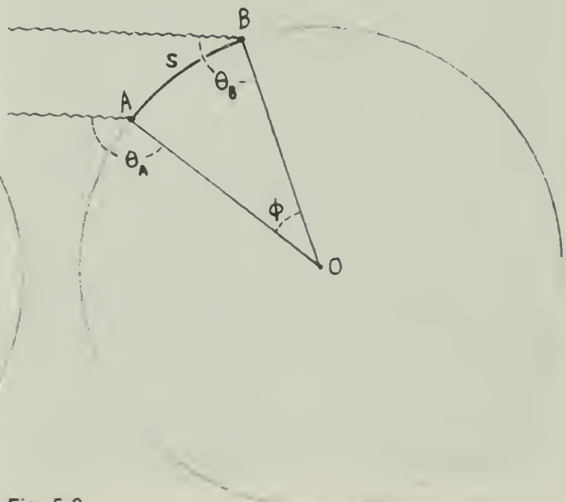

Fig. 5-3

Doing the Experiment

For best results, the two locations A and B should be directly north and south of each other, and the observations should be made when the star is near its highest point in the sky.

You will need some kind of instrument to measure the angle θ. Such an instrument is called an astrolabe. If your teacher does not have an astrolabe, you can make one fairly easily from a protractor, a small sighting tube, and a weighted string assembled according to the design in Fig. 5-4.

Align your astrolabe along the north-south line and measure the angle from the vertical to the star as it crosses the north-south line.

If the astrolabe is not aligned along the north-south line or meridian, the star will be observed before or after it is highest in the sky. An error of a few minutes from the time of crossing the meridian will make little difference in the angle measured.

An alternative method would be to measure the angle to the sun at local noon. This means the time when the sun is highest in the sky, and not necessarily 12 o'clock by standard time. (Remember that the sun, seen from the earth, is itself $\frac{1}{2}°$ wide.)

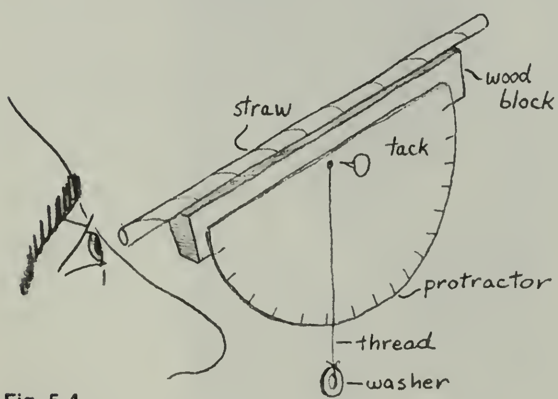

Fig. 5-4

DO NOT TRY SIGHTING DIRECTLY AT THE SUN. You may damage your eyes. Instead, have the sighting tube of your astrolabe pierce the center of a sheet of cardboard so that sunlight going through the sighting tube makes a bright spot on a shaded card that you hold up behind the tube.

An estimate of the uncertainty in your measurement of θ is important. Take several measurements on the same star (at the same time) and take the average value of θ. Use the spread in values of θ to estimate the uncertainty of your observations and of your result.

Your value for the earth's circumference depends also in knowing the over-the-earth distance between the two points of observation. You should get this distance from a map, using its scale of miles. For a description of what earth measurements over the years have disclosed about the earth's shape, see: "The Shape of the Earth," *Scientific American*, October, 1967, page 67.

Q1 How does the uncertainty of the over-the-earth distance compare with the uncertainty in your value for θ?

Q2 What is your calculated value for the circumference of the earth and what is the uncertainty of your value?

Q3 Astronomers have found that the average circumference of the earth is about 24,900 miles (40,000 km). What is the percentage error of your result?

Q4 Is this acceptable, in terms of the uncertainty of your measurement?

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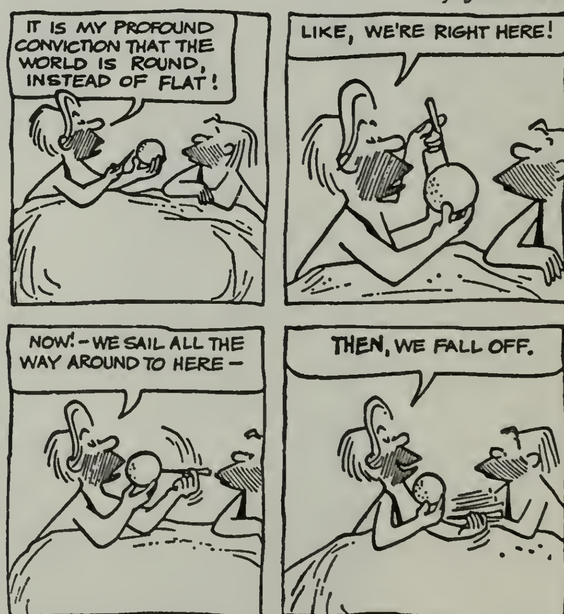


EXPERIMENT 16 THE HEIGHT OF PITON, A MOUNTAIN ON THE MOON

Closeup photographs of the moon's surface have been radioed back to earth from Lunar Orbiter spacecraft (Fig. 5-10) and from Surveyor vehicles that have made "soft landings" on the moon (Fig. 5-11, p. 137), and carried back by the Apollo astronauts. Scientists are discovering a great deal about the moon from such photographs, as well as from the landings made by astronauts in Apollo spacecraft.

But long before the space age, indeed since Galileo's time, astronomers have been learning about the moon's surface without even leaving the earth. In this experiment, you will use a photograph (Fig. 5-5) taken with a 36-inch telescope in California to estimate the height of a mountain on the moon. You will use a method similar in principle to one used by Galileo, although you should be able to get a more accurate value than he could working with his small telescope (and without photographs!).

The photograph of the moon in Fig. 5-5 was taken at the Lick Observatory very near the time of the third quarter. The photograph does not show the moon as you see it in the sky at third quarter because an astronomical telescope gives an inverted image – reversing top-and-bottom and left-and-right. (Thus north is at the bottom.) Fig. 5-6 is a 10X enlargement of the area within the white rectangle in Fig. 5-5.

Why Choose Piton?

Piton, a mountain in the moon's Northern Hemisphere, is fairly easy to measure because it is a slab-like pinnacle in an otherwise fairly flat area. When the photograph was made, with the moon near third quarter phase, Piton was quite close to the line separating the lighted portion from the darkened portion of the moon. (This line is called the *terminator*.)

You will find Piton to the south and west of the large, dark-floored crater, Plato (numbered 230 on your moon map) which is located at a longitude of −10° and a latitude of +50°.

Fig. 5-5

Fig. 5-6

Assumptions and Relations

Fig. 5-7 represents the third-quarter moon of radius r, with Piton P, its shadow of length l, at a distance d from the terminator.

The rays of light from the sun can be considered to be parallel because the moon is a great distance from the sun. Therefore, the

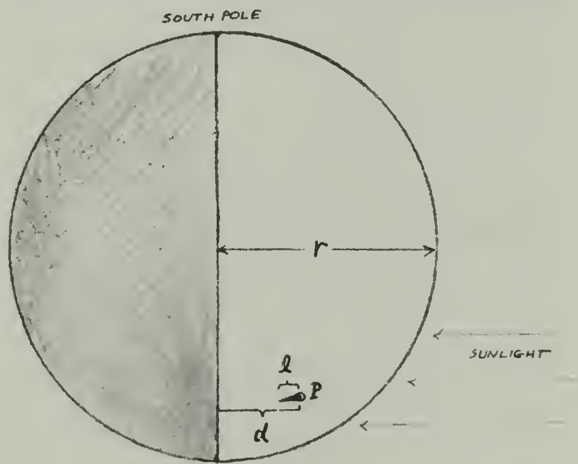


Fig. 5-7

angle at which the sun's rays strike Piton will not change if, in imagination, we rotate the moon on an axis that points toward the sun. In Fig. 5-8, the moon has been rotated just enough to put Piton on the lower edge. In this position it is easier to work out the geometry of the shadow.

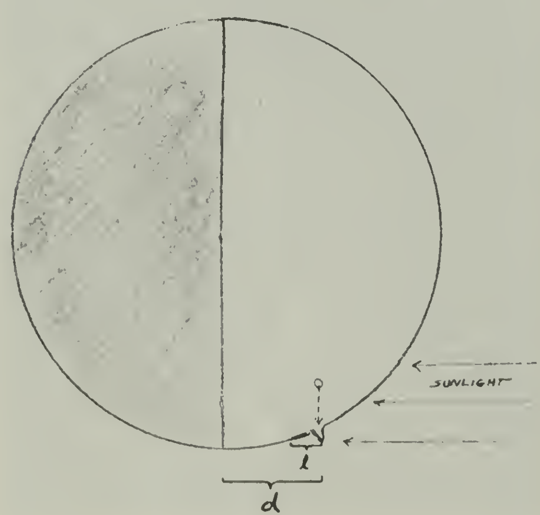


Fig. 5-8

Fig. 5-9 shows how the height of Piton can be found from similar triangles. h represents the height of the mountain, l is the apparent length of its shadow, d is the distance of the mountain from the terminator; r is a radius of the moon (drawn from Piton at P to the center of the moon's outline at O.)

It can be proven geometrically (and you can see from the drawing) that the small triangle BPA is similar to the large triangle PCO. The corresponding sides of similar triangles are proportional, so we can write

$$\frac{h}{l} = \frac{d}{r}$$

$$\text{and } h = \frac{l \times d}{r}$$

All of the quantities on the right can be measured from the photograph.

Fig. 5-9

The curvature of the moon's surface introduces some error into the calculations, but as long as the height and shadow are very small compared to the size of the moon, the error is not too great.

Measurements and Calculations

Unless specifically instructed by your teacher, you should work on a tracing of the moon picture rather than in the book itself. Trace the outline of the moon and the location of Piton. If the photograph was made when the moon was exactly at third quarter phase, then the moon was divided exactly in half by the terminator. The terminator appears ragged because highlands cast shadows across the lighted side and peaks stick up out of the shadow side. Estimate the best overall straight line for the terminator and add it to your tracing. Use a cm ruler to measure the length of Piton's shadow and the distance from the terminator to Piton's peak.

It probably will be easiest for you to do all the calculations in the scale of the photograph, find the height of Piton in cm, and then finally change to the real scale of the moon.

Q1 How high is Piton in cm on the photograph scale?

Q2 The diameter of the moon is 3,476 km. What is the scale of the photograph?

Q3 What value do you get for the actual height of Piton?

Q4 Which of your measurements is the least certain? What is your estimate of the uncertainty of your height for Piton?

Q5 Astronomers, using more complicated methods than you used, find Piton to be about 2.3 km high (and about 22 km across at its base). Does your value differ from the accepted value by more than your experimental uncertainty? If so, can you suggest why?

Fig. 5-10 A fifty square mile area of the moon's surface near the large crater, Goclenius. An unusual feature of this crater is the prominent rille that crosses the crater rim.

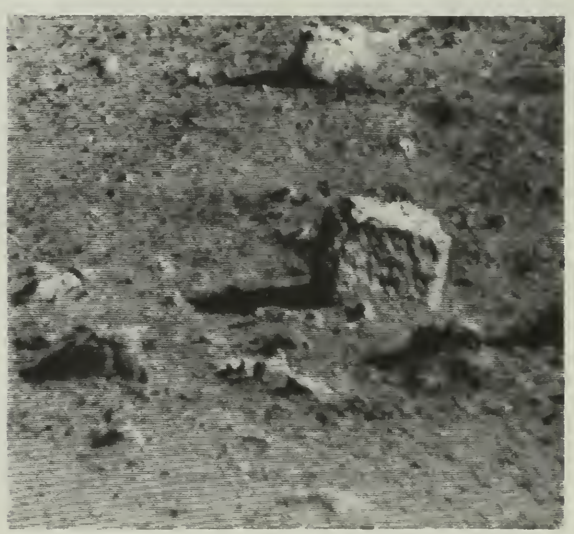


Fig. 5-11 A four-inch rock photographed on the lunar surface by Surveyor VII in 1968.

ACTIVITIES

MAKING ANGULAR MEASUREMENTS

For astronomical observations, and often for other purposes, you need to estimate the angle between two objects. You have several instant measuring devices handy once you calibrate them. Held out at arm's length in front of you, they include:

1. Your thumb,
2. Your fist not including thumb knuckles,
3. Two of your knuckles, and
4. The span of your hand from thumb-tip to tip of little finger when your hand is opened wide.

For a first approximation, your fist is about 8° and thumb-tip to little finger is between 15° and 20°.

However, since the length of people's arms and the size of their hands vary, you can calibrate yours using the following method.

To find the angular size of your thumb, fist, and hand span at your arm's length, you make use of one simple relationship. An object viewed from a distance that is 57.4 times its diameter covers an angle of 1°. For example, a one-inch coin viewed from 57.4 inches away has an angular size of 1°.

Set a 1″ ruler on the blackboard chalk tray. Stand with your eye at a distance of $57\frac{1}{2}$″ from the scale. From there observe how many inches of the scale are covered by your thumb, etc. Make sure that you have your arm straight out in front of your nose. Each inch covered corresponds to 1°. Find some convenient measuring dimensions on your hand.

A Mechanical Aid

You can use a 3″ × 5″ file card and a meter stick or yard stick to make a simple instrument for measuring angles. Remember that when an object with a given diameter is placed at a distance from your eye equal to 57.4 times its diameter, it cuts off an angle of 1°. This means that a one inch object placed at a distance of 57.4 inches from your eye would cut off an angle of 1°. An instrument 57.4 inches long would be a bit cumbersome, but we can scale down the measurements to a convenient size. A $\frac{1}{2}$ inch diameter object placed at a distance of 27.7 inches (call it 28 inches) from your eye, would cut off an angle of 1°. At this same distance, 28 inches, a 1 inch diameter object would cut off 2° and a $2\frac{1}{2}$ inch object 5°.

Now you can make a simple device that you can use to estimate angles of a few degrees.

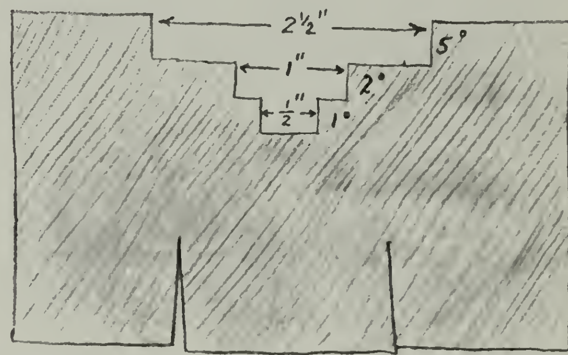
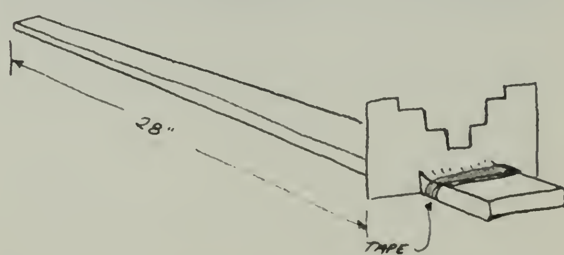


Fig. 5-12

Cut a series of step-wise slots as shown in Fig. 5-12, in the file card. Make the slots $\frac{1}{2}$ inch for 1°, one inch for 2°, and $2\frac{1}{2}$ inches for 5°. Mount the card vertically at the 28 inch mark on a yard stick. (If you use a meter stick, put the card at 57 cm and make the slots 1 cm wide for 1°, 2 cm for 2°, etc.) Cut flaps in the bottom of the card, fold them to fit along the stick and tape the card to the stick. Hold the zero end of the stick against your upper lip—and observe. (Keep a stiff upper lip!)



Things to Observe

1. What is the visual angle between the pointers of the Big Dipper?
2. What is the angular length of Orion's Belt?

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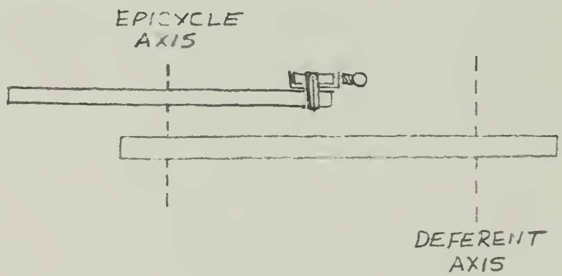
3. How many degrees away from the sun is the moon? Observe on several nights at sunset.
4. What is the angular diameter of the moon? Does it change between the time the moon rises and the time when it is highest in the sky on a given day? To most people, the moon seems larger when near the horizon. Is it? See: "The Moon Illusion," *Scientific American*, July 1962, p. 120.

EPICYCLES AND RETROGRADE MOTION

The hand-operated epicycle machine allows you to explore the motion produced by two circular motions. You can vary both the ratio of the turning rates and the ratio of the radii to find the forms of the different curves that may be traced out.

The epicycle machine has three possible gear ratios: 2 to 1 (producing two loops per revolution), 1 to 1 (one loop per revolution) and 1 to 2 (one loop per two revolutions). To change the ratio, simply slip the drive band to another set of pulleys. The belt should be twisted in a figure 8 so the deferent arm (the long arm) and the epicycle arm (the short arm) rotate in the same direction.



Tape a light source (pen-light cell, holder and bulb) securely to one end of the short, epicycle arm and counter-weight the other end of the arm with, say, another (unlit) light source. If you use a fairly high rate of rotation in a darkened room, you and other observers should be able to see the light source move in an epicycle.

The form of the curve traced out depends not only on the gear ratio but also on the relative lengths of the arms. As the light is moved closer to the center of the epicycle arm, the epicycle loop decreases in size until it becomes

Fig. 5-13

a cusp. When the light is very close to the center of the epicycle arm, as it would be for the motion of the moon around the earth, the curve will be a slightly distorted circle. (Fig. 5-14).

To relate this machine to the Ptolemaic model, in which planets move in epicycles around the earth as a center, you should really stand at the center of the deferent arm (earth) and view the lamp against a distant fixed background. The size of the machine, however, does not allow you to do this, so you must view the motion from the side. (Or, you can glue a spherical glass Christmas-tree ornament at the center of the machine; the reflection you see in the bulb is just what you would see if you were at the center.) The lamp then goes into retrograde motion each time an observer in front of the machine sees a loop. The retrograde motion is most pronounced with the light source far from the center of the epicycle axis.

Fig. 5-14

Photographing Epicycles

The motion of the light source can be photographed by mounting the epicycle machine on a turntable and holding the center pulley stationary with a clamp (Fig. 5-15). Alternatively, the machine can be held in a burette clamp on a ringstand and turned by hand.

Running the turntable for more than one revolution may show that the traces do not exactly overlap (Fig. 5-13). (This probably occurs because the drive band is not of uniform thickness, particularly at its joint, or because the pulley diameters are not in exact proportion.) As the joining seam in the band runs over either pulley, the ratio of speeds changes momentarily and a slight displacement of the axes takes place. By letting the turntable rotate for some time, the pattern will eventually begin to overlap.

A time photograph of this motion can reveal very interesting geometric patterns. You might enjoy taking such pictures as an after-class activity. Figures 5-16, a through d, show four examples of the many different patterns that can be produced.

Fig. 5-15 An epicycle demonstrator connected to a turntable.

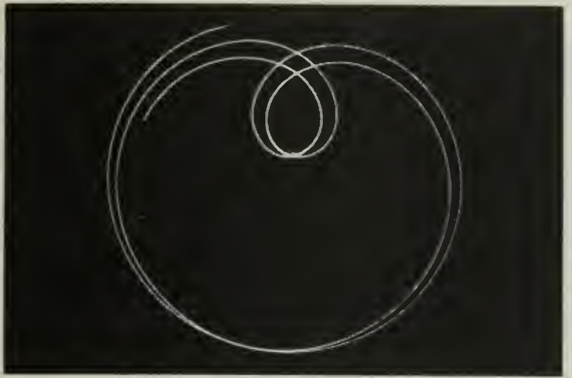


Fig. 5-16a

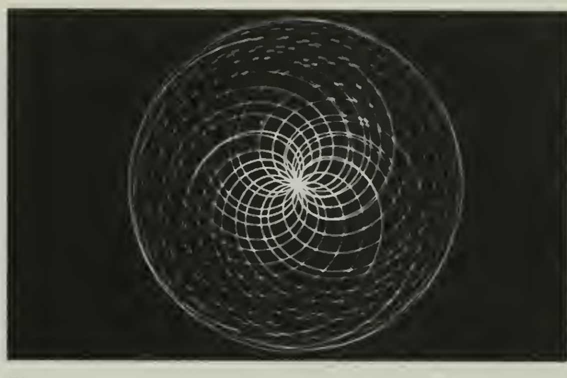


Fig. 5-16b

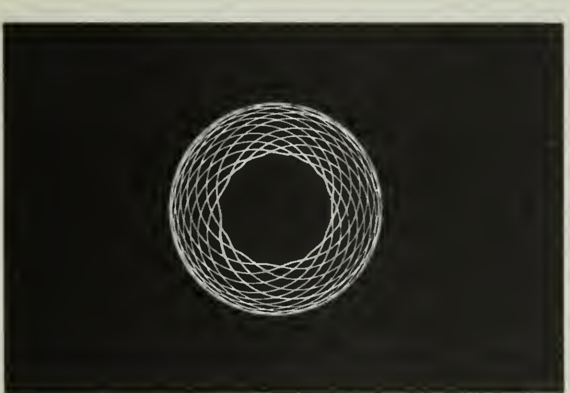


Fig. 5-16c

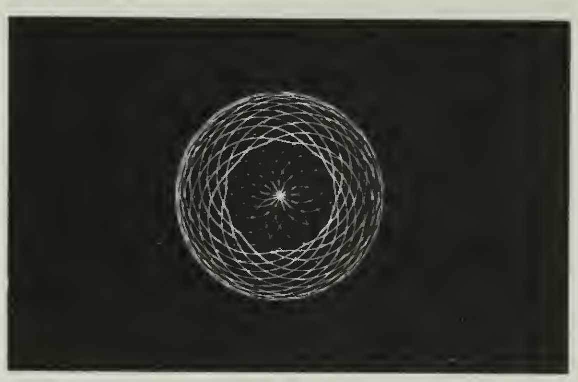


Fig. 5-16d

CELESTIAL SPHERE MODEL*

You can make a model of the celestial sphere with a round-bottom flask. With it, you can see how the appearance of the sky changes as you go northward or southward and how the stars appear to rise and set.

To make this model, you will need, in addition to the round-bottom flask, a one-hole rubber stopper to fit its neck, a piece of glass tubing, paint, a fine brush (or grease pencil), a star map or a table of star positions, and considerable patience.

On the bottom of the flask, locate the point opposite the center of the neck. Mark this point and label it "N" for north celestial pole. With a string or tape, determine the circumference of the flask—the greatest distance around it. This will be 360° in your model. Then, starting at the north celestial pole, mark points that are $\frac{1}{4}$ of the circumference, or 90°, from the North Pole point. These points lie around the flask on a line that is the celestial equator. You can mark the equator with a grease pencil (china marking pencil), or with paint.

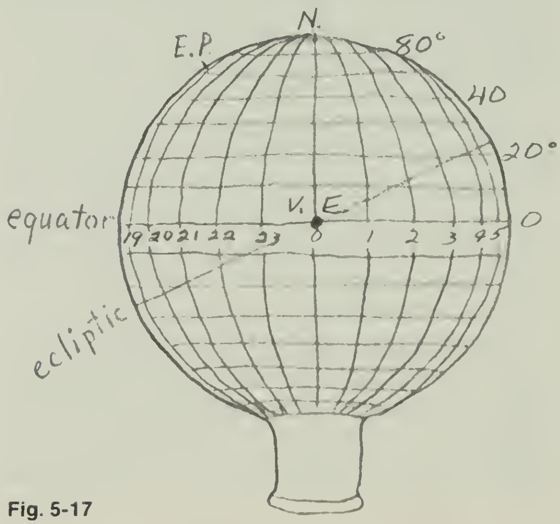


Fig. 5-17

*Adapted from *You and Science*, by Paul F. Brandwein, et al., copyright 1960 by Harcourt, Brace and World, Inc.

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To locate the stars accurately on your "globe of the sky," you will need a coordinate system. If you do not wish to have the coordinate system marked permanently on your model, put on the lines with a grease pencil.

Mark a point $23\frac{1}{2}°$ from the North Pole (about $\frac{1}{4}$ of 90°). This will be the *pole of the ecliptic* marked E.P. in Fig. 5-17. The ecliptic (path of the sun) will be a great circle 90° from the ecliptic pole. The point where the ecliptic crosses the equator from south to north is called the *vernal equinox*, the position of the sun on March 21. All positions in the sky, are located eastward from this point, and north or south from the equator.

To set up the north-south scale, measure off eight circles, about 10° apart, that run east and west in the northern hemisphere parallel to the equator. These lines are like altitude on the earth but are called *declination* in the sky. Repeat the construction of these lines of declination for the southern hemisphere.

A star's position, called its *right ascension*, is recorded in hours eastward from the vernal equinox. To set up the east-west scale, mark intervals of 1/24th of the total circumference starting at the vernal equinox. These marks are 15° apart (rather than 10°)—the sky turns through 15° each hour.

From a table of star positions or a star map, you can locate a star's coordinates, then mark the star on your globe. All east-west positions are expressed eastward, or to the right of the vernal equinox as you face your globe.

To finish the model, put the glass tube into the stopper so that it almost reaches across the flask and points to your North Pole point. Then put enough inky water in the flask so that, when you hold the neck straight down, the water just comes up to the line of the equator. For safety, wrap wire around the neck of the flask and over the stopper so it will not fall out (Fig. 5-18).

Now, as you tip the flask you have a model of the sky as you would see it from different latitudes in the Northern Hemisphere. If you were at the earth's North Pole, the north celestial pole would be directly overhead and you would see only the stars in the northern half of the sky. If you were at latitude 45°N, the north

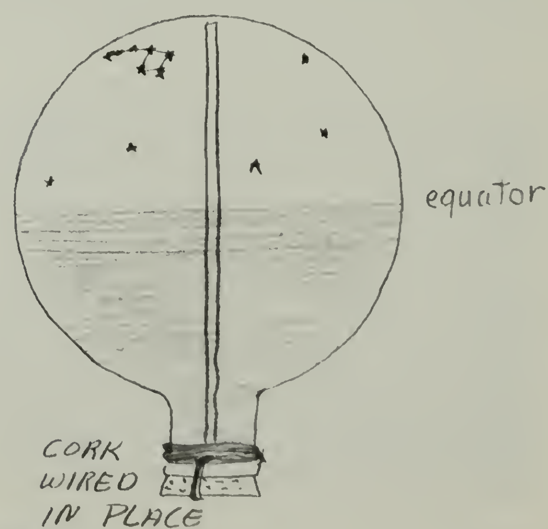


Fig. 5-18

celestial pole would be halfway between the horizon and the point directly overhead. You can simulate the appearance of the sky at 45°N by tipping the axis of your globe to 45° and rotating it. If you hold your globe with the axis horizontal, you would be able to see how the sky would appear if you were at the equator.

HOW LONG IS A SIDEREAL DAY?

A sidereal day is the time interval it takes a star to travel completely around the sky. To measure a sidereal day you need an electric clock and a screw eye.

Choose a neighboring roof, fence, etc., towards the west. Then fix a screw-eye as an eye-piece in some rigid support such as a post or a tree so that a bright star, when viewed through the screw-eye will be a little above the roof (Fig. 5-19).

Record the time when the star viewed through the screw-eye just disappears behind the roof, and again on the next night. How long did it take to go around? What is the uncertainty in your measurement? If you doubt your result, you can record times for several nights in a row and average the time intervals; this should give you a very accurate measure of a sidereal day. (If your result is not exactly 24 hours, calculate how many days would be needed for the error to add up to 24 hours.)

A scale model of the solar system

Object	Solar Distance		Diameter		Sample Object
	AU	Model (ft)	km (approx.)	Model (inches)	
Sun	------	------	1,400,000	3	tennis ball
Mercury	0.39		4,600		
Venus	0.72		12,000		
Earth	1.00	27	13,000		pinhead
Mars	1.52		6.600		
Jupiter	5.20		140,000		
Saturn	9.45		120,000		
Uranus	19.2		48,000		
Neptune	30.0		45,000		
Pluto	39.5		6,000		
Nearest star	2.7×10^5				

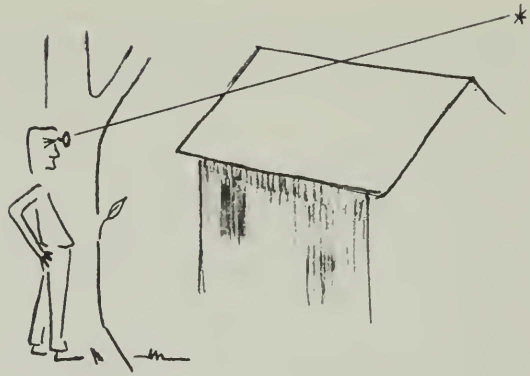


Fig. 5-19

SCALE MODEL OF THE SOLAR SYSTEM

Most drawings of the solar system are badly out of scale, because it is impossible to show both the sizes of the sun and planets and their relative distances on an ordinary-sized piece of paper. Constructing a simple scale model will help you develop a better picture of the real dimensions of the solar system.

Let a three-inch tennis ball represent the sun. The distance of the earth from the sun is 107 times the sun's diameter, or for this model, about 27 feet. (You can confirm this easily. In the sky the sun has a diameter of half a degree – about half the width of your thumb when held upright at arm's length in front of your nose. Check this, if you wish, by comparing your thumb to the angular diameter of the moon, which is nearly equal to that of the sun; both have diameters of $\frac{1}{2}°$. Now hold your thumb in the same upright position and walk away from the tennis ball until its diameter is about half the width of your thumb. You will be between 25 and 30 feet from the ball!) Since the diameter of the sun is about 1,400,000 kilometers (870,000 miles), in the model one inch represents about 464,000 kilometers. From this scale, the proper scaled distances and sizes of all the other planets can be derived.

The moon has an average distance of 384,000 kilometers from the earth and has a diameter of 3,476 kilometers. Where is it on the scale model? How large is it? Completion of the column for the scale-model distances in the table to the left will yield some surprising results.

The average distance between the earth and sun is called the "astronomical unit" (AU). This unit is used for describing distance within the solar system.

BUILD A SUNDIAL

If you are interested in building a sundial, there are numerous articles in the Amateur Scientist section of *Scientific American* that you will find helpful. See particularly the article in the issue of August 1959. Also see the issues of September 1953, October 1954, October 1959, or March 1964. The book *Sundials* by Mayall and Mayall (Charles T. Branford, Co., publishers, Boston) gives theory and building instructions for a wide variety of sundials. Encyclopedias also have helpful articles.

PLOT AN ANALEMMA

Have you seen an analemma? Examine a globe of the earth, and you will usually find a graduated scale in the shape of a figure 8, with dates on it. This figure is called an analemma. It is used to summarize the changing positions of the sun during the year.

You can plot your own analemma. Place a small square mirror on a horizontal surface so that the reflection of the sun at noon falls on a south-facing wall. Make observations each day at exactly the same time, such as noon, and mark the position of the reflection on a sheet of paper fastened to the wall. If you remove the paper each day, you must be sure to replace it in exactly the same position. Record the date beside the point. The north-south motion is most evident during September–October and March–April. You can find more about the east–west migration of the marks in astronomy texts and encyclopedias under the subject Equation of Time.

STONEHENGE

Stonehenge (pages 1 and 2 of your Unit 2 *Text*) has been a mystery for centuries. Some scientists have thought that it was a pagan temple, others that it was a monument to slaughtered chieftains. Legends invoked the power of Merlin to explain how the stones were brought to their present location. Recent studies indicate that Stonehenge may have been an astronomical observatory and eclipse computer.

Read "Stonehenge Physics," in the April, 1966 issue of *Physics Today: Stonehenge Decoded,* by G. S. Hawkins and J. B. White; or see *Scientific American,* June, 1953. Make a report and/or a model of Stonehenge for your class.

MOON CRATER NAMES

Prepare a report about how some of the moon craters were named. See Isaac Asimov's *Biographical Encyclopedia of Science and Technology* for material about some of the scientists whose names were used for craters.

LITERATURE

The astronomical models that you read about in Chapters 5 and 6, Unit 2, of the *Text* strongly influenced the Elizabethan view of the world and the universe. In spite of the ideas of Galileo and Copernicus, writers, philosophers, and theologians continued to use Aristotelian and Ptolemaic ideas in their works. In fact, there are many references to the crystal-sphere model of the universe in the writings of Shakespeare, Donne, and Milton. The references often are subtle because the ideas were commonly accepted by the people for whom the works were written.

For a quick overview of this idea, with reference to many authors of the period, read the paperbacks *The Elizabethan World Picture*, by E. M. W. Tillyard, Vintage Press, or Basil Willey, *Seventeenth Century Background*, Doubleday. See also the articles by H. Butterfield and B. Willey in Project Physics *Reader 1.*

An interesting specific example of the prevailing view, as expressed in literature, is found in Christopher Marlowe's *Doctor Faustus,* when Faustus sells his soul in return for the secrets of the universe. Speaking to the devil, Faustus says:

". . . Come, Mephistophilis, let us dispute again
And argue of divine astrology.
Tell me, are there many heavens above the moon?
Are all celestial bodies but one globe.
As is the substance of this centric earth? . . .

THE SIZE OF THE EARTH—SIMPLIFIED VERSION

Perhaps, for lack of a distant colleague, you were unable to determine the size of the earth as described in Experiment 13. You may still do so if you measure the maximum altitude of one of the objects on the following list and then use the attached data as described below.

In Santiago, Chile, Miss Maritza Campusano Reyes made the following observations of the maximum altitude of stars and of the sun: (all were observed *north* of her zenith)

Antares (Alpha Scorpio)		83.0°
Vega (Alpha Lyra)		17.5
Deneb (Alpha Cygnus)		11.5
Altair (Alpha Aquila)		47.5
Fomalhaut (Alpha Pisces Austr.)		86.5
Sun: October 1	59.4°	
15	64.8°	
November 1	70.7°	
15	74.8°	

Since Miss Reyes made her observations when the objects were highest in the sky, the values depend only upon her latitude and not upon her longitude or the time at which the observations were made.

From the map below, find how far north you are from Santiago. Next, measure the maximum altitude of one or more of these objects at your location. Then using the reasoning in Experiment 13, calculate a value for the circumference of the earth.

FILM STRIP RETROGRADE MOTION OF MARS

Photographs of the positions of Mars, from the files of the Harvard College Observatory, are shown for three oppositions of Mars, in 1941, 1943, and 1946. The first series of twelve frames shows the positions of Mars before and after the opposition of October 10, 1941. The series begins with a photograph on August 3, 1941 and ends with one on December 6, 1941. The second series shows positions of Mars before and after the opposition of December 5, 1943. This second series of seven photographs begins on October 28, 1943 and ends on February 19, 1944.

The third set of eleven pictures, which shows Mars during 1945-46, around the opposition of January 14, 1946, begins with October 16, 1945 and ends with February 23, 1946.

The film strip is used in the following way:

1. The star fields for each series of frames have been carefully positioned so that the star positions are nearly identical. If the frames of each series can be shown in rapid succession, the stars will be seen as stationary on the screen, while the motion of Mars among the stars is quite apparent. This would be like viewing a flip-book.

2. The frames can be projected on a paper screen where the positions of various stars and of Mars can be marked. If the star pattern for each frame is adjusted to match that plotted from the first frame of that series, the positions of Mars can be marked accurately for the various dates. A continuous line through these points will be a track for Mars. The dates of the turning points (when Mars begins and ends its retrograde motion) can be estimated. From these dates, the duration of the retrograde motion can be found. By use of the scale (10°) shown on one frame, the angular size of the retrograde loop can also be derived.

During 1943-44 and again in 1945-46, Mars and Jupiter came to opposition at approximately the same time. As a result, Jupiter appears in the frames and also shows its retrograde motion. Jupiter's oppositions were: January 11, 1943; February 11, 1944; March 13, 1945; and April 13, 1946. Jupiter's position can also be tracked, and the duration and size of its retrograde loop derived. The durations and angular displacements can be compared to the average values listed in Table 5.1 of the Unit 2 *Text*. This is the type of observational information which Ptolemy, Copernicus and Kepler attempted to explain by their theories.

The photographs were taken by the routine Harvard Sky Patrol with a camera of 6-inch focal length and a field of 55°. During each exposure, the camera was driven by a clockwork to follow the daily western motion of the stars and hold their images fixed on the photographic plate. Mars was never in the center of the field and was sometimes almost at the edge because the photographs were not made especially to show Mars. The planet just happened to be in the star fields being photographed.

The images of the stars and planets are not of equal brightness on all pictures because the sky was less clear on some nights and the exposures varied somewhat in duration. Also, the star images show distortions from limitations of the camera's lens. Despite these limitations, however, the pictures are adequate for the uses described above.

From a purely artistic point of view, some of the frames show beautiful pictures of the Milky Way in Taurus (1943) and Gemini (1945).

FILM LOOPS

FILM LOOP 10A RETROGRADE MOTION OF MARS AND MERCURY

To illustrate the retrograde motions of all the planets, the retrograde motions of Mercury and Mars are shown. The changing positions of each planet against the background of stars are shown during several months by animated drawings. Stars are represented by small disks whose sizes are proportional to the brightness of the stars.

Mercury first moves eastward, stops, and moves westward in retrograde motion. During the retrograde motion, Mercury passes between the earth and the sun, which has been moving steadily eastward. Mercury stops its westward motion and resumes its eastward motion following the sun. Time flashes appear for each five days. The star field includes portions of the constellations of Aries and Taurus; the familiar cluster of stars known as the Pleiades is in the upper left part of the field.

Mars similarly moves eastward across the star field, stops, moves westward in retrograde motion, stops, and resumes its eastward motion. Time flashes appear for each ten days. The star field included parts of the constellations of Leo and Cancer. The open star cluster at the upper right is Praesepe (the Beehive) which is faintly visible on a moonless night (and beautiful in a small telescope).

An angular scale (10°) allows the magnitude of the retrograde motions to be measured, while the time flashes permit a determination of the duration of those motions. The disks representing the planets change in brightness in the same manner as observed for the planets in the sky.

FILM LOOP 10 RETROGRADE MOTION —GEOCENTRIC MODEL

The film illustrates the motion of a planet such as Mars, as seen from the earth. It was made using a large "epicycle machine," as a model of the Ptolemaic system.

First, from above, you see the characteristic retrograde motion during the "loop" when the planet is closest to the earth. Then the studio lights go up and you see that the motion is due to the combination of two circular motions. One arm of the model rotates at the end of the other.

The earth, at the center of the model, is then replaced by a camera that points in a fixed direction in space. The camera views the motion of the planet relative to the fixed stars (so the rotation of the earth on its axis is being ignored). This is the same as if you were looking at the stars and planets from the earth toward one constellation of the zodiac, such as Sagittarius.

The planet, represented by a white globe, is seen along the plane of motion. The direct motion of the planet, relative to the fixed stars, is eastward, toward the left (as it would be if you were facing south). A planet's retrograde motion does not always occur at the same place in the sky, so some retrograde motions are not visible in the chosen direction of observation. To simulate observations of planets better, an additional three retrograde loops were photographed using smaller bulbs and slower speeds.

Note the changes in apparent brightness and angular size of the globe as it sweeps close to the camera. Actual planets appear only as points of light to the eye, but a marked change in brightness can be observed. This was not considered in the Ptolemaic system, which focused only on positions in the sky.

Another film loop, described on page 33, Chapter 6 of this *Handbook*, shows a similar model based on a heliocentric theory.

Chapter 6 Does the Earth Move?—The Work of Copernicus and Tycho

EXPERIMENT 17 THE SHAPE OF THE EARTH'S ORBIT

Ptolemy and most of the Greeks thought that the sun revolved around the earth. But after the time of Copernicus the idea gradually became accepted that the earth and other planets revolve around the sun. Although you probably believe the Copernican model, the evidence of your senses gives you no reason to prefer one model over the other.

With your unaided eyes you see the sun going around the sky each day in what appears to be a circle. This apparent motion of the sun is easily accounted for by imagining that it is the *earth* which rotates once a day. But the sun also has a *yearly* motion with respect to the stars. Even if we argue that the daily motion of objects in the sky is due to the turning of the earth, it is still possible to think of the earth as being at the center of the universe, and to imagine the sun moving in a year-long orbit around the earth. Simple measurements show that the sun's angular size increases and decreases slightly during the year as if it were alternately changing its distance from the earth. An interpretation that fits these observations is that the sun travels around the earth in a slightly off-center circle.

The purpose of this laboratory exercise is to plot the sun's apparent orbit with as much accuracy as possible.

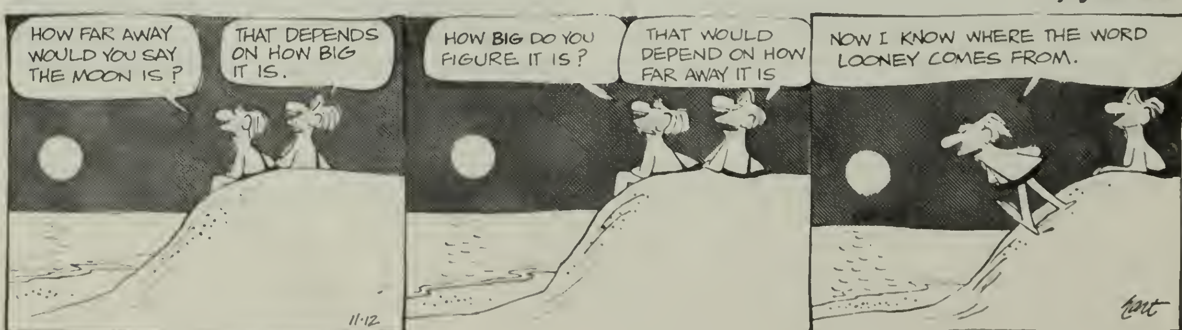
Plotting the Orbit

You know the sun's direction on each date that the sun is observed. From its observed diameter on that date you can find its relative distance from the earth. So, date by date, you can plot the sun's direction and relative distance. When you connect your plotted points by a smooth curve, you will have drawn the sun's apparent orbit.

Fig. 6-1 Frame 4 of the Sun Filmstrip.

For observations you will use a series of sun photographs taken by the U.S. Naval Observatory at approximately one-month intervals and printed on a film strip. Frame 4, in which the images of the sun in January and in July are placed adjacent to each other, has been reproduced in Fig. 6-1 so you can see how

B.C. **By John Hart**



much the apparent size of the sun changes during the year. Then note in Fig. 6-2 how the apparent size of an object is related to its distance from you.

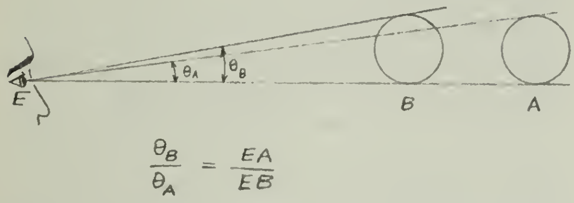


Fig. 6-2 When an object is closer to your eye, it looks bigger; it fills a larger angle as seen by your eye. In fact, the angles θ_A and θ_B are inversely proportional to the distances EA and EB:

$$\frac{\theta_B}{\theta_A} = \frac{EA}{EB}$$

In this drawing $EB = \frac{3}{4}$ EA, so angle $\theta_B = \frac{4}{3}$ angle θ_A.

Procedure

On a large sheet of graph paper (16″ × 20″, or four $8\frac{1}{2}$″ × 11″ pieces taped together) make a dot at the center to represent the earth. It is particularly important that the graph paper be this large if you are going on to plot the orbit of Mars (Experiments 17 and 19) which uses the results of the present experiment.

Take the 0° direction (toward a reference point among the stars) to be along the graph-paper lines toward the right. This will be the direction of the sun as seen from the earth on March 21. (Fig. 6-3) The dates of all the photographs and the directions to the sun, measured counterclockwise from this zero direction, are given in the table below. Use a protractor in order to draw accurately a fan of lines radiating from the earth in these different directions.

Fig. 6-3

Date	Direction from earth to sun	Date	Direction from earth to sun
March 21	000°	Oct. 4	191°
April 6	015	Nov. 3	220
May 6	045	Dec. 4	250
June 5	074	Jan. 4	283
July 5	102	Feb. 4	315
Aug. 5	132	March 7	346
Sept. 4	162		

Measure carefully the diameter of the projected image on each of the frames of the film strip. The apparent diameter of the sun depends inversely on how far away it is. You can get a set of relative distances to the sun by choosing some constant and then dividing it by the apparent diameters. An orbit with a radius of about 10 cm will be a particularly convenient size for later use. If you measure the sun's diameter to be about 50 cm, a convenient constant to choose would be 500, since $\frac{500}{50} = 10$. A larger image 51.0 cm in diameter leads to a smaller earth-sun distance:

$$\frac{500}{51.0} = 9.8 \text{ cm.}$$

Make a table of the relative distances for each of the thirteen dates.

Along each of the direction lines you have drawn, measure off the relative distance to the sun for that date. Through the points located in this way draw a smooth curve. This is the apparent orbit of the sun relative to the earth. (Since the distances are only relative, you cannot find the actual distance in miles from the earth to the sun from this plot.)

Q1 Is the orbit a circle? If so, where is the center of the circle? If the orbit is not a circle, what shape is it?

Q2 Locate the major axis of the orbit through the points where the sun passes closest to and farthest from the earth. What are the approximate dates of closest approach and greatest distance? What is the ratio of the largest distance to the smallest distance?

A Heliocentric System

Copernicus and his followers adopted the sun-centered model because they believed that the solar system could be described more simply that way. They had no new data that could not be accounted for by the old model.

Therefore, you should be able to use the same data to turn things around and plot the earth's orbit around the sun. Clearly there's going to be some similarity between the two plots.

You already have a table of the relative distances between the sun and the earth. The dates of largest and smallest distances from the earth won't change, and your table of relative distances is still valid because it wasn't based on which body was moving, only on the distance between them. Only the directions used in your plotting will change.

To figure out how the angles will change, remember that when the earth was at the center of the plot the sun was in the direction 0° (to the right) on March 21.

Q3 This being so, what is the direction of the earth as seen from the sun on that date? See if you can't answer this question for yourself before studying Fig. 6-4. Be sure you understand it before going on.

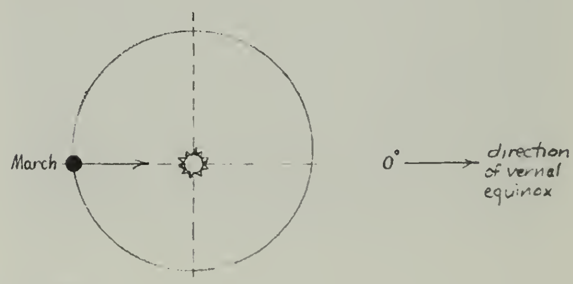


Fig. 6-4

At this stage the end is in sight. Perhaps you can see it already without doing any more plotting. But if not, here is what you can do.

If the sun is in the 0° direction from the earth, then from the sun the earth will appear to be in just the opposite direction, 180° away from 0°. You could make a new table of data giving the earth's apparent direction from the sun on the 12 dates, just by changing all the directions 180° and then making a new sun-centered plot. An easier way is to rotate your plot until top and bottom are reversed; this will change all of the directions by 180°. Relabel the 0° direction; since it is toward a reference point among the distant stars, it will still be toward the right. You can now label the center as the sun, and the orbit as the earth's.

CHAPTER 6

EXPERIMENT 18
USING LENSES TO MAKE A TELESCOPE

In this experiment, you will first examine some of the properties of single lenses. Then, you will combine these lenses to form a telescope, which you can use to observe the moon, the planets, and other heavenly (as well as earth-bound) objects.

The Simple Magnifier

You certainly know something about lenses already—for instance, that the best way to use a magnifier is to hold it immediately in front of the eye and then move the object you want to examine until its image appears in sharp focus.

Examine some objects through several different lenses. Try lenses of various shapes and sizes. Separate the lenses that magnify from those that don't. Describe the difference between lenses that magnify and those that do not.

Q1 Arrange the lenses in order of their magnifying powers. Which lens has the highest magnifying power?

Q2 What physical feature of a lens seems to determine its power or ability to magnify—is it diameter, thickness, shape, the curvature of its surface? To vary the diameter, simply put pieces of paper or cardboard with various sizes of holes in them over the lens.

Sketch side views of a high-power lens, of a low-power lens, and of the highest-power and lowest-power lenses you can imagine.

Real Images

With one of the lenses you have used, project an image of a ceiling light or an outdoor scene on a sheet of paper. Describe all the properties of the image that you can observe. An image that can be projected is called a *real image.*

Q3 Do all your lenses form real images?

Q4 How does the size of the image depend on the lens?

Q5 If you want to look at a real image without using the paper, where do you have to put your eye?

Q6 The image (or an interesting part of it) may be quite small. How can you use a second lens to inspect it more closely? Try it.

Q7 Try using other combinations of lenses. Which combination gives the greatest magnification?

Making a Telescope

With two lenses properly arranged, you can magnify distant objects. Figure 6-5 shows a simple assembly of two lenses to form a telescope. It consists of a large lens (called the *objective*) through which light enters and either of two interchangeable lenses for eyepieces.

The following notes will help you assemble your telescope.

1. If you lay the objective down on a flat clean surface, you will see that one surface is more curved than the other. The more curved surface should face the front of the telescope.
2. Clean dust, etc., off the lenses (using lens tissue or clean handkerchief) before assembling and try to keep fingerprints off it during assembly.
3. Wrap rubber bands around the slotted end of the main tube to give a convenient amount

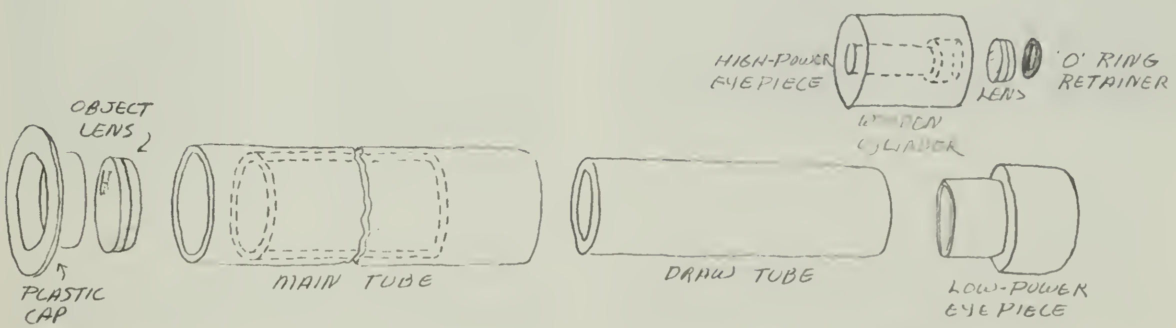


Fig. 6-5

of friction with the draw-tube—tight enough so as not to move once adjusted, but loose enough to adjust without sticking. Focus by sliding the draw tube with a rotating motion, not by moving the eyepiece in the tube.

4. To use high power satisfactorily, a steady support (a tripod) is essential.

5. Be sure that the lens lies flat in the high-power eyepiece.

Use your telescope to observe objects inside and outside the lab. Low power gives about 12× magnification. High power gives about 30× magnification.

Mounting the Telescope

If no tripod mount is available, the telescope can be held in your hands for low-power observations. Grasp the telescope as far forward and as far back as possible (Fig. 6-6) and brace both arms firmly against a car roof, telephone pole, or other rigid support.

With the higher power you must use a mounting. If a swivel-head camera tripod is available, the telescope can be held in a wooden saddle by rubber bands, and the saddle attached to the tripod head by the head's standard mounting screw. Because camera tripods are usually too short for comfortable viewing from a standing position, it is strongly recommended that you be seated in a reasonably comfortable chair.

Fig. 6-6

Aiming and Focusing

You may have trouble finding objects, especially with the high-power eyepiece. One technique is to sight over the tube, aiming slightly below the object, and then to tilt the tube up slowly while looking through it and sweeping left and right. To do this well, you will need some practice.

Focusing by pulling or pushing the sliding tube tends to move the whole telescope. To avoid this, rotate the sliding tube while moving it as if it were a screw.

Eyeglasses will keep your eye farther from the eyepiece than the best distance. Far-sighted or near-sighted observers are generally able to view more satisfactorily by removing their glasses and refocusing. Observers with astigmatism have to decide whether or not the distorted image (without glasses) is more annoying than the reduced field of view (with glasses).

Many observers find that they can keep their eye in line with the telescope while aiming and focusing if the brow and cheek rest lightly against the forefinger and thumb. (Fig. 6-6) When using a tripod mounting, remove your hands from the telescope while actually viewing to minimize shaking the instrument.

Limitations of Your Telescope

You can get some idea of how much fine detail to expect when observing the planets by comparing the angular sizes of the planets with the resolving power of the telescope. For a telescope with a 1 inch diameter object lens, to distinguish between two details, they must be at least 0.001° apart as seen from the location of the telescope. The low-power Project Physics eyepiece may not quite show this much detail, but the high power will be more than sufficient.

The angular sizes of the planets as viewed from the earth are:

Venus:	0.003°	(minimum)
	0.016	(maximum)
Mars:	0.002	(minimum)
	0.005	(maximum)
Jupiter:	0.012	(average)
Saturn:	0.005	(average)
Uranus:	0.001	(average)

Galileo's first telescope gave 3× magnification, and his "best" gave about 30× magnification. (But, he used a different kind of eyepiece that gave a much smaller field of view.) You should find it challenging to see whether you can observe all the phenomena he saw which are mentioned in Sec. 7.7 of the *Text*.

Observations You Can Make

The following group of suggested objects have been chosen because they are (1) fairly easy to find, (2) representative of what is to be seen in the sky, and (3) very interesting. You should observe all objects with the low power first and then the high power. For additional information on current objects to observe, see the paperback *New Handbook of the Heavens*, or the last few pages of each monthly issue of the magazines *Sky and Telescope, Natural History*, or *Science News*.

Venus: No features will be visible on this planet, but you can observe its phases, as shown in the photographs below (enlarged to equal sizes) and on page 72 of the Unit 2 *Text*. When Venus is very bright you may need to reduce the amount of light coming through the telescope in order to tell the true shape of the image. A paper lens cap with a round hole in the center will reduce the amount of light (and the resolution of detail!) You might also try using sunglasses as a filter.

Venus, photographed at Yerkes Observatory with the 82-inch reflector telescope.

Saturn: The planet is so large that you can resolve the projection of the rings beyond the disk, but you probably can't see the gap between the rings and the disk with your 30× telescope. Compare your observations to the sketches on page 73 of the *Text*.

Saturn photographed with the 100-inch telescope at Mount Wilson.

Jupiter: Observe the four satellites that Galileo discovered. Observe them several times, a few hours or a day apart, to see changes in their positions. By keeping detailed data over several months time, you can determine the period for each of the moons, the radii of their orbits, and then the mass of Jupiter. (See the notes for the Film Loop, "Jupiter Satellite Orbit," in Chapter 8 of this *Handbook* for directions on how to analyze your data.)

Jupiter is so large that some of the detail on its disk—like a broad, dark, equatorial cloud belt—can be detected (especially if you know it should be there!)

Jupiter photographed with the 200-inch telescope at Mount Palomar.

Moon: Moon features stand out mostly because of shadows. Best observations are made about the time of half-moon, that is, around the first and last quarter. Make sketches of your observations, and compare them to Galileo's sketch on page 66 of your *Text*. Look carefully for walls, mountains in the centers of craters, bright peaks on the dark side beyond the terminator, and craters in other craters.

B.C. By John Hart

By permission of John Hart and Field Enterprises, Inc.

The Pleiades: A beautiful little star cluster, this is located on the right shoulder of the bull in the constellation Taurus. These stars are almost directly overhead in the evening sky in December. The Pleiades were among the objects Galileo studied with his first telescope. He counted 36 stars, which the poet Tennyson described as "a swarm of fireflies tangled in a silver braid."

The Hyades: This group of stars is also in Taurus, near the star Aldebaran, which forms the bull's eye. Mainly, the Hyades look like a "v." The high-power may show that several stars are double.

The Great Nebula in Orion: Look about halfway down the row of stars that form the sword of Orion. It is in the southeastern sky during December and January. Use low power.

Algol: This famous variable star is in the constellation Perseus, south of Cassiopeia. Algol is high in the eastern sky in December, and nearly overhead during January. Generally it is a second-magnitude star, like the Pole Star. After remaining bright for almost $2\frac{1}{2}$ days, Algol fades for 5 hours and becomes a fourth-magnitude star, like the faint stars of the Little Dipper. Then, the variable star brightens during 5 hours to its normal brightness. From one minimum to the next, the period is 2 days, 20 hours, 49 minutes.

Great Nebula in Andromeda: Look high in the western sky in the early evening in December for this nebula, for by January it is low on the horizon. It will appear like a fuzzy patch of light, and is best viewed with *low* power. The light you see from this galaxy has been on its way for two million years.

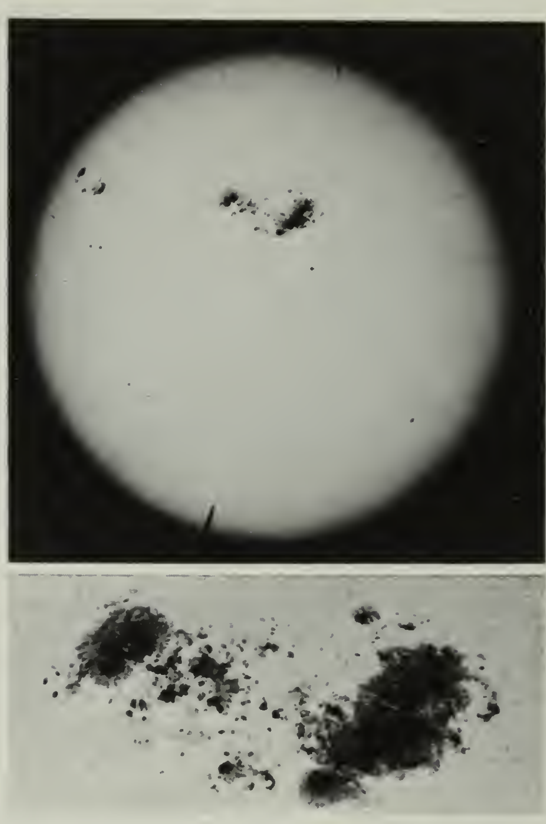
The Milky Way: This is particularly rich in Cassiopeia and Cygnus (if air pollution in your area allows it to be seen at all).

B.C. By John Hart

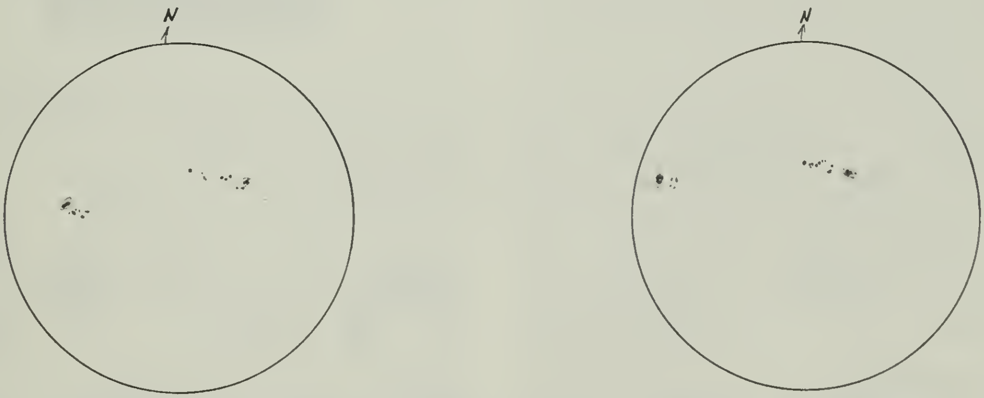
By permission of John Hart and Field Enterprises, Inc.

Fig. 6-7 Observing sunspots with a telescope.

Observing sunspots: DO NOT LOOK AT THE SUN THROUGH THE TELESCOPE. THE SUNLIGHT WILL INJURE YOUR EYES. Figure 6-7 shows an arrangement of a tripod, the low-power telescope, and a sheet of paper for projecting sunspots. Cut a hole in a piece of cardboard so it fits snugly over the object end of the telescope. This acts as a shield so there is a shadow area where you can view the sunspots. First focus the telescope, using the high-power eyepiece, on some distant object. Then, project the image of the sun on a piece of white paper a couple of feet behind the eye-piece. Focus the image by moving the draw-tube slightly further out. When the image is in focus, you may see some small dark spots on the paper. To tell marks on the paper from sunspots, jiggle the paper back and forth. How can you tell that the spots aren't on the lenses? By focusing the image farther from the telescope, you can make the image larger and not so bright. It may be easier to get the best focus by moving the paper rather than the eyepiece tube.



The sunspots of April 7, 1947.

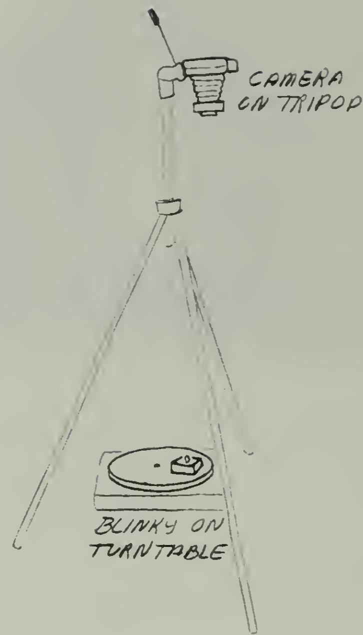


Drawings of the projected image of the sun on Aug. 26 and Aug. 27, 1966, drawn by an amateur astronomer in Walpole, Mass.

ACTIVITIES

TWO ACTIVITIES ON FRAMES OF REFERENCE

1. You and a classmate take hold of opposite ends of a meter stick or a piece of string a meter or two long. If you rotate about on one fixed spot so that you are always facing him while he walks around you in a circle, you will see him moving around you against a background of walls and furniture. But, how do you appear to him? Ask him to describe what he sees when he looks at you against the background of walls and furniture. How do your reports compare? In what direction did you see him move—toward your left or your right? In which direction did he see you move—toward his left or his right?

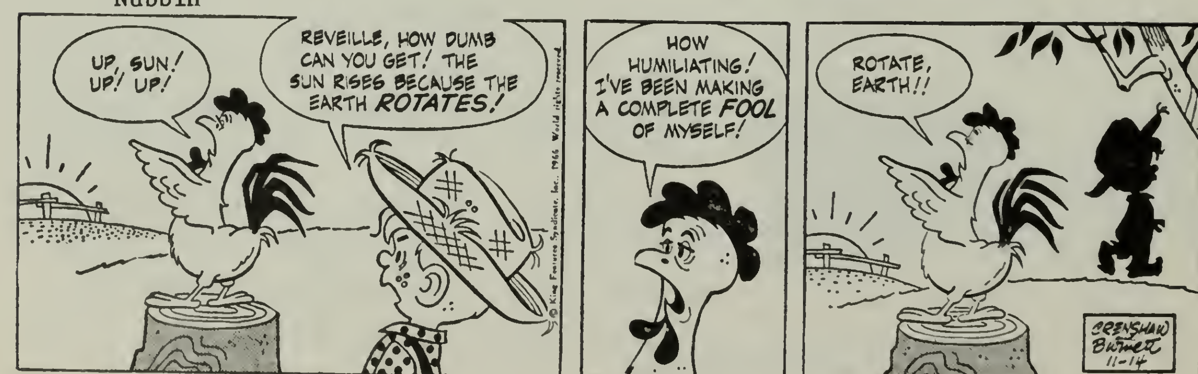


2. The second demonstration involves a camera, tripod, blinky, and turntable. Mount the camera on the tripod (using motor-strobe bracket if camera has no tripod connection) and put the blinky on a turntable. Aim the camera straight down.

Take a time exposure with the camera at rest and the blinky moving one revolution in a circle. If you do not use the turntable, move the blinky by hand around a circle drawn faintly on the background. Then take a second print, with the blinky at rest and the camera on time exposure moved steadily by hand about the axis of the tripod. Try to move the camera at the same rotational speed as the blinky moved in the first photo.

Can you tell, just by looking at the photos whether the camera or the blinky was moving?

Nubbin



FILM LOOP

FILM LOOP 11 RETROGRADE MOTION —HELIOCENTRIC MODEL

This film is based on a large heliocentric mechanical model. Globes represent the earth and a planet moving in concentric circles around the sun (represented by a yellow globe). The earth (represented by a light blue globe) passes inside a slower moving outer planet such as Mars (represented by an orange globe).

Then the earth is replaced by a camera having a 25° field of view. The camera points in a fixed direction in space, indicated by an arrow, thus ignoring the daily rotation of the earth and concentrating on the motion of the earth relative to the sun.

The view from the moving earth is shown for more than 1 year. First the sun is seen in direct motion, then Mars comes to opposition and undergoes a retrograde motion loop, and finally you see the sun again in direct motion.

Scenes are viewed from above and along the plane of motion. Retrograde motion occurs whenever Mars is in opposition, that is, whenever Mars is opposite the sun as viewed from the earth. But not all these oppositions take place when Mars is in the sector the camera sees. The time between oppositions averages about 2.1 years. The film shows that the earth moves about 2.1 times around its orbit between oppositions.

You can calculate this value. The earth makes one cycle around the sun per year and Mars makes one cycle around the sun every 1.88 years. So the frequencies of orbital motion are:

$$f_{earth} = 1 \text{ cyc/yr and } f_{mars} = 1 \text{ cyc/1.88 yr} = 0.532 \text{ cyc/yr}$$

The frequency of the earth relative to Mars is $f_{earth} - f_{mars}$:

$$f_{earth} - f_{mars} = 1.00 \text{ cyc/yr} - 0.532 \text{ cyc/yr} = 0.468 \text{ cyc/yr}$$

That is, the earth catches up with and passes Mars once every

$$\frac{1}{0.468} = 2.14 \text{ years.}$$

Note the increase in apparent size and brightness of the globe representing Mars when it is nearest the earth. Viewed with the naked eye, Mars shows a large variation in brightness (ratio of about 50:1) but always appears to be only a point of light. With the telescope we can see that the angular size also varies as predicted by the model.

The heliocentric model is in some ways simpler than the geocentric model of Ptolemy, and gives the general features observed for the planets: angular position, retrograde motion, and variation in brightness. However, detailed numerical agreement between theory and observation cannot be obtained using circular orbits.

A film of a similar model for the geocentric theory of Ptolemy is described on page 147, Chapter 5 of this *Handbook.*

Chapter 7 A New Universe Appears—the Work of Kepler and Galileo

EXPERIMENT 19 THE ORBIT OF MARS

In this laboratory activity you will derive an orbit for Mars around the sun by the same method that Kepler used in discovering that planetary orbits are elliptical. Since the observations are made from the earth, you will need the orbit of the earth that you developed in Experiment 17, "The Shape of the Earth's Orbit." Make sure that the plot you use for this experiment represents the orbit of the earth around the sun, not the sun around the earth.

If you did not do the earth-orbit experiment, you may use, for an approximate orbit, a circle of 10 cm radius drawn in the center of a large sheet of graph paper (16″ × 20″ or four $8\frac{1}{2}$″ × 11″ joined). Because the eccentricity of the earth's orbit is very small (0.017) you can place the sun at the center of the orbit without introducing a significant error in this experiment.

From the sun (at the center), draw a line to the right, parallel to the grid of the graph paper (Fig. 7-1). Label the line 0°. This line is directed toward a point on the celestial sphere called the vernal equinox and is the reference direction from which angles in the plane of the earth's orbit (the ecliptic plane) are measured. The earth crosses this line on September 23. When the earth is on the other side of its orbit on March 21, the sun is between the earth and the vernal equinox.

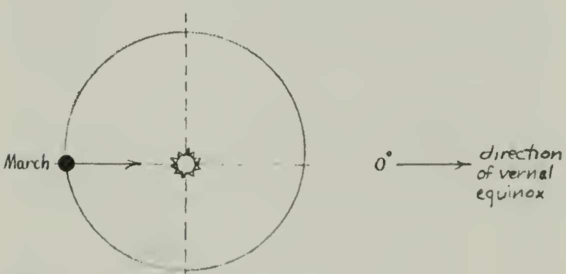


Fig. 7-1

Photographic Observations of Mars

You will use a booklet containing sixteen enlarged sections of photographs of the sky showing Mars among the stars at various dates between 1931 and 1950. All were made with the same small camera used for the Harvard Observatory Sky Patrol. In some of the photographs Mars was near the center of the field. In many other photographs Mars was near the edge of the field where the star images are distorted by the camera lens. Despite these distortions the photographs can be used to provide positions of Mars that are satisfactory for this study. Photograph P is a double exposure, but it is still quite satisfactory.

Changes in the positions of the stars relative to each other are extremely slow. Only a few stars near the sun have motions large enough to be detected after many years observations with the largest telescopes. Thus you can consider the pattern of stars as fixed.

Finding Mars' Location

Mars is continually moving among the stars but is always near the ecliptic. From several hundred thousand photographs at the Harvard Observatory sixteen were selected, with the aid of a computer, to provide pairs of photographs separated by 687 days—the period of Mars around the sun as determined by Copernicus. Thus, each pair of photographs shows Mars at one place in its orbit.

During these 687 days, the earth makes nearly two full cycles of its orbit, but the interval is short of two full years by 43 days. Therefore, the position of the earth, from which we can observe Mars, will not be the same for the two observations of each pair. If you can determine the direction from the earth towards Mars for each of the pairs of observations, the two sight lines must cross at a point on the orbit of Mars. (See Fig. 7-2.)

Coordinate System Used

When you look into the sky you see no coordinate system. Coordinate systems are created for various purposes. The one used here centers on the ecliptic. Remember that the ecliptic is the imaginary line on the celestial sphere along which the sun appears to move.

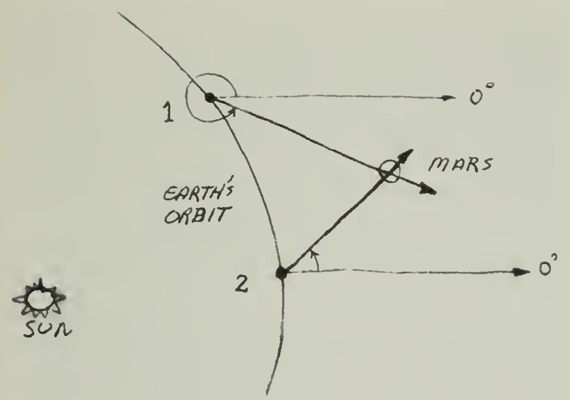


Fig. 7-2 Point 2 is the position of the earth 687 days after leaving point 1. In 687 days, Mars has made exactly one revolution and so has returned to the same point on the orbit. The intersection of the sight lines from the earth determines that point on Mars' orbit.

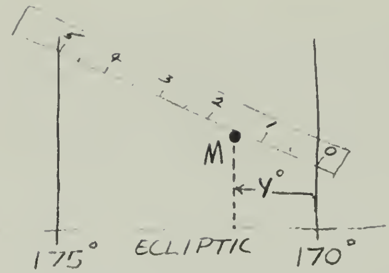


Fig. 7-3 Interpolation between coordinate lines. In the sketch, Mars (M), is at a distance y° from the 170° line. Take a piece of paper or card at least 10 cm long. Make a scale divided into 10 equal parts and label alternate marks), 1, 2, 3, 4, 5. This gives a scale in $\frac{1}{2}$° steps. Notice that the numbering goes from right to left on this scale. Place the scale so that the edge passes through the position of Mars. Now tilt the scale so that the 0 and 5 marks each fall on a grid line. Read off the value of y from the scale. In the sketch, $y = 1\frac{1}{2}$°, so that the longitude of M is $171\frac{1}{2}$°.

Along the ecliptic, *longitudes* are always measured eastward from the 0° point (the vernal equinox). This is toward the left on star maps. *Latitudes* are measured perpendicular to the ecliptic north or south to 90°. (The small movement of Mars above and below the ecliptic is considered in the Activity, "The Inclination of Mars' Orbit.")

To find the coordinates of a star or of Mars you must project the coordinate system upon the sky. To do this you are provided with transparent overlays that show the coordinate system of the ecliptic for each frame, A to P. The positions of various stars are circled. Adjust the overlay until it fits the star positions. Then you can read off the longitude and latitude of the position of Mars. Figure 7-3 shows how you can interpolate between marked coordinate lines. Because you are interested in only a small section of the sky on each photograph, you can draw each small section of the ecliptic as a straight line. For plotting, an accuracy of $\frac{1}{2}$° is satisfactory.

In a chart like the one shown in Figure 7-4, record the longitude and latitude of Mars for each photograph. For a simple plot of Mars' orbit around the sun you will use only the first column – the longitude of Mars. You will use the columns for latitude, Mars' distance from the sun, and the sun-centered coordinates if you do the Activity on the inclination, or tilt, of Mars' orbit on page 165.

Finding Mars' Orbit

When your chart is completed for all eight pairs of observations, you are ready to locate points on the orbit of Mars.

1. On the plot of the earth's orbit, locate the position of the earth for each date given in the

Fig. 7-4 Observed Positions of Mars

Frame	Date	Geocentric Long.	Geocentric Lat.	Mars to Earth Distance	Mars to Sun Distance	Heliocentric Long	Heliocentric Lat.
A	Mar. 21, 1931						
B	Feb. 5, 1933						
C	Apr. 20, 1933						
D	Mar. 8, 1935						
E	May 26, 1935						
F	Apr. 12, 1937						
G	Sept.16, 1939						
H	Aug. 4, 1941						
I	Nov. 22, 1941						
J	Oct. 11, 1943						
K	Jan. 21, 1944						
L	Dec. 9, 1945						
M	Mar. 19, 1946						
N	Feb. 3, 1948						
O	Apr. 4, 1948						
P	Feb. 21, 1950						

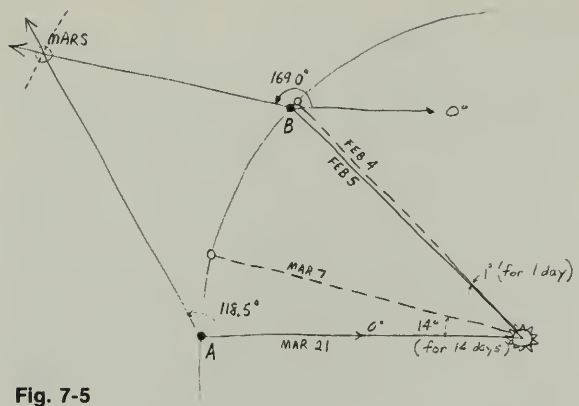


Fig. 7-5

16 photographs. You may do this by interpolating between the dates given for the earth's orbit experiment. Since the earth moves through 360° in about 365 days, you may use ±1° for each day ahead or behind the date given in the previous experiment. For example, frame A is dated March 21. The earth was at 166° on March 7: fourteen days later on March 21, the earth will have moved 14° from 166° to 180°. Always work from the earth-position date nearest the date of the Mars photograph.

2. Through each earth-position point draw a "0° line" parallel to the line you drew from the sun toward the vernal equinox (the grid on the graph paper is helpful). Use a protractor and a sharp pencil to mark the angle between the 0° line and the direction to Mars on that date as seen from the earth (longitude of Mars). The two lines drawn from the earth's positions for each pair of dates will intersect at a point. This is a point on Mars' orbit. Figure 7-5 shows one point on Mars' orbit obtained from the data of the first pair of photographs. By drawing the intersecting lines from the eight pairs of positions, you establish eight points on Mars' orbit.

3. You will notice that there are no points in one section of the orbit. You can fill in the missing part because the orbit is symmetrical about its major axis. Use a compass and, by trial and error, find a circle that best fits the plotted points. Perhaps you can borrow a French curve or long spline from the mechanical drawing or mathematics department.

Now that you have plotted the orbit, you have achieved what you set out to do: you have used Kepler's method to determine the path of Mars around the sun.

If you have time to go on, it is worthwhile to see how well your plot agrees with Kepler's generalization about planetary orbits.

Kepler's Laws from Your Plot

Q1 Does your plot agree with Kepler's conclusion that the orbit is an ellipse?

Photographs of Mars made with a 60 inch reflecting telescope (Mount Wilson and Palomar Observatories) during closest approach to the earth in 1956. Left: August 10; right: Sept. 11. Note the shrinking of the polar cap

Q2 What is the average sun-to-Mars distance in AU?

Q3 As seen from the sun, what is the direction (longitude) of Mars' nearest and farthest positions?

Q4 During what month is the earth closest to the orbit of Mars? What would be the minimum separation between the earth and Mars?

Q5 What is the eccentricity of the orbit of Mars?

Q6 Does your plot of Mars' orbit agree with Kepler's law of areas, which states that a line drawn from the sun to the planet sweeps out areas proportional to the time intervals? From your orbit, you see that Mars was at point B′ on February 5, 1933, and at point C′ on April 20, 1933, as shown in Fig. 7-6. There are eight such pairs of dates in your data. The time intervals are different for each pair.

Connect these pairs of positions with a line to the *sun* (Fig. 7-6). Find the areas of squares on the graph paper (count a square when more than half of it lies within the area). Divide the area (in squares) by the number of days in the interval to find an "area per day" value. Are these values nearly the same?

Q7 How much (by what percentage) do they vary?

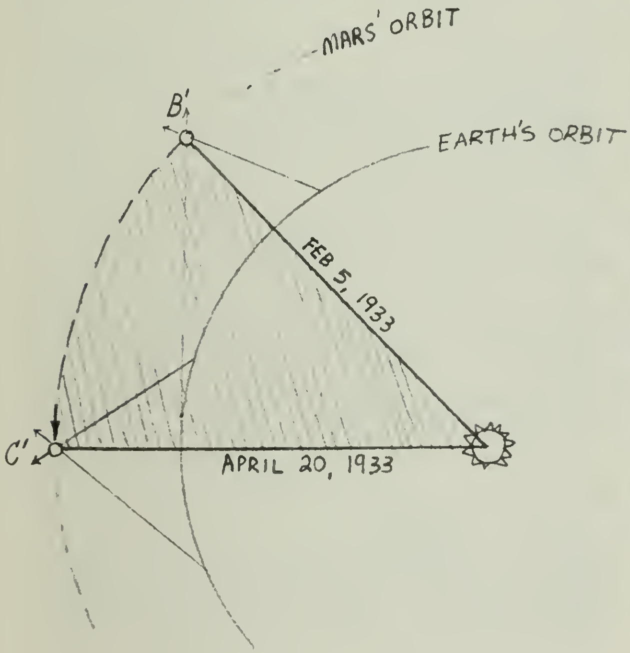
Q8 What is the uncertainty in your area measurements?

Q9 Is the uncertainty the same for large areas as for small?

Q10 Do your results bear out Kepler's law of areas?

This is by no means all that you can do with the photographs you used to make the plot of Mars' orbit. If you want to do more, look at the Activity, "The Inclination of Mars' Orbit."

Fig. 7-6 In this example, the time interval is 74 days.



Television picture of a 40 × 50 mile area just below Mars' equator, radioed from the Mariner 6 Mars probe during its 1969 fly-by.

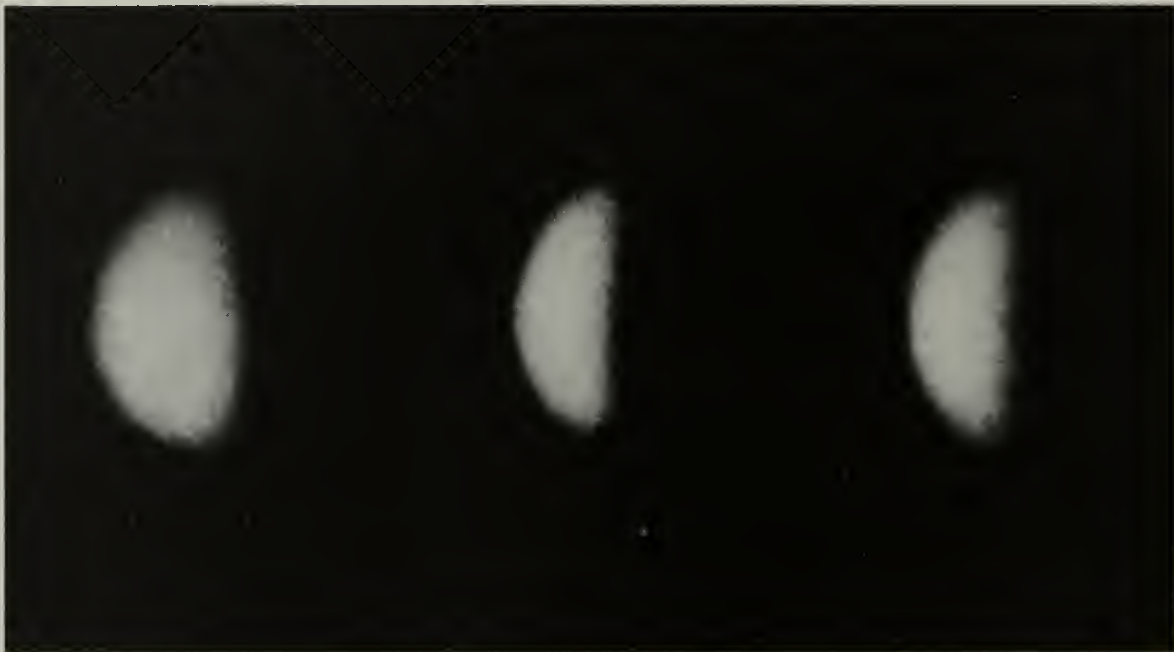


Fig. 7-7 Mercury, first quarter phase, taken June 7, 1934 at the Lowell Observatory, Flagstaff, Ariz.

EXPERIMENT 20
THE ORBIT OF MERCURY

Mercury, the innermost planet, is never very far from the sun in the sky. It can be seen only close to the horizon, just before sunrise or just after sunset, and viewing is made difficult by the glare of the sun. (Fig. 7-7.)

Except for Pluto, which differs in several respects from the other planets, Mercury has the most eccentric planetary orbit in our solar system ($e = 0.206$). The large eccentricity of Mercury's orbit has been of particular importance, since it has led to one of the tests for Einstein's General Theory of Relativity. For a planet with an orbit inside the earth's, there is a simpler way to plot the orbit than by the paired observations you used for Mars. In this experiment you will use this simpler method to get the approximate shape of Mercury's orbit.

Mercury's Elongations

Let us assume a heliocentric model for the solar system. Mercury's orbit can be found from Mercury's maximum angles of elongation east and west from the sun as seen from the earth on various known dates.

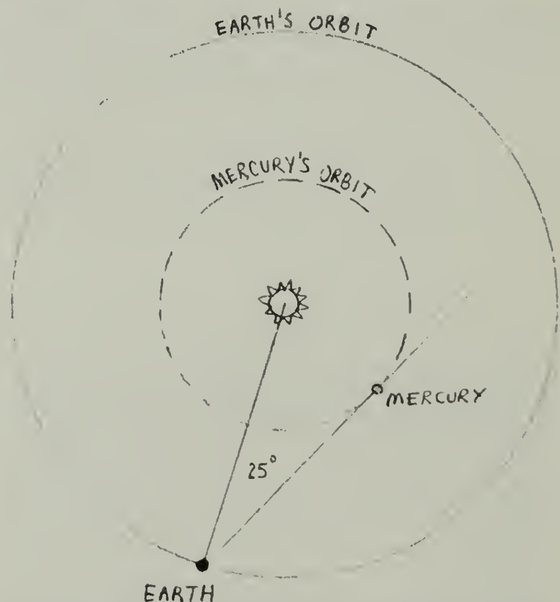


Fig. 7-8 The greatest western elongation of Mercury, May 25, 1964. The elongation had a value of 25° West.

The angle (Fig. 7-8), between the sun and Mercury as seen from the earth, is called the "elongation." Note that when the elongation reaches its maximum value, the sight lines from the earth are tangent to Mercury's orbit.

Since the orbits of Mercury and the earth are both elliptical, the greatest value of the elongation varies from revolution to revolution. The 28° elongation given for Mercury on page 14 of the *Text* refers to the maximum value. Table 1 gives the angles of a number of these greatest elongations.

TABLE 1 SOME DATES AND ANGLES OF GREATEST ELONGATION FOR MERCURY (from the American Ephemeris and Nautical Almanac)

Date	Elongation
Jan. 4, 1963	19° E
Feb. 14	26 W
Apr. 26	20 E
June 13	23 W
Aug. 24	27 E
Oct. 6	18 W
Dec. 18	20 E
Jan. 27, 1964	25 W
Apr. 8	19 E
May 25	25 W

Plotting the Orbit

You can work from the plot of the earth's orbit that you established in Experiment 17. Make sure that the plot you use for this experiment represents the orbit of the earth around the sun, not of the sun around the earth.

If you did not do the earth's orbit experiment, you may use, for an approximate earth orbit, a circle of 10 cm radius drawn in the center of a sheet of graph paper. Because the eccentricity of the earth's orbit is very small (0.017) you can place the sun at the center of the orbit without introducing a significant error in the experiment.

Draw a reference line horizontally from the center of the circle to the right. Label the line 0°. This line points toward the vernal equinox and is the reference from which the earth's position in its orbit on different dates can be established. The point where 0° line from the sun crosses the earth's orbit is the earth's position in its orbit on September 23.

The earth takes about 365 days to move once around its orbit (360°). Use the rate of approximately 1° per day, or 30° per month, to establish the position of the earth on each of the dates given in Table 1. Remember that the earth moves around this orbit in a *counterclockwise* direction, as viewed from the north celestial pole. Draw radial lines from the sun to each of the earth positions you have located.

Now draw sight lines from the earth's orbit for the elongation angles. Be sure to note, from Fig. 7-8, that for an *eastern* elongation, Mercury is to the *left* of the sun as seen from the earth. For a *western* elongation, Mercury is to the right of the sun.

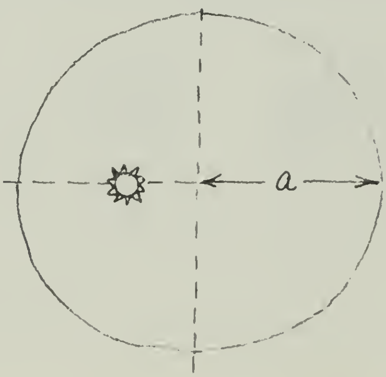
You know that on a date of greatest elongation Mercury is somewhere along the sight line, but you don't know exactly where on the line to place the planet. You also know that the sight line is tangent to the orbit. A reasonable assumption is to put Mercury at the point along the sight line closest to the sun.

You can now find the orbit of Mercury by drawing a smooth curve through, or close to, these points. Remember that the orbit must *touch* each sight line without crossing any of them.

Finding R_{av}

The average distance of a planet in an elliptical orbit is equal to one half the long diameter of the ellipse, the "semi-major axis."

To find the size of the semi-major axis *a* of Mercury's orbit, relative to the earth's semi-major axis, you must first find the aphelion and perihelion points of the orbit. You can use a drawing compass to find these points on the orbit farthest from and closest to the sun.



Measure the greatest diameter of the orbit along the line perihelion-sun-aphelion. Since 10.0 cm corresponds to one AU (the semi-major axis of the earth's orbit) you can now obtain the semi-major axis of Mercury's orbit in AU's.

Calculating Orbital Eccentricity

Eccentricity is defined as $e = c/a$ (Fig. 7-9). Since c, the distance from the center of Mercury's ellipse to the sun, is small on your plot, you lose accuracy if you try to measure it directly.

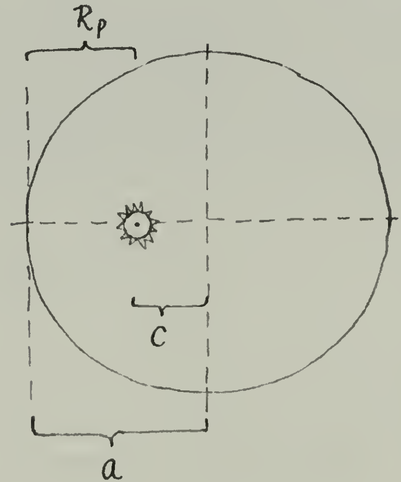


Fig. 7-9

From Fig. 7-9, you can see that c is the difference between Mercury's perihelion distance R_p and the semi-major axis a. That is:

$$c = a - R_p$$

So

$$e = \frac{c}{a} = \frac{a - R_p}{a} = 1 - \frac{R_p}{a}$$

You can measure R_p and a with reasonable accuracy from your plotted orbit. Compute e, and compare your value with the accepted value, $e \approx 0.206$.

Kepler's Second Law

You can test Kepler's equal-area law on your Mercury orbit in the same way as that described in Experiment 19, The Orbit of Mars. By counting squares you can find the area swept out by the radial line from the sun to Mercury between successive dates of observation, such as January 4 to February 14, and June 13 to August 24. Divide the area by the number of days in the interval to get the "area per day." This should be constant, if Kepler's law holds for your plot. Is it constant?

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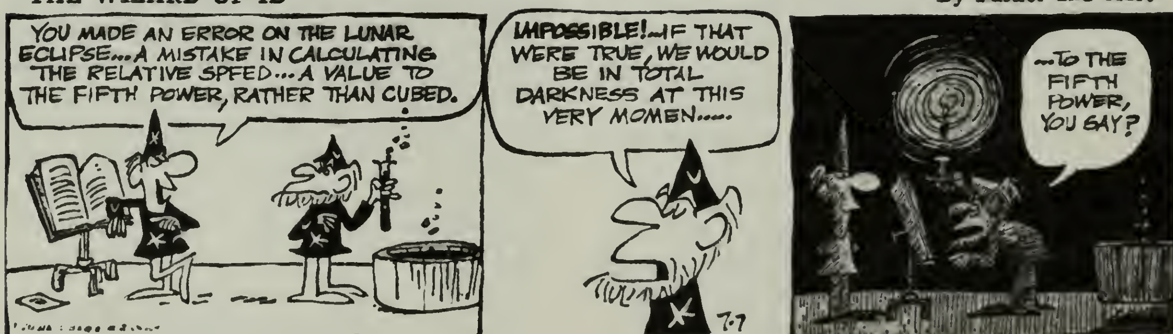


ACTIVITIES

THREE-DIMENSIONAL MODEL OF TWO ORBITS

You can make a three-dimensional model of two orbits quickly with two small pieces of cardboard (or 3″ × 5″ cards). On each card draw a circle or ellipse, but have one larger than the other. Mark clearly the position of the focus (sun) on each card. Make a straight cut *to the sun*, on one card from the left, on the other from the right. Slip the cards together until the sun-points coincide. (Fig. 7-10) Tilt the two cards (orbit planes) at various angles.

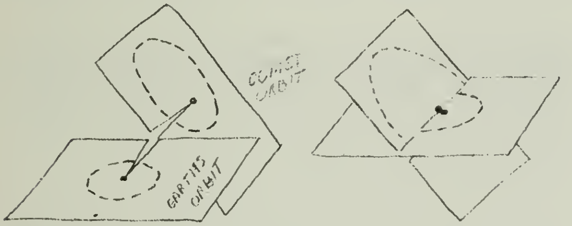


Fig. 7-10

INCLINATION OF MARS' ORBIT

When you plotted the orbit of Mars in Experiment 17, you ignored the slight movement of the planet above and below the ecliptic. This movement of Mars north and south of the ecliptic shows that the plane of its orbit is slightly inclined to the plane of the earth's orbit. In this activity, you may use the table of values for Mars latitude (which you made in Experiment 17) to determine the inclination of Mar's orbit.

Do the activity, "Three-dimensional model of two orbits," just before this activity, to see exactly what is meant by the inclination of orbits.

Theory

From each of the photographs in the set of 16 that you used in Experiment 17, you can find the observed latitude (angle from the ecliptic) of Mars at a particular point in its orbital plane. Each of these angles is measured on a photograph taken *from the earth.* As you can see from Fig. 7-11, however, it is the *sun*, not the earth, which is at the center

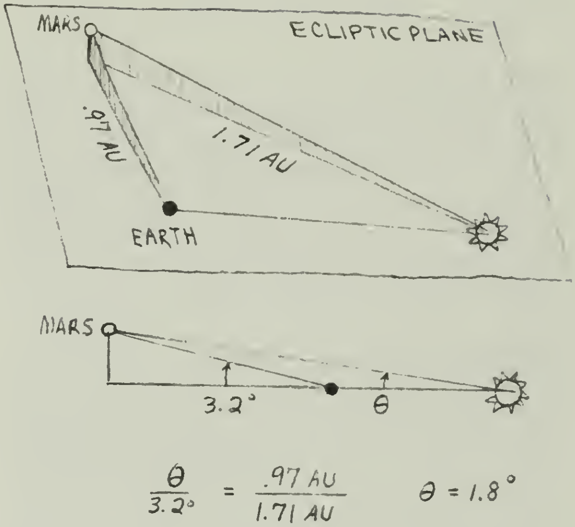


Fig. 7-11

of the orbit. The inclination of Mars' orbit must, therefore, be an angle measured *at the sun.* It is this angle (the heliocentric latitude) that you wish to find.

Figure 7-11 shows that Mars can be represented by the head of a pin whose point is stuck into the ecliptic plane. We see Mars from the earth to be north or south of the ecliptic, but we want the N-S angle of Mars as seen from the sun. The following example shows how you can derive the angles as if you were seeing them from the sun.

In Plate A (March 21, 1933), Mars was about 3.2° north of the ecliptic *as seen from the earth.* But the earth was considerably closer to Mars on this date than the sun was. Can you see how the angular elevation of Mars above the ecliptic plane as seen from the sun will therefore be considerably less than 3.2°?

For very small angles, the apparent angular sizes are inversely proportional to the distances. For example, if the sun were twice as far from Mars as the earth was, the angle at the sun would be $\frac{1}{2}$ the angle at the earth.

Measurement on the plot of Mars' orbit (Experiment 17) gives the earth-Mars distance as 9.7 cm (0.97 AU) and the distance sun-Mars as 17.1 cm (1.71 AU) on the date of the photograph. The heliocentric latitude of Mars is therefore

$$\frac{9.7}{17.1} \times 3.2°\text{N} = 1.8°\text{N}$$

You can check this value by finding the heliocentric latitude of this same point in Mars' orbit on photograph B (February 5, 1933). The earth was in a different place on this date so the geocentric latitude and the earth-Mars distance will both be different, but the heliocentric latitude should be the same to within your experimental uncertainty.

Making the Measurements

Turn to the table you made that is like Fig. 7-4 in Experiment 17, on which you recorded the geocentric latitudes λ_g of Mars. On your Mars' orbit plot from Experiment 17, measure the corresponding earth-Mars and sun-Mars distances and note them in the same table.

From these two sets of values, calculate the heliocentric latitudes as explained above. The values of heliocentric latitude calculated from the two plates in each pair (A and B, C and D, etc.) should agree within the limits of your experimental procedure.

On the plot of Mars' orbit, measure the *heliocentric longitude* λ_h for each of the eight Mars positions. Heliocentric longitude is measured from the sun, counterclockwise from the 0° direction (direction toward vernal equinox), as shown in Fig. 7-12.

Complete the table given in Fig. 7-4, Experiment 17, by entering the earth-to-Mars and sun-to-Mars distances, the geocentric and heliocentric latitudes, and the geocentric and heliocentric longitudes for all sixteen plates.

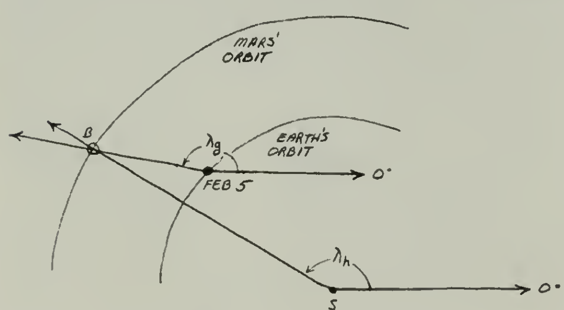


Fig. 7-12 On February 5, the heliocentric longitude (λ_h) of Point B on Mars' orbit is 150°; the geocentric longitude (λ_g) measured from the earth's position is 169°.

Make a graph, like Fig. 7-13, that shows how the heliocentric latitude of Mars changes with its heliocentric longitude.

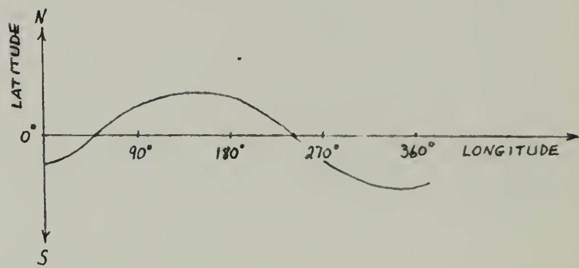


Fig. 7-13 Change of Mars' heliocentric latitude with heliocentric longitude. Label the ecliptic, latitude, ascending node, descending node and inclination of the orbit in this drawing.

From this graph, you can find two of the elements that describe the orbit of Mars with respect to the ecliptic. The point at which Mars crosses the ecliptic from south to north is called the ascending node. (The descending node, on the other side of the orbit, is the point at which Mars crosses the ecliptic from north to south.)

The angle between the plane of the earth's orbit and the plane of Mars' orbit is the inclination of Mars' orbit, *i*. When Mars reaches its maximum latitude above the ecliptic, which occurs at 90° beyond the ascending node, the planet's maximum latitude equals the inclination of the orbit, *i*.

Elements of an Orbit

Two angles, the longitude of the ascending node, Ω, and the inclination, *i*, locate the plane of Mars' orbit with respect to the plane of the ecliptic. One more angle is needed to orient the orbit of Mars in its orbital plane. This is the "argument of perihelion" ω, shown in Fig. 7-14 which is the angle in the *orbit plane* between the ascending node and perihelion point. On your plot of Mars' orbit measure the angle from the ascending node Ω to the direction of peri-

Fig. 8-2a

Fig. 8-2b

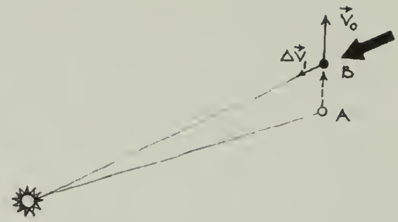


Fig. 8-2c

You can now proceed to plot an approximate comet orbit if you will make these additional assumptions:

1. The force on the comet is an attraction toward the sun.
2. The force of the blow varies inversely with the square of the comet's distance from the sun.
3. The blows occur regularly at equal time intervals, in this case, 60 days. The magnitude of each brief blow is assumed to equal the total effect of the continuous attraction of the sun throughout a 60-day interval.

Effect of the Central Force

From Newton's second law you know that the gravitational force will cause the comet to accelerate toward the sun. If a force $\vec{F}$ acts for a time interval Δt on a body of mass m, you know that

$$F = m\vec{a} = m\frac{\Delta\vec{v}}{\Delta t} \text{ and therefore}$$

$$\Delta\vec{v} = \frac{\vec{F}}{m}\Delta t$$

This equation relates the change in the body's velocity to its mass, the force, and the time for which it acts. The mass m is constant. So is Δt (assumption 3 above). The change in velocity is therefore proportional to the force, $\Delta\vec{v} \propto \vec{F}$. But remember that the force is *not* constant in magnitude; it varies inversely with the square of the distance from comet to sun.

Q4 Is the force of a blow given to the comet when it is near the sun greater or smaller than one given when the comet is far from the sun?

Q5 Which blow causes the biggest velocity change?

In Fig. 8-2a the vector $\vec{v}_0$ represents the comet's velocity at the point A. During the first 60 days, the comet moves from A to B (Fig. 8-2b). At B a blow causes a velocity change $\Delta\vec{v}_1$ (Fig. 8-2c). The new velocity after the blow is $\vec{v}_1 = \vec{v}_0 + \Delta\vec{v}_1$, and is found by completing the vector triangle (Fig. 8-2d).

The comet therefore leaves point B with velocity $\vec{v}_1$ and continues to move with this velocity for another 60-day interval. Because the time intervals between blows are always the same (60 days), the displacement along the path is proportional to the velocity, $\vec{v}$. You therefore use a length proportional to the comet's velocity to represent its displacement during each time interval. (Fig. 8-2e.)

Each new velocity is found, as above, by adding to the previous velocity the $\Delta\vec{v}$ given by the blow. In this way, step by step, the comet's orbit is built up.

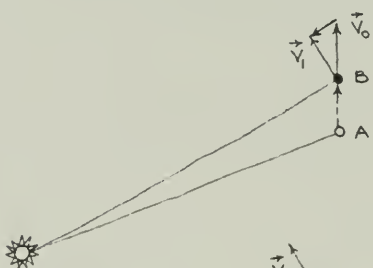


Fig. 8-2d

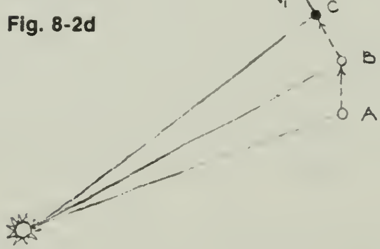


Fig. 8-2e

Scale of the Plot

The shape of the orbit depends on the initial position and velocity, and on the force acting. Assume that the comet is first spotted at a distance of 4 AU from the sun. Also assume that the comet's velocity at this point is $v = 2$ AU per year (about 20,000 miles per hour) at right angles to the sun-comet distance R.

The following scale factors will reduce the orbit to a scale that fits conveniently on a 16″ x 20″ piece of graph paper. (Make this up from four $8\frac{1}{2}$″ x 11″ pieces if necessary.)

1. Let 1 AU be scaled to 2.5 inches (or 6.5 cm) so that 4 AU becomes 10 inches (or about 25 cm).
2. Since the comet is hit every 60 days, it is convenient to express the velocity in AU per 60 days. Suppose you adopt a scale factor in which a velocity vector of 1 AU/60 days is represented by an arrow 2.5 inches (or 6.5 cm) long.

The comet's initial velocity of 2 AU per year can be given as 2/365 AU per day, or $2/365 \times 60 = 0.33$ AU per 60 days. This scales to an arrow 0.83 inches (or 2.11 cm) long. This is the *displacement* of the comet in the first 60 days.

Computing Δv

On the scale and with the 60-day iteration interval that has been chosen, the force field of the sun is such that the Δv given by a blow when the comet is 1 AU from the sun is 1 AU/60 days.

To avoid computing Δv for each value of R, you can plot Δv against R on a graph. Then for any value of R you can immediately find the value of Δv.

Table 1 gives values of R in AU and in inches and in centimeters to fit the scale of your orbit plot. The table also gives for each value of R the corresponding value of Δv in AU/60 days and in inches and in centimeters to fit the scale of your orbit plot.

Table 1 Scales for R and Δv

Distance from sun, R			Change in speed, Δv		
AU	inches	cm	AU/60 days	inches	cm
0.75	1.87	4.75	1.76	4.44	11.3
0.8	2.00	5.08	1.57	3.92	9.97
0.9	2.25	5.72	1.23	3.07	7.80
1.0	2.50	6.35	1.00	2.50	6.35
1.2	3.0	7.62	0.69	1.74	4.42
1.5	3.75	9.52	0.44	1.11	2.82
2.0	5.0	12.7	0.25	0.62	1.57
2.5	6.25	15.9	0.16	0.40	1.02
3.0	7.50	19.1	0.11	0.28	0.71
3.5	8.75	22.2	0.08	0.20	0.51
4.0	10.00	25.4	0.06	0.16	0.41

Graph these values on a separate sheet of paper at least 10 inches long, as illustrated in Fig. 8-3, and carefully connect the points with a smooth curve.

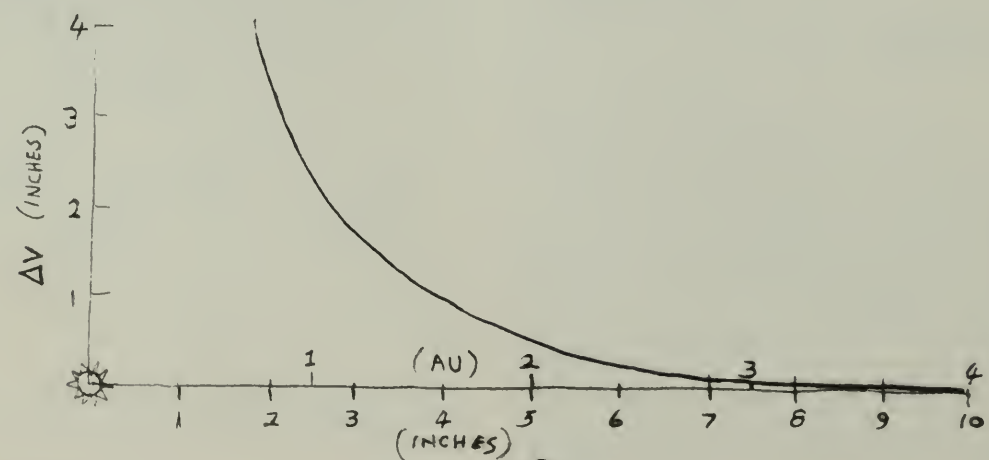


Fig. 8-3

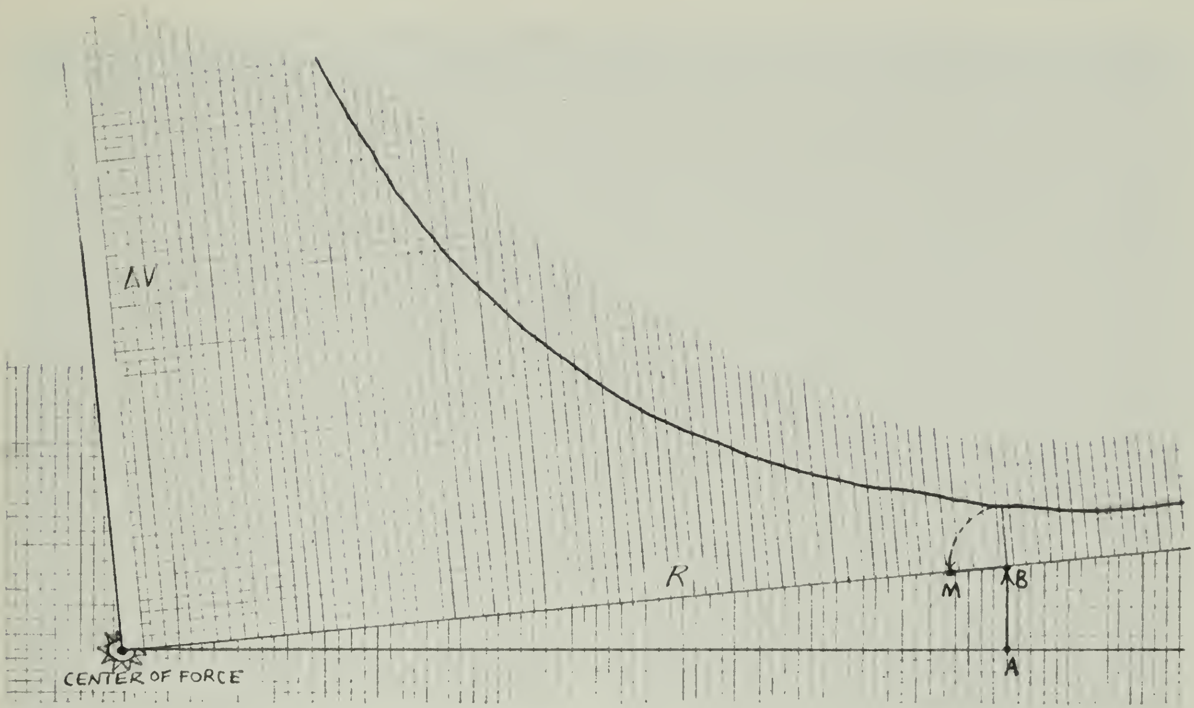
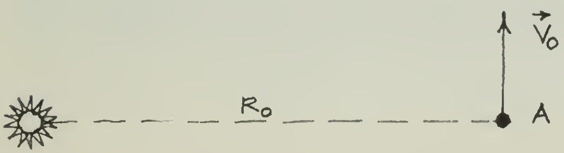


Fig. 8-4

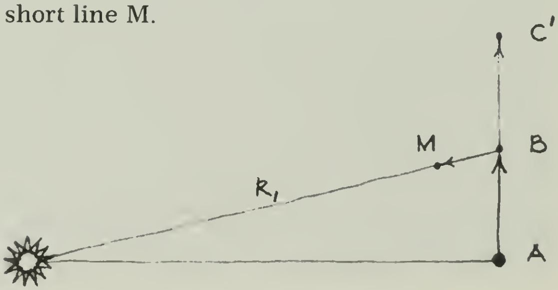
You can use this curve as a simple graphical computer. Cut off the bottom margin of the graph paper, or fold it under along the *R* axis. Lay this edge on the orbit plot and measure the distance from the sun to a blow point (such as *B* in Fig. 8-4). With dividers or a drawing compass pick off the value of Δv corresponding to this *R* and lay off this distance along the radius line toward the sun (see Fig. 8-4).

Making the Plot

1. Mark the position of the sun S halfway up the large graph paper (held horizontally) and 12 inches (or 30 cm) from the right edge.
2. Locate a point 10 inches (or 25 cm), 4 AU, that is, to the right from the sun S. This is point A where you first find the comet.



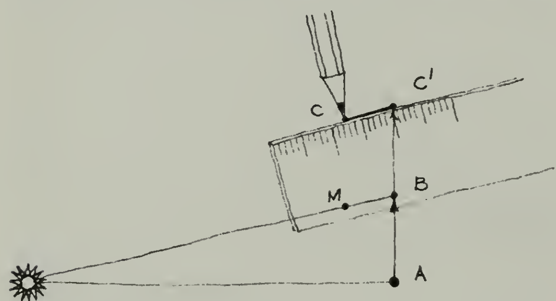
3. To represent the comet's initial velocity draw vector AB perpendicular to SA. B is the comet's position at the end of the first 60-day interval. At B a blow is struck which causes a change in velocity Δv_1.
4. Use your Δv graph to measure the distance of B from the sun at S, and to find Δv_1 for this distance (Fig. 8-4).
5. The force, and therefore the change in velocity, is always directed toward the sun. From B lay off $\Delta \vec{v}_1$ toward S. Call the end of this short line M.



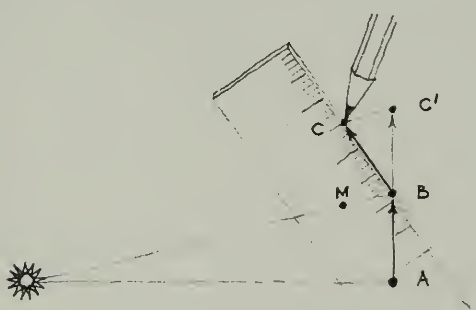
6. Draw the line BC′, which is a continuation of AB and has the same length as AB. That is

where the comet would have gone in the next 60 days if there had been no blow at B.

7. The new velocity after the blow is the vector sum of the old velocity (represented by BC′) and $\Delta\vec{v}$ (represented by BM). To find the new velocity $\vec{v}_1$ draw the line C′C parallel to BM



and of equal length. The line BC represents the new velocity vector $\vec{v}_1$, the velocity with which the comet leaves point B.



8. Again the comet moves with uniform velocity for 60 days, arriving at point C. Its displacement in that time is $\Delta\vec{d}_1 = \vec{v}_1 \times 60$ days, and because of the scale factor chosen, the displacement is represented by the line BC.

9. Repeat steps 1 through 8 to establish point D and so forth, for at least 14 or 15 steps (25 steps gives the complete orbit).

10. Connect points A, B, C . . . with a smooth curve. Your plot is finished.

Prepare for Discussion

Since you derived the orbit of this comet, you may name the comet.

Q6 From your plot, find the perihelion distance.

Q7 Find the center of the orbit and calculate the eccentricity of the orbit.

Q8 What is the period of revolution of your comet? (Refer to *Text*, Sec. 7.3.)

Q9 How does the comet's speed change with its distance from the sun?

If you have worked this far, you have learned a great deal about the motion of this comet. It is interesting to go on to see how well the orbit obtained by iteration obeys Kepler's laws.

Q10 Is Kepler's law of ellipses confirmed? (Can you think of a way to test your curve to see how nearly it is an ellipse?)

Q11 Is Kepler's law of equal areas confirmed?

To answer this remember that the time interval between blows is 60 days, so the comet is at positions B, C, D . . . , etc., after equal time intervals. Draw a line from the sun to each of these points (include A), and you have a set of triangles.

Find the area of each triangle. The area A of a triangle is given by $A = \frac{1}{2}ab$ where a and b are altitude and base, respectively. Or you can count squares to find the areas.

More Things to Do

1. The graphical technique you have practiced can be used for many problems. You can use it to find out what happens if different initial speeds and/or directions are used. You may wish to use the $1/R^2$ graph, or you may construct a new graph. To do this, use a different law (for example, force proportional to $1/R^3$, or to $1/R$ or to R) to produce different paths; actual gravitational forces are *not* represented by such force laws.

2. If you use the same force graph but reverse the direction of the force to make it a repulsion, you can examine how bodies move under such a force. Do you know of the existence of any such repulsive force?

Spiral nebula in the constellation *Leo*, photographed by the 200-inch telescope at Mount Palomar.

ACTIVITIES

MODEL OF THE ORBIT OF HALLEY'S COMET

Halley's comet is referred to several times in your *Text*. You will find that its orbit has a number of interesting features if you construct a model of it.

Since the orbit of the earth around the sun lies in one plane and the orbit of Halley's comet lies in another plane intersecting it, you will need two large pieces of stiff cardboard for planes, on which to plot these orbits.

The Earth's Orbit

Make the earth's orbit first. In the center of one piece of cardboard, draw a circle with a radius of 5 cm (1 AU) for the orbit of the earth. On the same piece of cardboard, also draw approximate (circular) orbits for Mercury (radius 0.4 AU) and Venus (radius 0.7 AU). For this plot, you can consider that all of these planets lie roughly in the one plane. Draw a line from the sun at the center and mark this line as 0° longitude.

The table on page 149 of this *Handbook* lists the apparent position of the sun in the sky on thirteen dates. By adding 180° to each of the tabled values, you can get the position of the earth in its orbit on those dates. Mark these positions on your drawing of the earth's orbit. (If you wish to mark more than those thirteen positions, you can do so by using the technique described on page 160.)

The Comet's Orbit

Figure 8-9 shows the positions of Halley's comet near the sun in its orbit, which is very nearly a parabola. You will construct your own orbit of Halley's comet by tracing Fig. 8-9 and mounting the tracing on stiff cardboard.

Combining the Two Orbits

Now you have the two orbits, the comet's and the earth's in their planes, each of which contains the sun. You need only to fit the two together in accordance with the elements of orbits shown in Fig. 7-1 that you may have used in the activity on the "Inclination of Mars Orbit" in Chapter 7.

The line along which the comet's orbital plane cuts the ecliptic plane is called the "line of nodes." Since you have the major axis drawn, you can locate the ascending node, in the orbital plane, by measuring ω, the angle from perihelion in a direction *opposite* to the comet's motion (see Fig. 8-9).

To fit the two orbits together, cut a narrow slit in the ecliptic plane (earth's orbit) along the line of the *ascending* node in as far as the sun. The longitude of the comet's ascending node Ω was at 57° as shown in Fig. 8-5. Then slit the comet's orbital plane on the side of the *descending* node in as far as the sun (see Fig. 8-6). Slip one plane into the other along the cuts until the sun-points on the two planes come together.

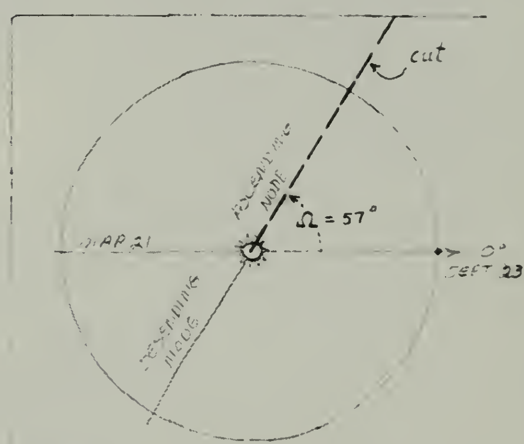


Fig. 8-5

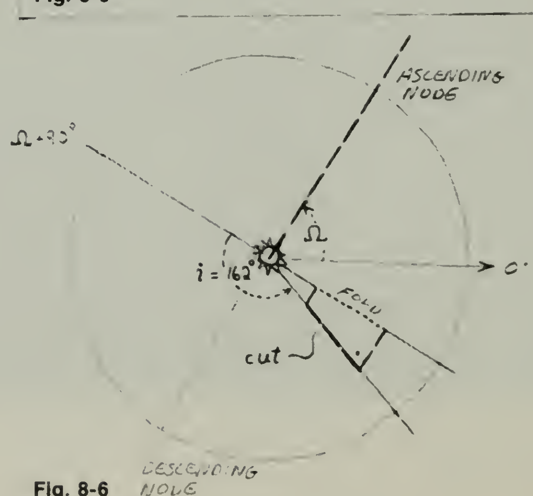


Fig. 8-6

To establish the model in three dimensions you must now fit the two planes together at the correct angle. Remember that the inclination i, 162°, is measured upward (northward) from the ecliptic in the direction of $\Omega + 90°$ (see Fig. 8-7). When you fit the two planes together you will find that the comet's orbit is on the underside of the cardboard. The simplest way to transfer the orbit to the top of the cardboard is to prick through with a pin at enough points so that you can draw a smooth curve through them. Also, you can construct a small tab to support the orbital plane in the correct position.

Halley's comet moves in the opposite sense to the earth and other planets. Whereas the earth and planets move counterclockwise when viewed from above (north of) the ecliptic, Halley's comet moves clockwise.

If you have persevered this far, and your model is a fairly accurate one, it should be easy to explain the comet's motion through the sky shown in Fig. 8-8. The dotted line in the figure is the ecliptic.

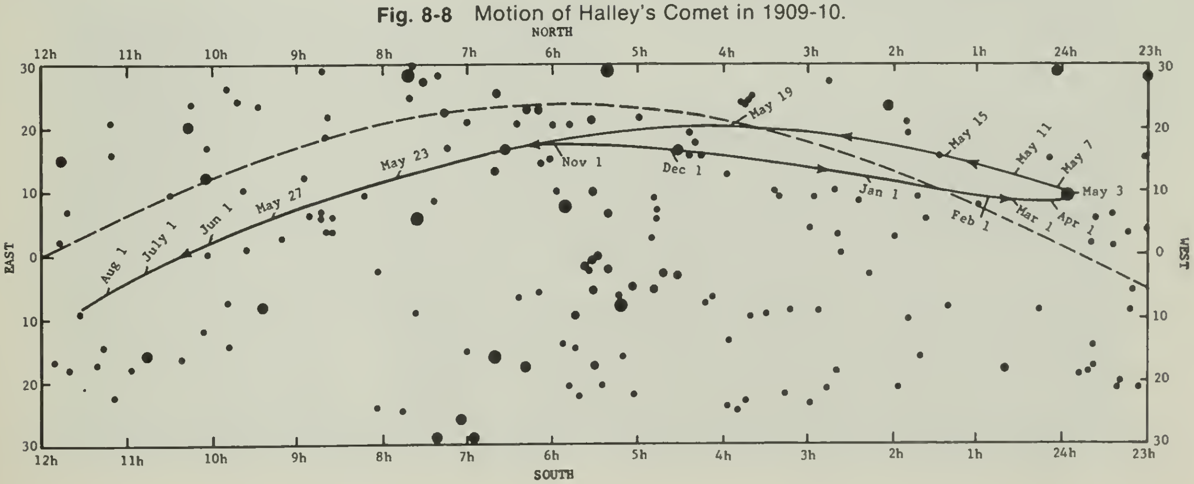
With your model of the comet orbit you can now answer some very puzzling questions about the behavior of Halley's comet in 1910.

1. Why did the comet appear to move westward for many months?
2. How could the comet hold nearly a stationary place in the sky during the month of April 1910?

Fig. 8-7

3. After remaining nearly stationary for a month, how did the comet move nearly halfway across the sky during the month of May 1910?
4. What was the position of the comet in space relative to the earth on May 19th?
5. If the comet's tail was many millions of miles long on May 19th, is it likely that the earth passed through part of the tail?
6. Were people worried about the effect a comet's tail might have on life on the earth? (See newspapers and magazines of 1910!)
7. Did anything unusual happen? How dense is the material in a comet's tail? Would you expect anything to have happened?

Fig. 8-8 Motion of Halley's Comet in 1909-10.



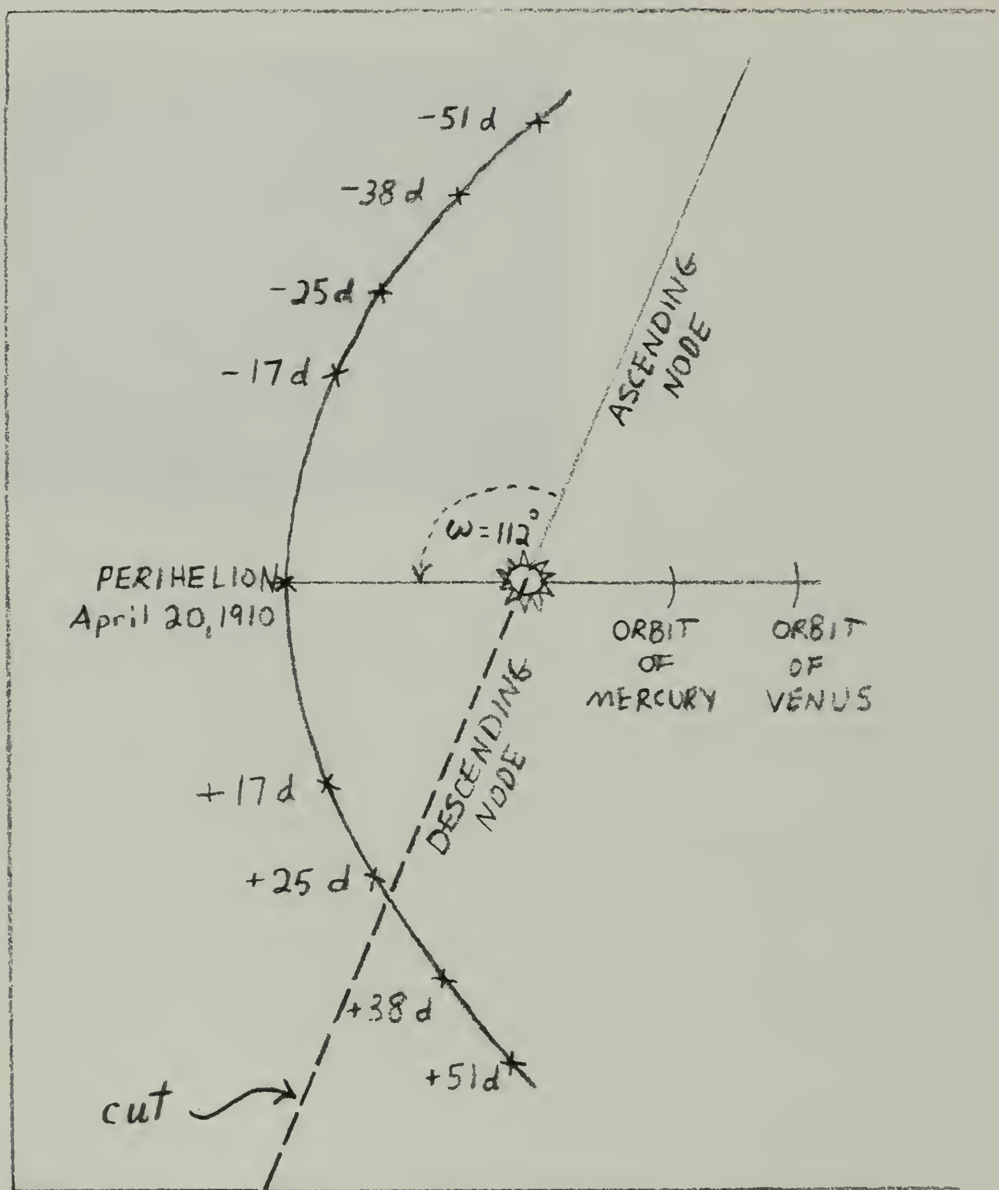


Fig. 8-9

The elements of Halley's comet are, approximately:

a (semi-major axis)	17.9 AU
e (eccentricity)	0.967
i (inclination Forbit plane)	162°
Ω (longitude of ascending node)	057°
ω (angle to perihelion)	112°
T (perihelion date)	April 20, 1910

From these data we can calculate that the period is 76 years, and is 0.59 AU the perihelion distance.

OTHER COMET ORBITS

If you enjoyed making a model of the orbit of Halley's comet, you may want to make models of some other comet orbits. Data are given below for several others of interest.

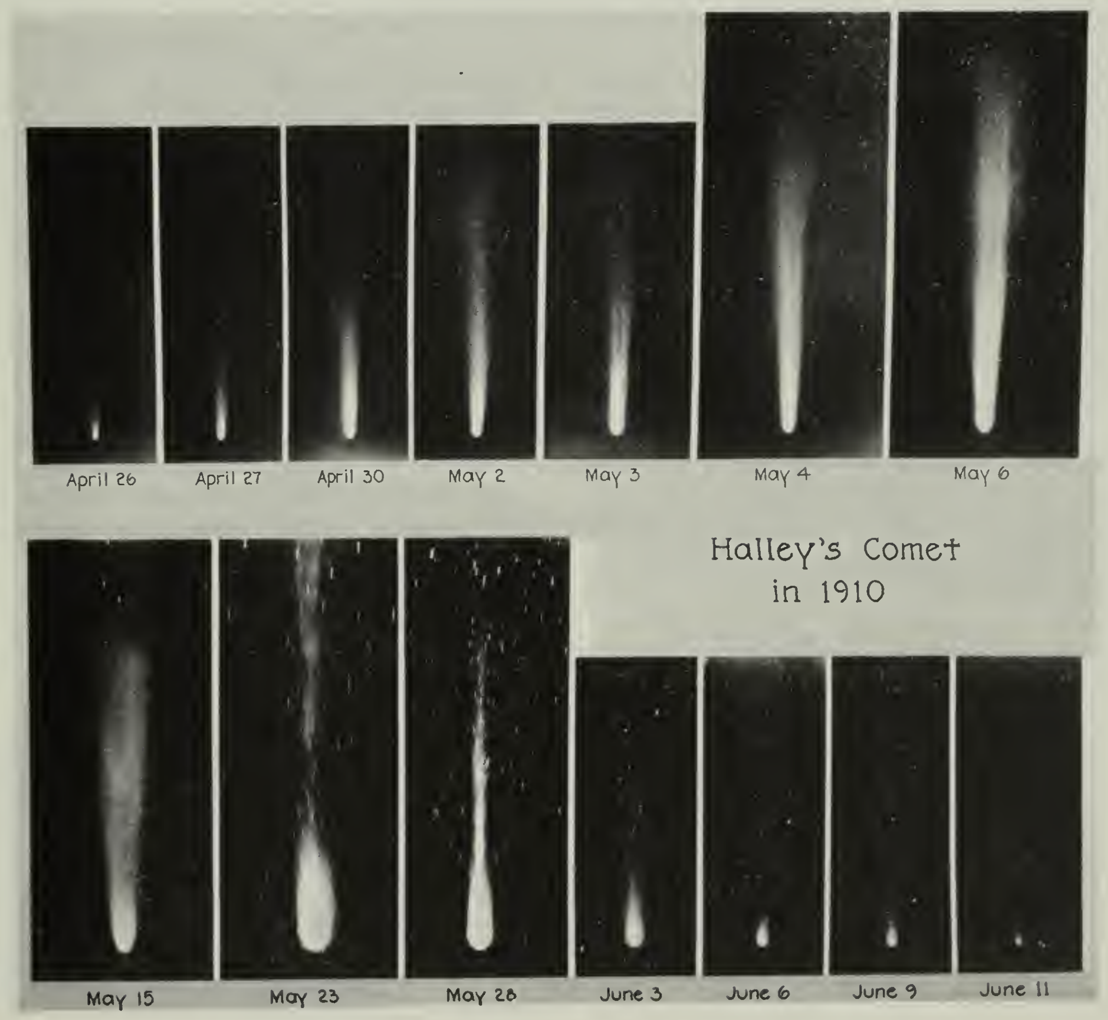
Encke's comet is interesting because it has the shortest period known for a comet, only 3.3 years. In many ways it is representative of all short-period comet orbits. All have orbits of low inclination and pass near the orbit of Jupiter, where they are often strongly deviated. The full ellipse can be drawn at the scale of 10 cm for 1 AU. The orbital elements for Encke's comet are:

$a = 2.22$ AU
$e = 0.85$
$i = 15°$
$\Omega = 335°$
$\omega = 185°$

From these data we can calculate that the perihelion distance R_r is 0.33 AU and the aphelion distance R_a is 4.11 AU.

The comet of 1680 is discussed extensively in Newton's *Principia,* where approximate orbital elements are given. The best parabolic orbital elements known are:

T = Dec. 18, 1680
$\omega = 350.7°$
$\Omega = 272.2°$
$i = 60.16°$
$R_p = 0.00626$ AU



M. Babinet prévenu par sa portière de la visite de la comète. A lithograph by the French artist Honoré Daumier (1808-1879) Museum of Fine Arts, Boston.

Note that this comet passed very close to the sun. At perihelion it must have been exposed to intense destructive forces like the comet of 1965.

Comet Candy (1960N) had the following parabolic orbital elements:

$T = $ Feb. 8, 1961
$\omega = 136.3°$
$\Omega = 176.6$
$i = 150.9$
$R_p = 1.06$ AU

FORCES ON A PENDULUM

If a pendulum is drawn aside and released with a small sideways push, it will move in an almost elliptical path. This looks vaguely like the motion of a planet about the sun, but there are some differences.

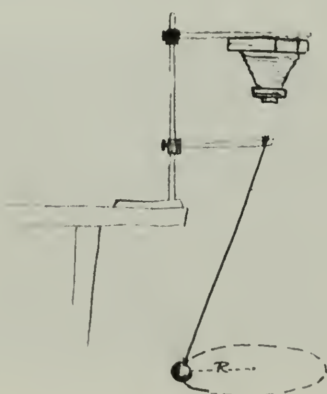


Fig. 8-10

To investigate the shape of the pendulum orbit and see whether the motion follows the law of areas, you can make a strobe photo with the setup shown in Fig. 8-10. Use either an electronic strobe flashing from the side, or use a small light and AA battery cell on the pendulum and a motor strobe disk in front of the lens. If you put the tape over one slot of a 12-slot disk to make it half as wide as the rest, it will make every 12th dot fainter giving a handy time marker, as shown in Fig. 8-11. You can also set the camera on its back on the floor with the motor strobe above it, and suspend the pendulum overhead.

Are the motions and the forces similar for the pendulum and the planets? The center of force for planets is located at one focus of the ellipse. Where is the center of force for the

Fig. 8-11

$$\frac{(m_1 + m_2)_{\text{pair}}}{m_{\text{sun}}} = \left(\frac{T_E}{T_{\text{pair}}}\right)^2 \left(\frac{R_{\text{pair}}}{R_E}\right)^3$$

The arithmetic is greatly simplified if we take the periods in years and the distances in astronomical units (A.U.) which are both units for the earth. The period of Kruger 60 is about 45 years. The mean distance of the components can be found in seconds of arc from the diagram above. The mean separation is

$$\frac{\text{max} + \text{min}}{2} = \frac{3.4 \text{ seconds} + 1.4 \text{ seconds}}{2}$$

$$= \frac{4.8 \text{ seconds}}{2} = 2.4 \text{ seconds.}$$

Earlier we found that the distance from the sun to the pair is nearly 8.7×10^5 A.U. Then the mean angular separation of 2.4 seconds equals

$$\frac{2.4 \times 8.7 \times 10^5 \text{ A.U.}}{2.1 \times 10^5} = 10 \text{ A.U.}$$

or the stars are separated from each other by about the same distance as Saturn is from the sun.

Now, upon substituting the numbers into the equation we have

$$\frac{(m_1 + m_2)_{\text{pair}}}{m_{\text{sun}}} = \frac{1}{45}^{2}\ \frac{10^3}{1}$$

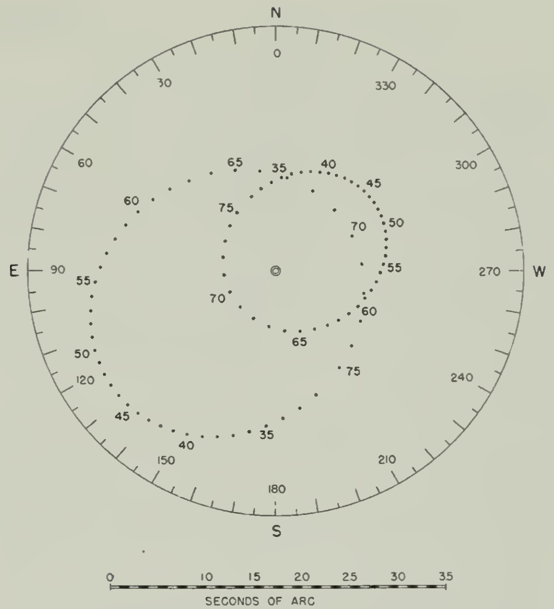
$$= \frac{1000}{2025} = 0.50,$$

or, the two stars together have about half the mass of the sun.

We can even separate this mass into the two components. In the diagram of motions relative to the center of mass we see that one star has a smaller motion, and we conclude that it must be more massive. For the positions of 1970 (or those observed a cycle earlier in 1925) the less massive star is 1.7 times farther than the other from the center of mass. So the masses of the two stars are in the ratio 1.7:1. Of the total mass of the pair, the less massive star has

$$\frac{1}{1 + 1.7} \times 0.5 = 0.18$$

the mass of the sun, while the other star has 0.32 the mass of the sun. The more massive star is more than four times brighter than the smaller star. Both stars are red dwarfs, less massive and considerably cooler than the sun.



Kruger 60's components trace elliptical orbits, indicated by dots, around their center of mass, marked by a double circle. For the years 1932 to 1975, each dot is plotted on September 1. The outer circle is calibrated in degrees, so the position angle of the companion may be read directly, through the next decade. (Positions after 1965 by extrapolation from data for 1932 to 1965).

Peanuts By Charles M. Schulz

CHAPTER 8

FILM LOOPS

FILM LOOP 12
JUPITER SATELLITE ORBIT

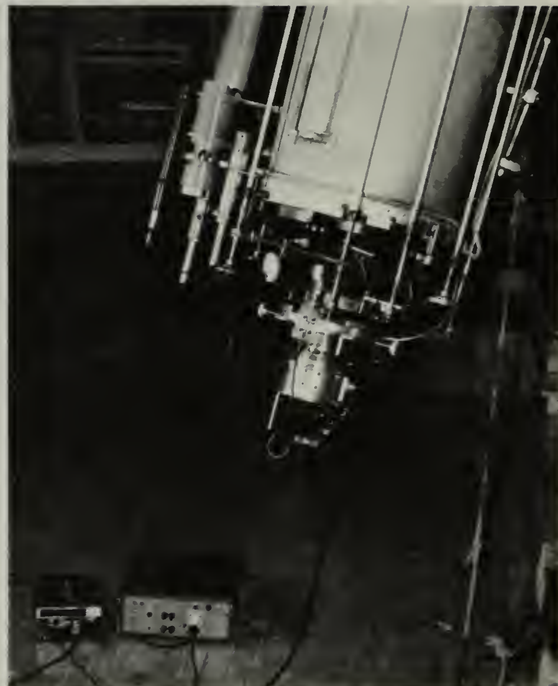
This time-lapse study of the orbit of Jupiter's satellite, Io, was filmed at the Lowell Observatory in Flagstaff, Arizona, using a 24-inch refractor telescope.

Exposures were made at 1-minute intervals during seven nights in 1967. An almost complete orbit of Io is reconstructed using all these exposures.

The film first shows a segment of the orbit as photographed at the telescope; a clock shows the passage of time. Due to small errors in guiding the telescope, and atmospheric turbulence, the highly magnified images of Jupiter and its satellites dance about. To remove this unsteadiness, each image—over 2100 of them!—was optically centered in the frame. The stabilized images were joined to give a continuous record of the motion of Io. Some variation in brightness was caused by haze or cloudiness.

The four Galilean satellites are listed in Table 1. On Feb. 3, 1967, they had the configuration shown in Fig. 8-12. The satellites move nearly in a plane which we view almost edge-on; thus they seem to move back and forth along a line. The field of view is large enough to include the entire orbits of I and II, but III and IV are outside the camera field when they are farthest from Jupiter.

The position of Io in the last frame of the Jan. 29 segment matches the position in the first frame of the Feb. 7 segment. However, since these were photographed 9 days apart, the other three satellites had moved varying distances, so you see them pop in and out while the image of Io is continuous. Lines identify Io in each section. Fix your attention on the steady motion of Io and ignore the comings and goings of the other satellites.



Business end of the 24-inch refractor at Lowell Observatory.

TABLE 1
SATELLITES OF JUPITER

	NAME	PERIOD	RADIUS OF ORBIT (miles)	ECCENTRICITY OF ORBIT	DIAMETER (miles)
I	Io	$1^d\ 18^h\ 28^m$	262,000	0.0000	2,000
II	Europa	$3^d\ 13^h\ 14^m$	417,000	0.0003	1,800
III	Ganymede	$7^d\ 3^h\ 43^m$	666,000	0.0015	3,100
IV	Callisto	$16^d\ 16^h\ 32^m$	1,171,000	0.0075	2,800

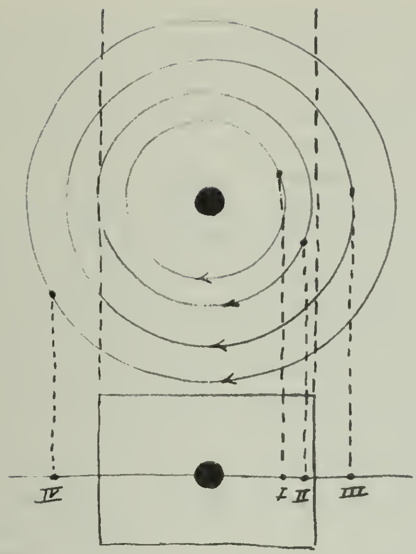


Fig. 8-12

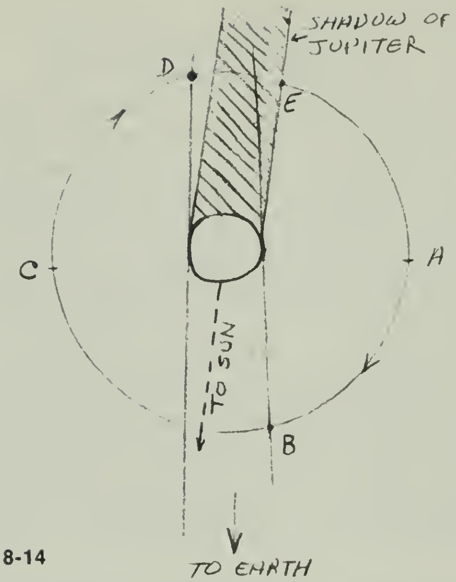


Fig. 8-14

Interesting Features of the Film

1. At the start Io appears almost stationary at the right, at its greatest elongation; another satellite is moving toward the left and overtakes it.

2. As Io moves toward the left (Fig. 8-13), it passes in front of Jupiter, a *transit*. Another satellite, *Ganymede*, has a transit at about the same time. Another satellite moves toward the right and disappears behind Jupiter, an *occultation*. It is a very active scene! If you look closely during the transit, you may see the shadow of Ganymede and perhaps that of Io, on the left part of Jupiter's surface.

Fig. 8-13 Still photograph from Film Loop 12 showing the positions of three satellites of Jupiter at the start of the transit and occultation sequence. Satellite IV is out of the picture, far to the right of Jupiter.

3. Near the end of the film, Io (moving toward the right) disappears; an occulation begins. Look for Io's reappearance—it emerges from an eclipse and appears to the right of Jupiter. Note that Io is out of sight part of the time because it is behind Jupiter as viewed from the earth and part of the time because it is in Jupiter's shadow. It cannot be seen as it moves from O to E in Fig. 8-14.

4. Jupiter is seen as a flattened circle because its rapid rotation period (9 h 55 m) has caused it to flatten at the poles and bulge at the equator. The effect is quite noticeable: the equatorial diameter 89,200 miles and the polar diameter is 83,400 miles.

Measurements

1. *Period of orbit.* Time the motion between transit and occulation (from B to D in Fig. 8-14), half a revolution, to find the period. The film is projected at about 18 frames/sec, so that the speed-up factor is 18×60, or 1080. How can you calibrate your projector more accurately? (There are 3969 frames in the loop.) How does your result for the period compare with the value given in the table?

2. *Radius of orbit.* Project on paper and mark the two extreme positions of the satellite,

farthest to the right (at A) and farthest to the left (at C). To find the radius in miles, use Jupiter's equatorial diameter for a scale.

3) *Mass of Jupiter.* You can use your values for the orbit radius and period to calculate the mass of Jupiter relative to that of the sun (a similar calculation based on the satellite Callisto is given in SG 8.9 of the *Text*). How does your experimental result compare with the accepted value, which is $m_j/m_s = 1/1048$?

FILM LOOP 13 PROGRAM ORBIT I

A student (right, Fig. 8-15) is plotting the orbit of a planet, using a stepwise approximation. His teacher (left) is preparing the computer program for the same problem. The computer and the student follow a similar procedure.

Fig. 8-15

The computer "language" used was FORTRAN. The FORTRAN program (on a stack of punched cards) consists of the "rules of the game": the laws of motion and of gravitation. These describe precisely how the calculation is to be done. The program is translated and stored in the computer's memory before it is executed.

The calculation begins with the choice of initial position and velocity of the planet. The initial position values of X and Y are selected and also the initial components of velocity XVEL and YVEL. (XVEL is the name of a single variable, not a product of four variables X, V, E, and L.)

Then the program instructs the computer to calculate the force on the planet from the sun from the inverse-square law of gravitation. Newton's laws of motion are used to calculate how far and in what direction the planet moves after each blow.

The computer's calculations can be displayed in several ways. A table of X and Y values can be typed or printed. An X-Y plotter can draw a graph from the values, similar to the hand-constructed graph made by the student. The computer results can also be shown on a cathode ray tube (CRT), similar to that in a television set, in the form of a visual trace. In this film, the X-Y plotter was the mode of display used.

The dialogue between the computer and the operator for trial 1 is as follows. The numerical values are entered at the computer typewriter by the operator after the computer types the messages requesting them.

Computer:	GIVE ME INITIAL POSITION IN AU . . .
Operator:	X = 4 Y = 0
Computer:	GIVE ME INITIAL VELOCITY IN AU/YR . . .
Operator:	XVEL = 0 YVEL = 2
Computer:	GIVE ME CALCULATION STEP IN DAYS . . .
Operator:	60.
Computer:	GIVE ME NUMBER OF STEPS FOR EACH POINT PLOTTED . . .
Operator:	1.
Computer:	GIVE ME DISPLAY MODE . . .
Operator:	X-Y PLOTTER.

You can see that the orbit displayed on the X-Y plotter, like the student's graph, does not close. This is surprising, as you know that the orbits of planets are closed. Both orbits fail to close exactly. Perhaps too much error is introduced by using such large steps in the step-by-step approximation. The blows may be too infrequent near perihelion, where the force is largest, to be a good approximation to a continuously acting force. In the Film Loop, "Program Orbit II," the calculations are based upon smaller steps, and you can see if this explanation is reasonable.

FILM LOOP 14 PROGRAM ORBIT II

In this continuation of the film "Program Orbit I," a computer is again used to plot a planetary orbit with a force inversely proportional to the square of the distance. The computer program adopts Newton's laws of motion. At equal intervals, blows act on the body. We guessed that the orbit calculated in the previous film failed to close because the blows were spaced too far apart. You could calculate the orbit using many more blows, but to do this by hand would require much more time and effort. In the computer calculation we need only specify a smaller time interval between the calculated points. The laws of motion are the same as before, so the same program is used.

A portion of the "dialogue" between the computer and the operator for trial 2 is as follows:

Computer: GIVE ME CALCULATION STEP IN DAYS . . .
Operator: 3.
Computer: GIVE ME NUMBER OF STEPS FOR EACH POINT PLOTTED . . .
Operator: 7.
Computer: GIVE ME DISPLAY MODE . . .
Operator: X-Y PLOTTER.

Points are now calculated every 3 days (20 times as many calculations as for trial 1 on the "Program Orbit I" film), but, to avoid a graph with too many points, only 1 out of 7 of the calculated points is plotted.

The computer output in this film can also be displayed on the face of a cathode ray tube (CRT). The CRT display has the advantage of speed and flexibility and we will use it in the other loops in this series, *Film loops* 15, 16 and 17. On the other hand, the permanent record produced by the X-Y plotter is sometimes very convenient.

Orbit Program

The computer program for orbits is written in FORTRAN II and includes "ACCEPT" (data) statements used on an IBM 1620 input typewriter. (Example at the right.)

```
      PROGRAM ORBIT
C
C         HARVARD PROJECT PHYSICS ORBIT PROGRAM.
C         EMPIRICAL VERIFICATION OF KEPLERS LAWS
C         FROM NEWTONS LAW OF UNIVERSAL GRAVITATION.
C
      G=40.
    4 CALL MARKF(0.,0.)
    6 PRINT 7
    7 FORMAT(9HGIVE ME Y )
      X=0.
      ACCEPT 5,Y
      PRINT 8
    8 FORMAT(12HGIVE ME XVEL)
    5 FORMAT(F10.6)
      ACCEPT 5,XVEL
      YVEL=0.
      PRINT 9
    9 FORMAT(49HGIVE ME DELTA IN DAYS,  AND NUMBER BETWEEN PRINTS)
      ACCEPT 5,DELTA
      DELTA=DELTA/365.25
      ACCEPT 5,PRINT
      IPRINT = PRINT
      INDEX = 0
      NFALLS = 0
   13 CALL MARKF(X,Y)
      PRINT 10,X,Y
   15 IF(SENSE SWITCH 3) 20,16
   20 PRINT 21
   10 FORMAT(2F7.3)
      NFALLS = NFALLS + IPRINT
   21 FORMAT(23HTURN OFF SENSE SWITCH 3 )
   22 CONTINUE
      IF(SENSE SWITCH 3) 22,4
   16 RADIUS = SQRTF(X*X + Y*Y)
      ACCEL = -G/(RADIUS*RADIUS)
      XACCEL = (X/RADIUS)*ACCEL
      YACCEL = (Y/RADIUS)*ACCEL
C  FIRST TIME THROUGH WE WANT TO GO ONLY 1/2 DELTA
      IF(INDEX) 17,17,18
   17 XVEL = XVEL + 0.5 * XACCEL * DELTA
      YVEL = YVEL + 0.5 * YACCEL * DELTA
      GO TO 19
C  DELTA V = ACCELLERATION TIMES DELTA T
   18 XVEL = XVEL + XACCEL * DELTA
      YVEL = YVEL + YACCEL * DELTA
C  DELTA X = XVELOCITY TIMES DELTA T
   19 X = X + XVEL * DELTA
      Y = Y + YVEL * DELTA
      INDEX = INDEX + 1
      IF(INDEX - NFALLS) 15,15,13
      END
```

With slight modification it worked on a CDC 3100 and CDC 3200, as shown in the film loops 13 and 14, "Program Orbit I" and "Program Orbit II." With additional slight modifications (in statement 16 and the three succeeding statements) it can be used for other force laws. The method of computation is the scheme used in Project Physics *Reader 1* "Newton's Laws of Dynamics." A similar program is presented and explained in *FORTRAN for Physics* (Alfred M. Bork, Addison-Wesley, 1967).

Note that it is necessary to have a subroutine MARK. In our case we used it to plot the points on an X-Y plotter, but MARK could be replaced by a PRINT statement to print the X and Y coordinates.

FILM LOOP 15 CENTRAL FORCES—ITERATED BLOWS

In Chapter 8 and in Experiment 19 and Film Loop 13 on the stepwise approximation or orbits we find that Kepler's law of areas applied to objects acted on by a central force. The force in each case was attractive and was either constant or varied smoothly according to some pattern. But suppose the central force is repulsive; that is, directed *away* from the center? or sometimes attractive and sometimes repulsive? And what if the amount of force applied each time varies unsystematically? Under these circumstances would the law of areas still hold? You can use this film to find out.

The film was made by photographing the face of a cathode ray tube (CRT) which displayed the output of a computer. It is important to realize the role of the computer program in this film: it controlled the change in direction and change in speed of the "object" as a result of a "blow." This is how the computer program uses Newton's laws of motion to predict the result of applying a brief impulsive force, or blow. The program remained the same for all parts of the loop, just as Newton's laws remain the same during all experiments in a laboratory. However, at one place in the program, the operator had to specify how he wanted the force to vary.

Fig. 8-16

Random Blows

The photograph (Fig. 8-16) shows part of the motion of the body as blows are repeatedly applied at equal time intervals. No one decided in advance how great each blow was to be. The computer was programmed to select a number at random to represent the magnitude of the blow. The directions toward or away from the center were also selected at random, although a slight preference for attractive blows was built in so the pattern would be likely to stay on the face of the CRT. The dots appear at equal time intervals. The intensity and direction of each blow is represented by the length of line at the point of the blow.

Study the photograph. How many blows were attractive? How many were repulsive? Were any blows so small as to be negligible?

You can see if the law of areas applies to this random motion. Project the film on a piece of paper, mark the center and mark the points where the blows were applied. Now measure the areas of the triangles. Does the moving body sweep over equal areas in equal time intervals?

Force Proportional to Distance

If a weight on a string is pulled back and released with a sideways shove, it moves in an elliptical orbit with the force center (lowest point) at the center of the ellipse. A similar path is traced on the CRT in this segment of the film. Notice how the force varies at different distances from the center. A smooth orbit is approximated by the computer by having the blows come at shorter time intervals. In 2(a), 4 blows are used for a full orbit; in 2(b) there are 9 blows, and in 2(c), 20 blows which give a good approximation to the ellipse that is observed with this force. Geometrically, how does this orbit differ from planetary orbits? How is it different physically?

Inverse-square Force

A similar program is used with two planets simultaneously, but with a force on each varying *inversely* as the *square* of the distance from a force center. Unlike the real situation, the program assumes that the planets do not exert forces on one another. For the resulting ellipses, the force center is at one *focus* (Kep-

CHAPTER 8

ler's first law), not at the center of the ellipse as in the previous case.

In this film, the computer has done thousands of times faster what you could do if you had enormous patience and time. With the computer you can change conditions easily, and thus investigate many different cases and display the results. And, once told what to do, the computer makes fewer calculation errors than a person!

FILM LOOP 16 KEPLER'S LAWS

A computer program similar to that used in the film "Central forces—iterated blows" causes the computer to display the motion of two planets. Blows directed toward a center (the sun), act on each planet in equal time intervals. The force exerted by the planets on one another is ignored in the program; each is attracted only by the sun, by a force which varies inversely as the square of the distance from the sun.

Initial positions and initial velocities for the planets were selected. The positions of the planets are shown as dots on the face of the cathode ray tube at regular intervals. (Many more points were calculated between those displayed.)

You can check Kepler's three laws by projecting on paper and marking successive positions of the planets. The law of areas can be verified by drawing triangles and measuring areas. Find the areas swept out in at least three places: near perihelion, near aphelion, and at a point approximately midway between perihelion and aphelion.

Kepler's third law holds that in any given planetary system the squares of the periods of the planets are proportional to the cubes of their average distances from the object around which they are orbiting. In symbols,

$$T^2 \propto R_{av}^3$$

where T is the period and R_{av} is the average distance. Thus in any one system, the value of T^2/R_{av}^3 ought to be the same for all planets.

We can use this film to check Kepler's law of periods by measuring T and for each of the two orbits shown, and then computing T^2/R_{av}^3

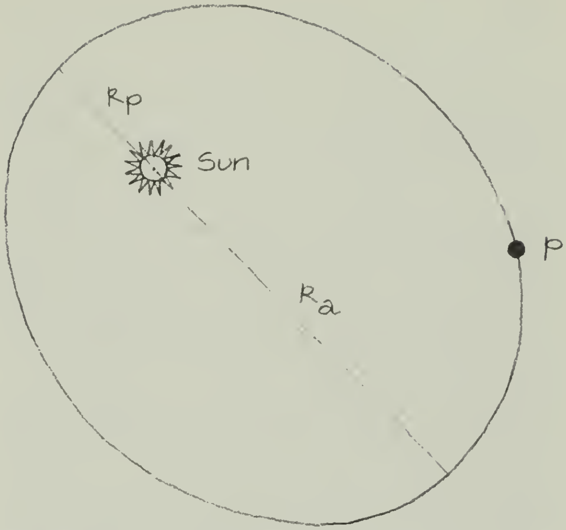


Fig. 8-17 The mean distance R_{av} of a planet P orbiting about the sun is $(R_p + R_a)/2$.

for each. To measure the periods of revolution, use a clock or watch with a sweep second hand. Another way is to count the number of plotted points in each orbit. To find R_{av} for each orbit, measure the perihelion and aphelion distances (R_p and R_a) and take their average (Fig. 8-17).

How close is the agreement between your two values of T^2/R_{av}^3? Which is the greater source of error, the measurement of T or of R_{av}?

To check Kepler's first law, see if the orbit is an ellipse with the sun at a focus. You can use string and thumbtacks to draw an ellipse. Locate the empty focus, symmetrical with respect to the sun's position. Place tacks in a

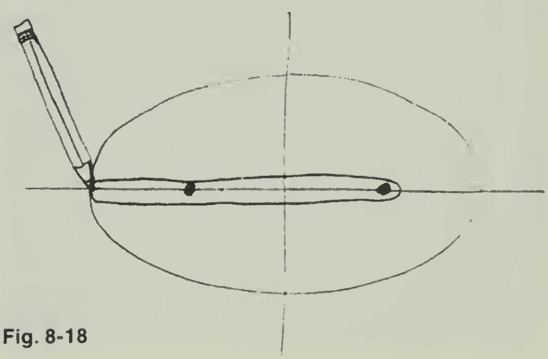


Fig. 8-18

CHAPTER 8

board at these two points. Make a loop of string as shown in Fig. 8-18.

Put your pencil in the string loop and draw the ellipse, keeping the string taut. Does the ellipse match the observed orbit of the planet? What other methods can be used to find if a curve is a good approximation to an ellipse?

You might ask whether checking Kepler's laws for these orbits is just busy-work, since the computer already "knew" Kepler's laws and used them in calculating the orbits. But the computer was *not* given instructions for Kepler's laws. What you are checking is whether Newton's laws lead to motions that fit Kepler's descriptive laws. The computer "knew" (through the program we gave it) only Newton's laws of motion and the inverse-square law of gravitation. This computation is exactly what Newton did, but without the aid of a computer to do the routine work.

FILM LOOP 17 UNUSUAL ORBITS

In this film a modification of the computer program described in "Central forces – iterated blows" is used. There are two sequences: the first shows the effect of a disturbing force on an orbit produced by a central inverse-square force; the second shows an orbit produced by an inverse-cube force.

The word "perturbation" refers to a small variation in the motion of a celestial body caused by the gravitational attraction of another body. For example, the planet Neptune was discovered because of the perturbation it caused in the orbit of Uranus. The main force on Uranus is the gravitational pull of the sun, and the force exerted on it by Neptune causes a perturbation which changes the orbit of Uranus very slightly. By working backward, astronomers were able to predict the position and mass of the unknown planet from its small effect on the orbit of Uranus. This spectacular "astronomy of the invisible" was rightly regarded as a triumph for the Newtonian law of universal gravitation.

Typically a planet's entire orbit rotates slowly, because of the small pulls of other planets and the retarding force of friction due to dust in space. This effect is called "advance of perihelion." (Fig. 8-19.) Mercury's perihelion advances about 500 seconds of arc, ($\frac{1}{7}°$) per century. Most of this was explained by perturbations due to the other planets. However, about 43 seconds per century remained unexplained. When Einstein reexamined the nature of space and time in developing the theory of relativity, he developed a new gravitational theory that modified Newton's theory in cru-

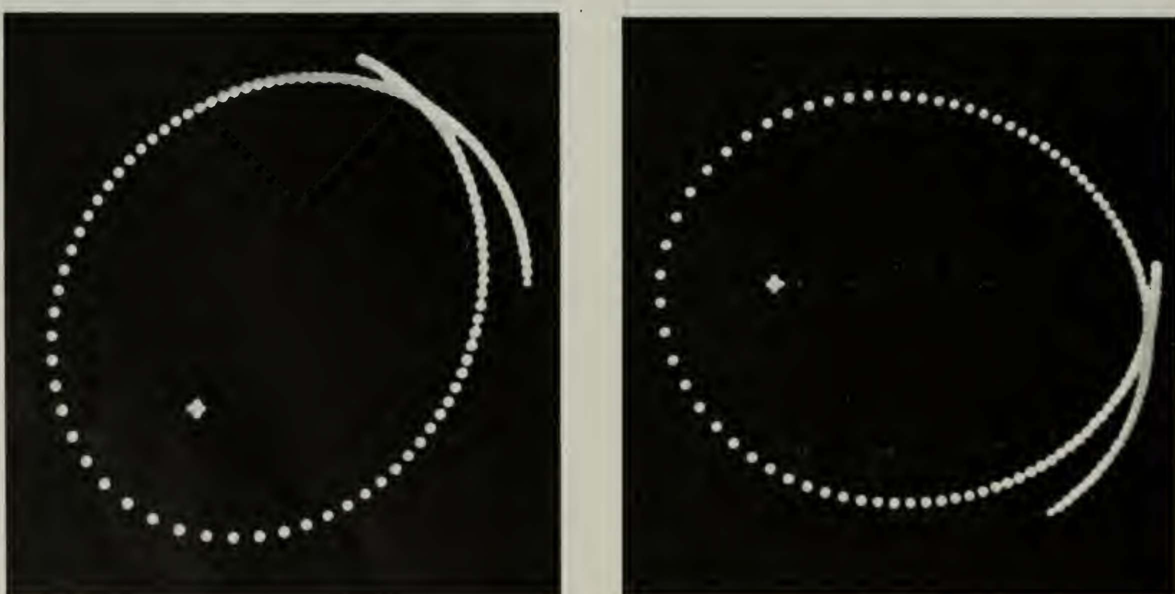


Fig. 8–19

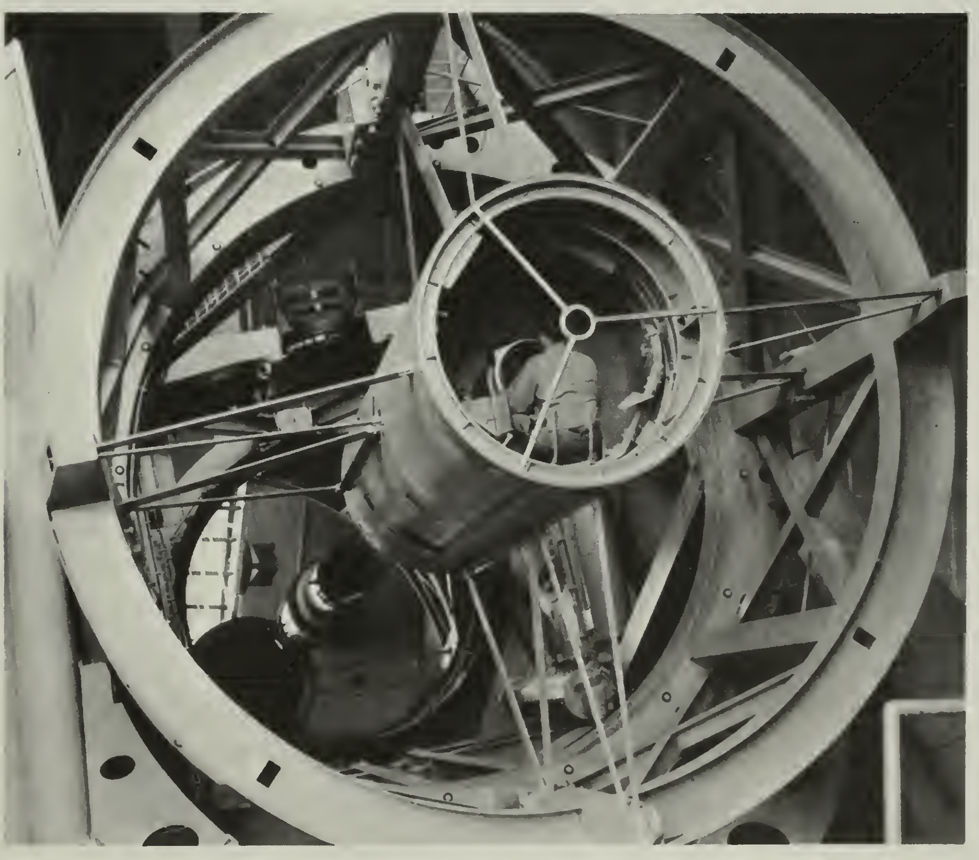
cial ways. Relativity theory is important for bodies moving at high speeds or near massive bodies. Mercury's orbit is closest to the sun and therefore most affected by Einstein's extension of the law of gravitation. Relativity was successful in explaining the extra 43 seconds per century of advance of Mercury's perehelion. But recently this "success" has again been questioned, with the suggestion that the extra 43 seconds may be explained instead by a slight bulge of the sun at its equator.

The first sequence shows the advance of perihelion due to a small force proportional to the distance R, added to the usual inverse-square force. The "dialogue" between operator and computer starts as follows:

PRECESSION PROGRAM WILL USE
  ACCEL = G/(R*R) + P*R
GIVE ME PERTURBATION P.
  P = 0.66666.
GIVE ME INITIAL POSITION IN AU
  X = 2.
  Y = 0.
GIVE ME INITIAL VELOCITY IN AU/YR
  XVEL = 0.
  YVEL = 3.

The symbol * means multiplication in the Fortran language used in the program. Thus $G/(R*R)$ is the inverse-square force, and $P*R$ is the perturbing force, proportional to R.

In the second part of the film, the force is an inverse-cube force. The orbit resulting from the inverse-cube attractive force, as from most force laws, is not closed. The planet spirals into the sun in a "catastrophic" orbit. As the planet approaches the sun, it speeds up, so points are separated by a large fraction of a revolution. Different initial positions and velocities would lead to quite different orbits.



Man in observation chamber of the 200-inch reflecting telescope on Mt. Palomar.

SATELLITES OF THE PLANETS

		DISCOVERY	AVERAGE RADIUS OF ORBIT	PERIOD OF REVOLUTION			DIAMETER
EARTH:	Moon		238,857 miles	27d	7h	43m	2160 miles
MARS:	Phobos	1877, Hall	5,800	0	7	39	10?
	Deimos	1877, Hall	14,600	1	6	18	5?
JUPITER:	V	1892, Barnard	113,000	0	11	53	150?
	1 (Io)	1610, Galileo	262,000	1	18	28	2000
	II (Europa)	1610, Galileo	417,000	3	13	14	1800
	III (Ganymede)	1610, Galileo	666,000	7	3	43	3100
	IV (Callisto)	1610, Galileo	1,170,000	16	16	32	2800
	VI	1904, Perrine	7,120,000	250	14		100?
	VII	1905, Perrine	7,290,000	259	14		35?
	X	1938, Nicholson	7,300,000	260	12		15?
	XII	1951, Nicholson	13,000,000	625			14?
	XI	1938, Nicholson	14,000,000	700			19?
	VIII	1908, Melotte	14,600,000	739			35?
	IX	1914, Nicholson	14,700,000	758			17?
SATURN:	Mimas	1789, Herschel	115,000	0	22	37	300?
	Enceladus	1789, Herschel	148,000	1	8	53	350
	Tethys	1684, Cassini	183,000	1	21	18	500
	Dione	1684, Cassini	234,000	2	17	41	500
	Rhea	1672, Cassini	327,000	4	12	25	1000
	Titan	1655, Huygens	759,000	15	22	41	2850
	Hyperion	1848, Bond	920,000	21	6	38	300?
	Phoebe	1898, Pickering	8,034,000	550			200?
	Iapetus	1671, Cassini	2,210,000	79	7	56	800
URANUS:	Miranda	1948, Kuiper	81,000	1	9	56	
	Ariel	1851, Lassell	119,000	2	12	29	600?
	Umbriel	1851, Lassell	166,000	4	3	28	400?
	Titania	1787, Herschel	272,000	8	16	56	1000?
	Oberon	1787, Herschel	364,000	13	11	7	900?
NEPTUNE:	Triton	1846, Lassell	220,000	5	21	3	2350
	Nereid	1949, Kuiper	3,440,000	359	10		200?

THE SOLAR SYSTEM

	RADIUS	MASS	AVERAGE RADIUS OF ORBIT	PERIOD OF REVOLUTION
Sun	6.95×10^{8} meters	1.98×10^{30} kilograms	—	—
Moon	1.74×10^{6}	7.34×10^{22}	3.8×10^{8} meters	2.36×10^{6} seconds
Mercury	2.57×10^{6}	3.28×10^{23}	5.79×10^{10}	7.60×10^{6}
Venus	6.31×10^{6}	4.83×10^{24}	1.08×10^{11}	1.94×10^{7}
Earth	6.38×10^{6}	5.98×10^{24}	1.49×10^{11}	3.16×10^{7}
Mars	3.43×10^{6}	6.37×10^{23}	2.28×10^{11}	5.94×10^{7}
Jupiter	7.18×10^{7}	1.90×10^{27}	7.78×10^{11}	3.74×10^{8}
Saturn	6.03×10^{7}	5.67×10^{26}	1.43×10^{12}	9.30×10^{8}
Uranus	2.67×10^{7}	8.80×10^{25}	2.87×10^{12}	2.66×10^{9}
Neptune	2.48×10^{7}	1.03×10^{26}	4.50×10^{12}	5.20×10^{9}
Pluto	?	?	5.9×10^{12}	7.28×10^{9}

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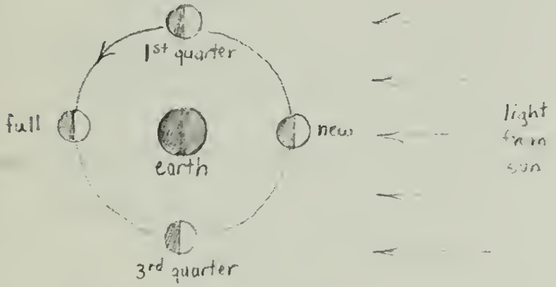
Answers to End-of-Section Questions

Chapter 5

Q1 The sun would set 4 minutes later each day.
Q2 Calendars were needed to schedule agricultural activities and religious rites.
Q3 The sun has a westward motion each day, an eastward motion with respect to the fixed stars and a north-south variation.
Q4



Q5 Eclipses do not occur each month, because the moon and the earth do not have the same planes of orbit.
Q6 Mercury and Venus are always found near the sun, either a little ahead of it or a little behind it.
Q7 When in opposition, a planet is opposite the sun; therefore the planet would rise at sunset and be on the north-south line at midnight.
Q8 After they have been farthest east of the sun and are visible in the evening sky.
Q9 When they are near opposition.
Q10 No, they are always close to the ecliptic.
Q11 How may the irregular motions of the planets be accounted for by combinations of constant speeds along circles?
Q12 Many of their written records have been destroyed by fire, weathering and decay.
Q13 Only perfect circles and uniform speeds were suitable for the perfect and changeless heavenly bodies.
Q14 A geocentric system is an earth-centered system. The yearly motion of the sun is accounted for by assuming that it is attached to a separate sphere which moves contrary to the motion of the stars.
Q15 The first solution, as proposed by Eudoxus, consisted of a system of transparent crystalline spheres which turned at various rates around various axes.
Q16 Aristarchus assumed that the earth rotated daily—which accounted for all the daily motions observed in the sky. He also assumed that the earth revolved around the sun—which accounted for the many annual changes observed in the sky.
Q17 When the earth moved between one of these planets and the sun (with the planet being observed in opposition), the earth would be moving faster than the planet. So the planet would appear to us to be moving westward.
Q18 The direction to the stars should show an annual shift—the annual parallax. (This involves a very small angle and so could not be observed with instruments available to the Greeks. It was first observed in 1836 A.D.)
Q19 Aristarchus was considered to be impious because he suggested that the earth, the abode of human life, might not be at the center of the universe.

His system was neglected for a number of reasons: (1) "Religious"—it displaced man from the center of the universe.
(2) Scientific—stellar parallax was not observed.
(3) Practical—it predicted celestial events no better than other, less offensive, theories.

Chapter 6

Q1 The lack of uniform velocity associated with equants was (1) not sufficiently absolute, (2) not sufficiently pleasing to the mind.
Q2 (a) P, C
(b) P, C
(c) P
(d) C
(e) P, C
(f) C
Q3 The relative size of the planetary orbits as compared with the distance between the earth and the sun. These were related to the calculated periods of revolution about the sun.
Q4 (b) and (d)
Q5 2° in both cases
Q6 No; precise computations required more small motions than in the system of Ptolemy.
Q7 Both systems were about equally successful in explaining observed phenomena.
Q8 The position of man and his abode, the earth, were important in interpreting the divine plan of the universe.
Q9 They are equally valid; for practical purposes we prefer the Copernican for its simplicity.
Q10 He challenged the earth-centered world outlook of his time and opened the way for later modifications and improvements by Kepler, Galileo, and Newton.
Q11 The appearance in 1572 of a "new star" of varying brightness.
Q12 It included expensive equipment and facilities and involved the coordinated work of a staff of people.
Q13 They showed that comets were distant astronomical objects, not local phenomena as had been believed.
Q14 He made them larger and sturdier and devised scales with which angle measurements could be read more precisely.

Q15 He analyzed the probable errors inherent in each piece of his equipment; also he made corrections for the effects of atmospheric refraction.
Q16 He kept the earth fixed as did Ptolemy and he had the planets going around the sun as did Copernicus.

Chapter 7

Q1 Finding out the correct motion of Mars through the heavens.
Q2 By means of circular motion, Kepler could not make the position of Mars agree with Tycho Brahe's observations. (There was a discrepancy of 8 minutes of arc in latitude.)
Q3 By means of triangulation, based on observations of the directions of Mars and the sun 687 days apart, he was able to plot the orbit of the earth.
Q4 A line drawn from the sun to a planet sweeps out equal areas during equal time intervals.
Q5 Where it is closest to the sun.
Q6 Mars has the largest eccentricity of the planets Kepler could study.
Q7 (a) E
(b) A
(c) A + E (+ date of passage of perihelion, for example)
Q8 The square of the period of any planet is proportional to the cube of its average distance to the sun.
Q9 Kepler based his laws upon observations, and expressed them in a mathematical form.
Q10 Popular language, concise mathematical expression.
Q11 Both the heliocentric and Tychonic theories.
Q12 The sunspots and the mountains on the moon refuted the Ptolemaic assertion that all heavenly bodies were perfect spheres.
Q13 Galileo's observations of the satellites of Jupiter showed that there could be motions around centers other than the earth. This contradicted basic assumptions in the physics of Aristotle and the astronomy of Ptolemy. Galileo was encouraged to continue and sharpen his attacks on those earlier theories.
Q14 No, they only supported a belief which he already held.
Q15 Some believed that distortions in the telescope (which were plentiful) could have caused the peculiar observations. Others believed that established physics, religion, and philosophy far outweighed a few odd observations.
Q16 b, c (d is not an unreasonable answer since it was by writing in Italian that he stirred up many people.)

Chapter 8

Q1 The forces exerted on the planets are always directed toward the single point where the sun is located.
Q2 The formula for centripetal acceleration
Q3 That the orbit was circular
Q4 No, he included the more general case of all conic sections (ellipses, parabolas and hyperbolas as well as circles).
Q5 That one law would be sufficient to account for both.
Q6 He thought it was magnetic and acted tangentially.
Q7 The physics of motion on the earth and in the heavens under one universal law of gravitation.
Q8 No, he thought it was sufficient to simply describe and apply it.
Q9 An all pervasive ether transmitted the force through larger distances.
Q10 He did not wish to use an hypothesis which could not be tested.
Q11 Phenomenological and thematic
Q12 (a) The forces are equal.
(b) The accelerations are inversely proportional to the masses.
Q13 (a) $2F$
(b) $3F$
(c) $6F$
Q14 (b) $F_{AB} = 4F_{CD}$
Q15 The values of the constant in Kepler's third law $T^2/R^3 = k$ as applies to satellites of each of the two planets to be compared.
Q16 The numerical value of G
Q17 F_{grav}, m_1, m_2, R
Q18 The period of the moon and the distance between the centers of the earth and the moon or the ratio T^2/R^3.
Q19 Similar information about Saturn and at least one of its satellites.
Q20 1/1000; that is, inversely proportional to the masses.
Q21 On the near side the water Is pulled away from the solid earth; on the far side the solid earth is pulled away from the water. Since $F \propto 1/R^2$ the larger R is, the smaller the corresponding F.
Q22 All of them
Q23 As the moon orbits its distance to the sun is continually changing, thus affecting the net force on the moon due to the sun and the earth. Also the earth is not a perfect sphere.
Q24 (a), (b), and (c)
Q25 Influence of sun and shape of earth
Q26 Comets travel on very elongated ellipses.
Q27 No

Brief Answers to Study Guide Questions

Chapter 5

5.1 Information
5.2 Discussion
5.3 (a) 674 seconds
(b) 0.0021%
5.4 Table
5.5 Discussion
5.6 Discussion
5.7 Discussion
5.8 102°, 78°, 78°, 102° starting with the upper right quadrant.
5.9 (a) 15°
(b) Geometric proof and calculation; about 8000 miles.
5.10 a, b, c, d, e, f
5.11 Discussion
5.12 Discussion

Chapter 6

6.1 Information
6.2 Diagram construction
6.3 Discussion
6.4 11 times; derivation
6.5 Discussion
6.6 Discussion
6.7 2.8×10^5 AU
6.8 Discussion
6.9 Discussion
6.10 Discussion
6.11 Discussion
6.12 Discussion

Chapter 7

7.1 Information
7.2 About 1/8 of a degree; about 1/100th of an inch; roughly 1/20th of a degree.
7.3 Discussion
7.4 4%
7.5 Discussion
7.6 a + c
7.7 Discussion
7.8 0.209
7.9 .594/1
7.10 Analysis
7.11 (a) 17.9 AU
(b) 35.3 AU
(c) 0.54 AU
(d) 66/1
7.12 $T = 249$ years
7.13 $K = 1.0$ for all three planets
7.14 Discussion
7.15 (a) sketch
(b) R_{av}: 3.4 mm, 5.2 mm, 8.2 mm, 4.6 mm
T: 44^h, 84^h, 168^h, 384^h
(c) K: 485, 495, 501, 470 Hr^2/mm^3
7.16 Discussion
7.17 Discussion
7.18 Discussion

Chapter 8

8.1 Information
8.2 Yes, to about 1% agreement
8.3 Discussion
8.4 Discussion
8.5 Derivation
8.6 $T^2 = \left(\frac{4\pi^2}{G}\right)\frac{R^3}{m}$
8.7 About 170 times as great
8.8 Discussion
8.9 (a) 1.05×10^3 days^2/AU^3
(b) discussion
(c) discussion
8.10 26,500 mi, or 42,600 km
8.11 5.98×10^{24} kg
8.12 6.04×10^{24} kg
8.13 (a) 5.52×10^3 kg/m^3
(b) discussion
8.14 7.30×10^{22} kg
8.15 Pluto has no known satellite
8.16 (a) 5.99×10^3 sec, or 1.66 hours
(b) 3.55 km/sec
(c) collisions
8.17 Table
8.18 17.7 AU, 0.60 AU, 34.8 AU
8.19 Derivations
8.20 Discussion
8.21 Discussion. No.
8.22 Discussion
8.23 It is useful today.
8.24 Discussion

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